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TORONTO

A CLASS BOOK

OF

PHYSICS

BY

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PREFACE

THERE is a diversity of opinion as to the value of text-books in the teaching of Science, and of practice as to their use. Prof. Armstrong has urged that "each child should write its own text-book and be taught to regard it as a holy possession"; and in a few schools this exalted doctrine is accepted in theory if not in fact. The principle is sound enough when applied by the private tutor (and the authors have no desire to belittle it) but it cannot be adapted satisfactorily to the work-a-day conditions of science teaching in schools where a large number of pupils of varying capacity and limited resource are receiving instruction in classes. In most cases, the time available for a science course will not permit the go-as-you-please pace postulated by some educational reformers as essential to good work, even if it be assumed that each pupil not only realises the necessity of working out his own intellectual salvation but is capable also of constructing his own road to it. How few pupils there are who possess the motive and purpose required for successful scientific study without assistance from a text-book is known only to the practical teacher.

Text-books may not be essential in the early work in science, but after pupils have acquired some familiarity with the scientific method it is desirable to fix ideas by more systematic study. Without practical experience there is no real scientific knowledge; but, on the other hand, unless laboratory work is accompanied by descriptive reading or lectures, it usually ends in a nebulous state of mind equally dispiriting to the teacher and pupil. The study of science is best encouraged by a right

combination of experiment, discussion, and reading, and the only limit to the amount of either is that of time. The exigencies of the school time-table do not permit the pursuit of an indefinite course of practical work or the leisurely consultation of authoritative treatises upon the subjects under consideration. For this reason books are demanded which present more or less concisely the essential principles of a branch of science and weave the scattered threads of thought into a fabric of definite pattern and reasonable dimensions.

It is not pretended that this volume is other than a text-book designed to facilitate the work of the teacher and concentrate the attention of the pupil: its purpose is not so much to inspire as to instruct. Between the prolix popular work, the treatise for reference, and the students' manual there are sharp lines of distinction; and though each has a place in scientific literature their respective merits must be judged by different standards. A text-book should be concise in description and precise in instructions for practical work: it should provide not only substance and guidance for study, but also exercises to test the increase of effective mental equipment and to cultivate the art of clear expression. In class teaching it is not sufficient to prescribe topics for reading or experiment: there must be some means of determining whether the work has been performed. A well kept record of laboratory experiments is doubtless an excellent index of progress made, but unless it is supplemented by exercises intended to test the ability to apply the results and conclusions arrived at to the solution of related problems, and the interpretation of wider experience, it may prove to be a vain thing. Such a record is a measure of the kinetic energy of a pupil's laboratory performances but not of the extent to which the energy has been transformed into potential power. It is good discipline and helpful teaching, therefore, by oral or written questions to make a periodic valuation of the capacity of students to comprehend the full meaning of the work done.

Considerations of time and space determine the scope of a

science course and of a text-book; and in neither case should sins of omission be judged so severely as those of commission. In most schools the chief work in science consists of the subjects of fundamental physical measurements and heat included in Parts I., II., and III. of the complete volume. Practical acquaintance with these aspects of Physics is a necessary preliminary or accompaniment to the successful study of physical or chemical science; and this must be the excuse for the apparently excessive amount of space devoted to them. Light, Sound, Magnetism and Electricity are regarded now as special subjects to be taken up selectively after a foundation has been laid in physical principles. Local circumstances and the requirements of examining bodies decide which of these subjects shall be studied, but in any case it is hoped that a satisfactory first course of systematic work will be found in the following pages.

The standard of each Part of the volume is roughly that which may be expected reasonably of pupils from about fourteen to sixteen years of age. Probably few pupils will work through the complete course, but an endeavour has been made to provide for the requirements of students who intend to present themselves as candidates in elementary examinations in any branch of physics.

By the kind consent of Mr. A. T. Simmons, parts of books of which he is joint author have been adapted for use in the present volume. Adequate thanks cannot be expressed for the self-abnegation thus manifested by the friend and long-time colleague of the authors. Most of the illustrations are new, but a few are based upon figures in other books published by Messrs. Macmillan & Co., Ltd., to whom the authors are glad to acknowledge their indebtedness.

In almost every case the exercises at the end of the chapters are from papers set at various examinations. Among the papers from which suitable questions have been selected are those of Oxford and Cambridge Locals, London University Matriculation and School examinations, Intermediate Scholarships of the

London County Council, and the Board of Education examinations. Typical papers set at School-leaving and Matriculation Examinations of Indian Universities are given at the end of the volume. It will be noticed that the questions are frequently of the nature of problems to be solved—either practically or otherwise—and cannot be answered by the mere repetition of the substance of the text preceding them. For permission to reproduce questions from examination papers thanks are expressed gladly to the Controller of H.M. Stationery Office, the Senate of the University of London, the Delegates of the Oxford Local Examinations, the Cambridge Local Examinations and Lectures Syndicate, the London County Council Education Committee, and other examining bodies whose papers have been used.

R. A. GREGORY.
H. E. HADLEY.

CONTENTS

PART I FUNDAMENTAL MEASUREMENTS

PART II. HYDROSTATICS AND MECHANICS

PART III. HEAT

PART IV. LIGHT

PART V. SOUND

PART VI. MAGNETISM

PART VII. STATIC ELECTRICITY

PART VIII VOLTAIC ELECTRICITY

PART I.

FUNDAMENTAL MEASUREMENTS.

CHAPTER I.

MEASUREMENT OF LENGTH, ANGLE, AND TIME

Units.—The science of physics is based largely upon exact measurement, and to render such measurements intelligible they must be stated in terms of convenient or conventional standards Thus, to express the magnitude of any measurable quantity by a number, it is necessary to decide upon a **unit**, such as a unit of length, surface, volume, weight, or time, as the case may be, with which the measurement obtained may be compared. What are known as fundamental quantities are length, mass and time, and from units based upon these other physical units are derived.

MEASUREMENT OF LENGTH.

The British and Metric systems of length.—In the British Empire the standard of length adopted is the length between two marks on a certain bronze bar deposited with the Board of Trade, the bar being at a certain fixed temperature when the measurement is made. This length is quite arbitrary and is called a **yard** (yd). The yard is subdivided into three equal parts, each of which is a **foot** (ft). The foot is divided in its turn into twelve equal parts, called **inches** (in.).

In France and many other countries, and for scientific work generally, what are known as metric measures are used. The standard length in this system is the **metre**, or the distance at

a particular temperature between the ends of a certain platinum rod deposited in the national archives at Sèvres. This standard is equal in length to 39·37079 inches. The metre is subdivided into ten equal parts, each of which is called a **decimetre**, tenth the part of the decimetre is called a **centimetre**, and the tenth part of the centimetre is known as a **millimetre** Thus we get

10 millimetres (mm) = 1 centimetre (cm)

10 centimetres, 100 millimetres = 1 decimetre (dm.).

10 decimetres, 100 centimetres, 1,000 millimetres = 1 **metre** (m).

The multiples of the metre are named **deka-**, **hekto-**, and **kilometres.** Their value is seen from the following table:

10 metres = 1 dekametre.
100 metres = 1 hektometre.
1,000 metres = 1 **kilometre**

(*N.B.*—Dekametre and Hektometre are spelt Decametre and Hectometre frequently.)

The kilometre is equal to about three-fifths of a mile.

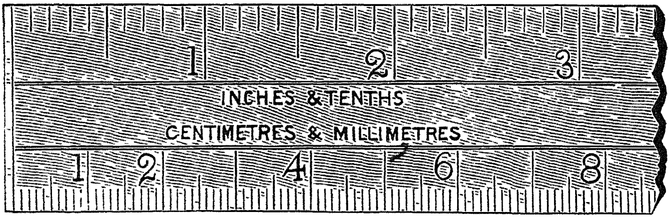

FIG. 1.—British and metric scales.

Exact relations between measures in the British and Metric systems of units are shown in the following tables, and also in Fig 1:

METRIC TO BRITISH		BRITISH TO METRIC.	
1 centimetre	= 0·394 inch.	1 inch	= 2·54 centimetres
1 metre	= 39·370 inches	1 yard	= 0·914 metre
	= 1·094 yards.	1 mile	= 1609·00 metres
1 kilometre	= 0·621 mile.		= 1 609 kilometres

METHODS OF MEASURING LENGTH.

Measurement of straight lines.—As an example we may consider the measurement of the dimensions of a rectangular wooden block by means of an ordinary wooden scale divided into inches and tenths or into centimetres and millimetres. The following precautions must be observed.

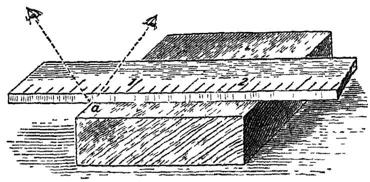

FIG 2.—Wrong method of using a scale.

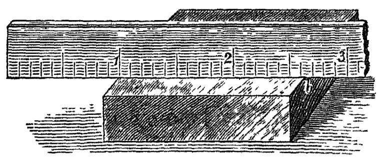

FIG 3.—Correct way to use a scale.

(*a*) *The scale must be held so that the divisions are actually in contact with the line to be measured* (Fig. 3). This precaution is necessary in order to avoid errors due to *parallax*, which can be understood by reference to Fig. 2, where the scale reading of the point *a* evidently depends upon the position of the observer's eye.

(*b*) *The zero end of the scale must not be used*, since the end is frequently more or less worn away; some definite division, other than the zero, of the scale should be used (Fig. 3).

(*c*) Since the position of the point under observation may not coincide with any one division of the scale, *it is necessary to estimate fractions of a scale division*, thus the scale reading of the point *b* (Fig. 3) is between 2.6 and 2·7, and by regarding each division as divided into 10 equal parts, it is evident that the reading is expressed more accurately by the number 2·63.

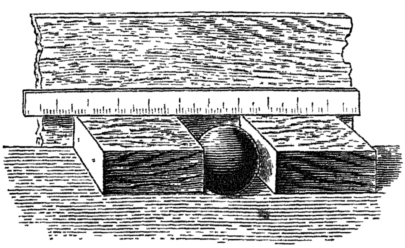

FIG 4.—Method of measuring the diameter of a sphere.

Fig. 4 indicates a method of using a rigid scale for the measurement of the diameter of a sphere.

Measurement of curved lines.—All curved lines can be regarded as made up of a very large number of straight lines. If

an infinite number of such small straight lines could be taken, there would be no difference between the sum of their lengths and the length of the curved line (Fig. 5) The procedure is as follows:

EXPT. 1 —**By dividers.** Open the dividers until the points are about 5 mm. apart; make a fine pencil-mark on the curve and place one

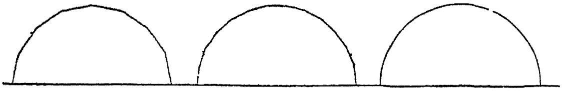

FIG 5 —Curved lines produced by many short lines.

point of the dividers on the pencil-mark, place the second leg on the curve, raise the first leg and rotate the dividers on the second leg until it is on the curve and beyond the second leg; repeat this process, while counting the number of lengths measured by the dividers, until the end of the curve is reached. Any portion of the curve, less than 5 mm., which remains, must be measured separately by readjusting the dividers. The length of the curve is given by the product of the distance between the divider points and the number of lengths measured by the divider.

EXPT. 2.—**By cotton thread.** Cut one end of a piece of cotton thread cleanly with scissors, and place the end in contact with a pencil-mark on the curve (Fig. 6). Make the thread coincide as nearly as you can with a small part of the curve, and place the nail of the first finger of your right hand upon it Now release your left-hand finger and carefully place it at the point where your right-hand finger is held, then, using your right hand, go on to make some more of the thread exactly coincide with another small length of curve. Repeat this until you have completed the whole curve. Measure the length of thread with a mm. scale.

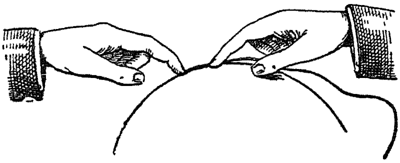

FIG. 6 —Measurement of a curved line.

EXPT. 3 —**By a strip of paper.** Wrap a strip of paper closely round a wooden cylinder (Fig. 7), and make a small hole with a pin at a place where the paper overlaps. Unroll the paper and measure the distance between the two holes. This gives the distance round the cylinder, that is, its circumference.

EXPT. 4 —**Circumference and diameter of a circle.** Cut out from a sheet of thin cardboard two discs of 4 cm. and 6 cm radius. Make

a pencil mark near to the edge of each disc; place one of the discs in a vertical position with its pencil-mark touching an observed division of a millimetre scale; roll the disc along the scale until the mark again touches the scale The difference between the two scale readings gives the length of the **circumference** of the disc Determine the circumference of the other disc in the same manner. Tabulate your results, thus:

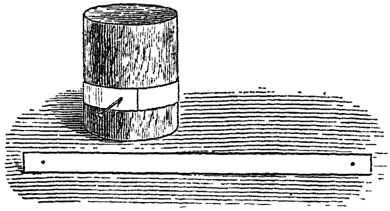

FIG. 7.—Measurement of the circumference of a cylinder

Diameter	Circumference.	$\frac{\text{Circumference}}{\text{Diameter}}$
1.		
2		

The ratio, in the last column, should be a *constant* quantity: it is usually denoted by the Greek letter π. Hence

$$\textbf{length of circumference} = \pi \times \textbf{diameter} = 2\pi \times \textbf{radius}$$

The vernier.—This device enables lengths to be measured accurately to a given fraction of the shortest division on the scale used The method was devised by Paul Vernier in 1630: it consists in the addition of a second scale, the divisions of which bear a simple relationship to those of the standard scale with which it is used. The simplest vernier scale is one which enables lengths to be read to $\frac{1}{10}$ of a scale division; and, in this case, the vernier scale is constructed either by dividing 9 divisions of the standard scale into 10 equal parts, or by dividing 11 divisions of the standard scale into 10 equal parts. The former, which is more usually adopted, is termed a *No. 1 vernier*, and the latter a *No. 2 vernier.*

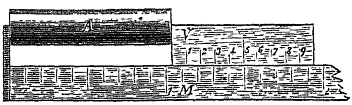

FIG 8 —Use of a No 1 vernier.

Fig. 8 represents the method of using a No. 1 vernier for measuring the length of an object A: M is an inch scale divided

into tenths, and V is the vernier scale. The length of A is evidently between 1 inch and 1·1 inch, and the vernier scale enables the amount by which the length exceeds 1 inch to be measured accurately. It will be noticed that the divisions of the two scales *coincide at one point only*, which is, approximately, the 4th of the vernier scale. Since 1 division of scale V is equal to $\frac{9}{10}$ of one division of scale M, *i.e.* to 0·09 inch, the 3rd division of V is 0·01 inch in advance of the division 1·3 on scale M, and

the 2nd division of V is 0·02 inch in advance of the division 1·2 on scale M,

the 1st division of V is 0·03 inch in advance of the division 1·1 on scale M,

the zero division of V is 0·04 inch in advance of the division 1·0 on scale M.

The last quantity, 0·04 inch, is the fraction of a division which had to be measured. Hence, the length of A is 1·04 inches.

An alternative method of reasoning is as follows.

The length of A + the length of 4 vernier divisions = 1·4 inches.

Hence, the length of A = 1·4 − (4 × 0·09)
= 1·04 inches

It is evident that the following rule may be adopted in using such a vernier scale: *Note where divisions on the two scales coincide, the number attached to the division of the vernier scale which coincides gives the numeral in the second place of decimals.*

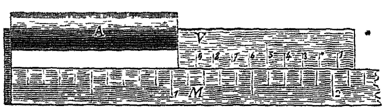

FIG. 9.—Use of a No. 2 vernier

It may be desired to use a vernier reading to a smaller fraction than 1/10 of a scale division; in this case, 19 scale divisions may be divided into 20 equal parts, giving a vernier-scale reading to 1/20 of a scale division. Generally speaking, in order to read to $1/n$ of a scale division, $(n-1)$ divisions must be divided into n parts.

Fig. 9 represents a No. 2 vernier, in which each division of the vernier scale is equal to 1·1 divisions of the standard scale M. It will be noticed that the divisions on the scale V are numbered from right to left. If the 4th division on scale V coincides with a division on scale M, then

the length of A + the length of 6 vernier divisions = 1·7 inches.

Hence, the length of A = 1·7 − (6 × 0·11)
= 1·04 inches.

The same rule, therefore, as given above when using a No 1 vernier, may be adopted in the case of a No. 2 vernier, providing that the scale divisions are numbered in the reverse direction.

The slide calipers.—Fig. 10 represents a simple form of slide calipers. The calipers consist of a thin steel rod with a jaw A

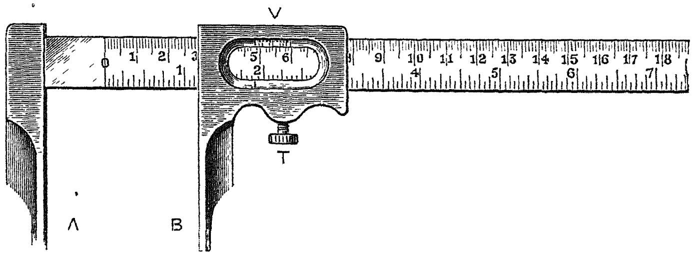

FIG 10.—Slide calipers

fixed at one end; B is a movable jaw with a vernier scale V which traverses a scale etched on the steel rod. The movable jaw can be fixed by means of the screw T. When the faces of the jaws are in contact, the zero division of the vernier scale should coincide with the zero division of the fixed scale The dimensions of an object are measured by noting how far the zero division of the vernier scale has moved along the fixed scale when the jaws have been separated until the object is *just* touched by the faces of the two jaws Before taking a measurement, it is necessary to note whether the fixed scale is divided into millimetres or into parts of an inch, and to what fraction of a scale division the vernier is intended to read. The instrument shown in Fig. 10 has both metric and British scales.

The screw-gauge.—The screw-gauge (Fig 11) furnishes a very accurate means for measuring the dimensions of small objects. It consists of a fixed frame F, attached to which is a hollow cylinder C. A screw thread is cut on the inside surface of C.

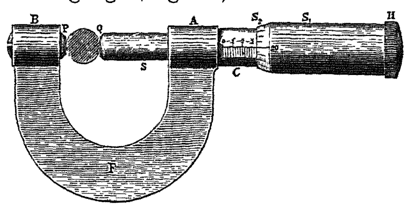

FIG 11 —Screw-gauge

The shaft S is the continuation of a screw which travels along the

threads cut within C, and a sleeve S_1 is attached to the head H of the screw. The edge S_2 of the sleeve is divided into a definite number, usually 50 or 100, of equal parts The end of the shaft S is a truly planed surface Q, and a similar surface P is obtained on the end of a fixed screw *f* carried by the other limb B of the fixed frame. The screw *f* is adjusted, once for all, so that, when the edge of S_2 coincides with the zero division of the scale on C and when the zero division of the scale S_2 coincides with the base line of the scale on C, the two plane faces P and Q are in contact.

Before taking a measurement, it is necessary to observe whether the scale on C is divided into millimetres or into tenths of an inch. The *pitch* of the screw S—*i.e.* the distance through which Q advances or recedes by one complete rotation of H—must then be determined. This is obtained by observing whether one division, or only *half* a division, of the scale on C is uncovered when H is rotated backwards by one complete revolution. Finally, the scale-value of one division of the scale S_2 is required.

As a general rule, the pitch of the screw S is 0·5 mm., and S_2 is divided into 50 equal parts; hence 1 division of

$$S_2 = \tfrac{1}{50} \times 0{\cdot}5 = 0{\cdot}01 \text{ mm.}$$

The object to be measured is placed between the faces P and Q, and the milled head H is rotated until the object is *lightly* gripped between the faces. The readings of the scales on C and on the sleeve at S_2 enable the size of the object to be measured

The spherometer.—The principle of this instrument closely resembles that of the micrometer screw-gauge. The instrument consists of a tripod, the legs of which are of equal length and are adjusted relatively to each other so that the three points occupy the corners of an equilateral triangle. A fine screw, which works through the centre of the tripod, terminates above in a milled head and a large circular disc, the edge of which is divided into 100 equal parts. A vertical scale, usually divided into millimetres, is fixed to one arm of the tripod, and with its divisions close to the edge of the disc.

Before using the instrument, it is necessary to determine the *pitch* of the screw: this may be equal to 1 mm. or to 0·5 mm. This is done by reading the position of the disc's edge on the vertical scale, and then rotating the disc through an observed

number of complete turns; the difference in the scale-reading divided by the number of turns gives the pitch of the screw. If the pitch of the screw is 0·5 mm., and if the disc is divided into 100 equal parts, then 1 division of the disc is equal to 0·005 mm.

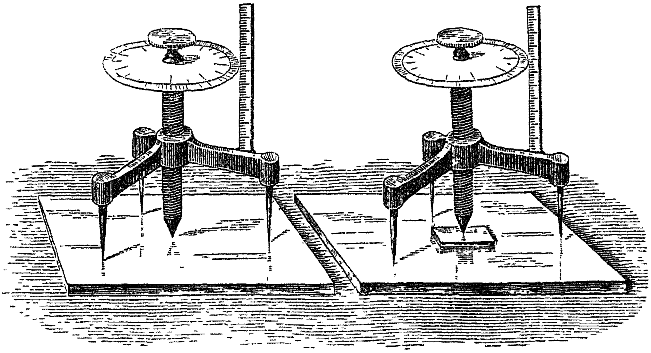

FIG 12 —Use of a spherometer.

In measuring the thickness of an object, the following procedure is adopted · Place the instrument on a truly horizontal surface, *e g* a sheet of plate glass, and rotate the screw downwards until its point *just* touches the surface (Fig. 12); this is determined most accurately by placing the thumb and first finger against opposite sides of one leg of the tripod, and endeavouring to make the instrument rotate round the centre leg, if the latter projects downwards too far the instrument readily rotates, but if contact is not complete there is an unmistakable sense of resistance to rotation. Having made this adjustment, take the reading of the scale and disc. Now raise the screw considerably, place the object underneath the screw point, and rotate the screw downwards until contact is just made again. The difference between the two sets of readings gives the thickness of the object.

The wedge.—The wedge gauge is useful in order to determine the internal diameter of a tube with a circular bore. A

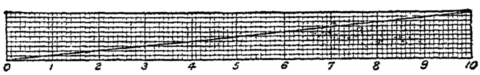

FIG 13 —Construction of a measuring wedge

simple form may be made by cutting, from squared paper or thin sheet metal, a right-angled triangle with a base 10 cm long

and a perpendicular 1 cm. long, as indicated in Fig 13. If the acute angle is pressed into the tube until it occupies a diameter of the bore, the diameter is given by $\frac{1}{10}$ of the length of that part of the base which has been introduced within the tube.

Measurement of Angles.

Unit of angular measurement.—The general plan adopted in measuring angles is to divide a circle into 360 equal parts, and to call each part a **degree** (1°). Thus, a movable hand pivoted at the centre of a circle has traced out an angle of one degree when it has gone 'round $\frac{1}{360}$th part of a complete revolution. When it has performed one quarter of its journey round, it has made an angle of ninety degrees, or a **right angle**, as it is called.

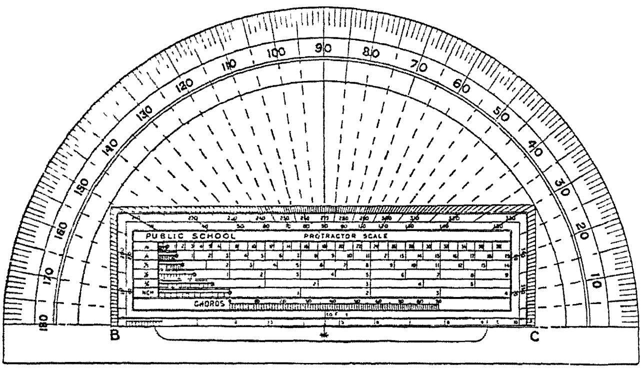

Fig. 14.—Semi-circular and rectangular forms of protractor, used for the measurement of angles.

The minute hand of a watch or clock moves through 360 degrees in an hour, or ninety degrees in every quarter of an hour, and this is true whatever the size of the timepiece. This illustrates the important fact that the size of an angle is quite independent of the length of the lines between which it is contained. All circles contain 360 degrees. All right angles contain ninety degrees, so there are four right angles to every circle. In accurate measurement, parts of a degree are required, and the sub-divisions used are that one degree (1°) equals sixty minutes, and one minute (1′) equal sixty seconds (60″).

The magnitude of an angle can be found by means of a protractor, two forms of which are shown in Fig. 14. The simplest form is a semi-circle divided into degrees, but a more common form is oblong in shape. The marks upon the edge of a protractor of this kind are obtained from the corresponding divisions on a semi-circle in the manner represented in the illustration.

Measurement of Time.

The earth's rotation—The apparent daily motion of the sun and stars across the sky is a direct consequence of the earth's rotation on its axis. The sun appears regularly to go through certain periodic changes of position. It rises, travels higher and higher into the sky, reaches its highest position, sinks lower and lower, and finally sets. When the sun is at its highest altitude on any day it is due south, and is said **to south** or be **southing.** The interval of time between the sun's highest position on any one day and its corresponding position on the next succeeding day is an **apparent solar day**

Mean solar day.—The length of days measured by the sun varies throughout the year, hence no single one of these days will do for a convenient standard of time. But if the lengths of all the days in the year be added together, or the length of a year measured by the sun be divided by the number of days in the year, we obtain an interval of time which is always the same. Such a day, which is of course an imaginary one, is called a **mean solar day** Sometimes the mean solar day is longer than the solar day, sometimes it is shorter, and occasionally both days are exactly the same length. Solar time is known as **apparent time,** and clock time as **mean time.**

Sidereal day.—As in the case of the sun, so it is with most of the stars, they rise, south, and set. But whereas with the sun the interval between two successive southings varies throughout the year, it is found that the time which elapses between two succeeding southings of a star at any season of the year is always the same. This interval constitutes **a star** or **sidereal day.**

Period of rotation of the earth.—As the apparent motions of stars across the sky are produced by the rotation of the earth,

it is evident that the exact time of rotation can be determined by finding the interval which elapses between two successive returns of any particular star to the same point of the sky. A star may, indeed, be regarded as a fixed reference mark under which the earth turns; so that by observation of it we are able to determine the time taken by the earth to spin round once. The interval between two successive transits of the same star, that is, a sidereal day, is the time of such rotation. No matter which star is selected for observation the interval is the same, thus showing that the earth is a rigid body, and that all parts of its surface have the same angular velocity.

Units of time.—The sidereal day, like the mean solar day, is subdivided into hours, minutes, and seconds, but as the latter is four minutes longer than the former, the units are not of the same value. We may take either the **mean solar second** as the unit of time or the **sidereal second**. In the former case, the unit is founded on the average length of the solar day, and in the latter upon the length of the invariable star day, or the time of rotation of the earth upon its axis. But in either case the second, that is, the unit of time, is the 86,400th part of the day used.

In physical measurements the unit of time adopted is the mean solar second, that is, it is the 86,400th part of the average time required by the earth to make one complete rotation on its axis relatively to the sun considered as a fixed point of reference.

Instruments for measuring time.—We need only concern ourselves with the modern contrivances for measuring time, viz., clocks and watches. It will be sufficient to regard these as instruments for measuring intervals of time in terms of the mean solar day to which attention has been directed. In a clock the rate is regulated usually by means of the pendulum, the properties of which can be best understood by an experiment.

The simple pendulum.—A **simple pendulum** may be defined as a **heavy particle suspended by a weightless thread**. An approximation to this ideal is obtained by suspending a small metal sphere by a very thin thread. In the arrangement represented in Fig. 15, stout cotton is *threaded* along the axis of a cork, which serves

as the carrier for the pendulum. The *bob* consists of a truly turned solid brass sphere through which a very narrow hole is bored along a diameter; this hole is bored out to larger size for a short distance from one end. In fitting up the pendulum the cotton is threaded through the bob, held with the wide end of the bore downwards; the cotton is knotted sufficiently for the knot to pass *just* within the wide end of the bore. The use of a cork, as suggested, allows the more accurate determination of the point of support, and the length of the pendulum is varied readily by pulling more or less of the cotton through the cork. The diagram indicates how the length of the pendulum may be measured by means of a metre scale and a wooden cube. The true length of the pendulum is approximately the distance from the point of support to the *centre* of the bob, and this length is best obtained by measuring the distance to the bottom of the bob and subtracting from this the radius of the bob. In the following experiment a cheap stop-watch is desirable.

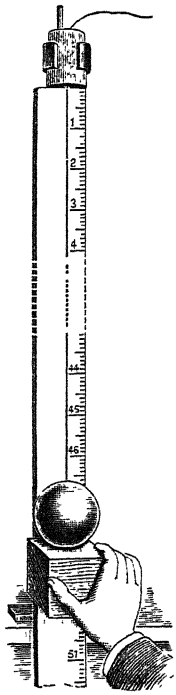

FIG 15.—A simple pendulum

EXPT. 5.—**Length and rate of swing of a pendulum.** Support immediately behind the thread of the pendulum a piece of cardboard on which a vertical pencil line is drawn. Sit down in front of the pendulum, and, *using one eye only*, adjust the position of the cardboard so that the thread exactly covers the pencil line. Keep the eye in this same position during the following observation. Set the pendulum swinging through a small angle not exceeding 15°. A single swing from side to side is termed usually a **vibration**, and a swing-swang, or complete movement to and fro, is an **oscillation**. The extent of a swing is termed the **amplitude** of vibration. Note the time indicated by the stop-watch, and start the watch just as the thread is passing in front of the pencil line. Count the number of subsequent passages of the thread until at least 50 vibrations have been completed, and stop the watch at an instant when the pendulum is passing the pencil line. Read the watch, and calculate the time interval between two consecutive passages of the pendulum in front of the pencil line. Repeat the experiment; and if this result differs from the first determination by more than 0.01 second, take a third observation. Measure the length of the pendulum. Alter the length of the pendulum.

and determine the time of vibration again in the same manner. Repeat this for different lengths, varying from 20 cm. to 120 cm. Tabulate the results thus:

Length	Time of vibration	$\sqrt{\text{Length}}$	$\frac{\sqrt{\text{Length}}}{\text{Time of vibration}}$

Plot on squared paper the *length* and the *time of vibration*, taking the latter as ordinates. Similarly plot the $\sqrt{\textit{length}}$ and the *time of vibration*, or *length* and *time*^{2}. From the curves obtained deduce the relationship between the time of vibration and the length of the pendulum.

The following readings were obtained with a simple pendulum constructed in the manner described above:

Length	Time of vibration.	Length	Time of vibration
20 cm	0·45 sec.	88 cm.	0 94 sec
30 „	0 55 „	95 „	0·98 „
42 „	0 65 „	102 „	1 01 „
55 „	0 74 „	115 „	1 07 „
70 „	0 835 „	130 „	1 14 „

Fig. 16 represents how these readings may be plotted on squared paper. Two **axes**, OX and OY, are drawn at right angles to each other, with the point O near to the left-hand bottom corner of the paper. The line OX is called the axis of **abscissae**, and OY the axis of **ordinates**. For the purpose of the present experiment, the horizontal scale along OX is taken to represent the *length* of the pendulum, and the scale along OY is taken to represent the time of vibration.

In the diagram, each division of OX represents 2 cm., and each division of OY represents 0·02 sec. In general, the value attached to each scale division should be chosen so that the lengths of the two scales utilised for plotting the observations are nearly equal, and so that the lengths are as great as the paper will allow.

Bearing in mind the simplicity of the apparatus, the liable error of observation, and the possibility of irregularity in the squared paper, it cannot be expected that all the plotted readings

may coincide with the curve (or straight line) which represents a theoretically accurate result Having plotted the points which indicate the observations taken, a line should be drawn which, as nearly as possible, passes through each point.

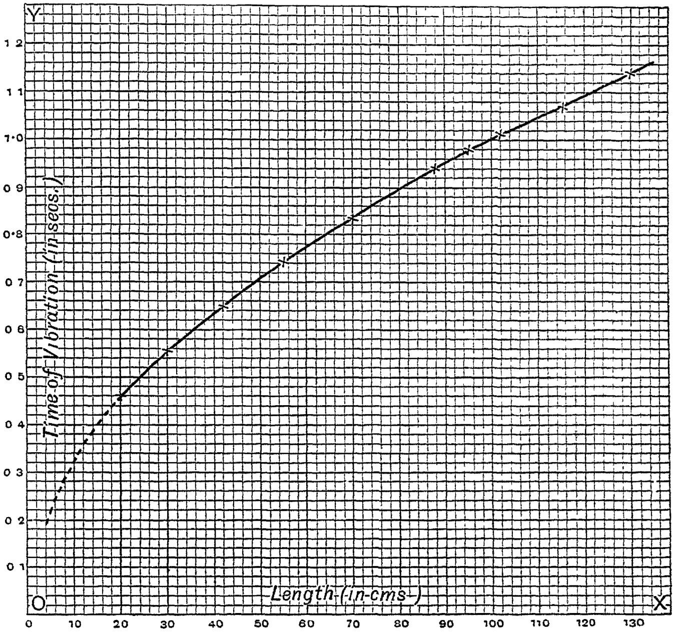

FIG 16 —Graphical representation of the relation between the length and time of vibration of a simple pendulum

EXPT. 6.—**Amplitude of swing.** Draw the pendulum ball aside, say ten inches from the vertical, and find the times required to complete twenty swings. Then draw it only three inches aside and again find the time of twenty vibrations. Hence determine whether the time is independent of the amplitude

EXPT 7.—**Mass of pendulum bob.** Determine the time required for a certain number of swings when balls of the same size and shape, but different masses, are suspended from the same length of thread. It will be found that within certain limits the mass of the bob does not affect the rate of swing.

EXERCISES ON CHAPTER I.

1. Convert 1 kilometre to yards, feet, and inches

2. Mont Blanc is 15,780 ft high. Express this in metres (to 1 place of decimals).

3. How many pieces of string, each 2·8 metres long, can be cut from a length of 6780 cm.? What length would be left over?

4. If the height of a column of mercury is 760 mm, what is it in inches?

5. What fraction is (*a*) 1 mm of 1 inch, (*b*) 1 decimetre of 1 foot, (*c*) 1 centimetre of 1 inch?

6. Measure the length of this page in inches and centimetres, and use the results to find the number of centimetres equal to one inch

7. What is meant by a unit of length, and why is it necessary to have such a unit?

8. Measure the length of three straight lines in inches and in centimetres, and calculate the mean value of one inch in centimetres

9. Draw a circle and measure the lengths of the circumference and the diameter. How many times does the circumference contain the diameter?

10. Measure the diameter and circumference of a cylinder, and calculate the ratio of the diameter to the circumference

11. Construct a vernier to read to one-tenth of a scale division.

12. Construct a graduated wedge to measure the diameter of a small tube

13. Draw a triangle and measure the value of each of its angles in degrees. Find the sum of the three angles.

14. Draw two lines crossing one another at any angle. Measure the four angles thus obtained and find their sum

15. What is the difference between an apparent solar day and a mean solar day?

16. What is the unit of time and how is it related to the period of the earth's rotation?

17. Describe a simple pendulum, and state the relationship between its length and the rate at which it swings

18. What is sidereal and what is mean time?

19. What do you understand by 'amplitude' and time of swing or oscillation as applied to a pendulum? What relation exists between time of oscillation and (*a*) the amplitude, (*b*) the length of the pendulum?

If the time of oscillation of a pendulum 100 cm long is t sec., state the time of oscillation when (1) the amplitude is doubled, (2) the weight of the bob is doubled. What must be the length of pendulum whose time of oscillation is $t/2$ sec.?

CHAPTER II.

MEASUREMENT OF AREA

Area.—In order to express an **area** (or, *extent of surface*) it is not sufficient to measure one length only, two lengths must be considered, viz length and width. We may select any **unit of surface,** *e g.* a sheet of foolscap paper; and the area of a surface, such as a table-top, might be measured by determining how many similar sheets were necessary in order to cover completely the top of the table the number of such units required would be given by multiplying the number of sheets in each row by the number of rows. In practice, instead of using such an arbitrary unit we use units derived from the standard units of length, viz the **square foot** or the **square centimetre,** according to whether the British or metric system is used

EXPT. 8.—**British and Metric Measures of Area.** Draw a square decimetre and divide it into square centimetres. Prove, by counting, that the area of the square is equal to its length multiplied by its height Now draw two or three oblongs and determine their areas by means of this rule Determine the areas of the square and oblongs both in square inches and square centimetres, and use your results to find the number of square centimetres in one square inch, thus

Area of a given rectangle in square inches	Area of same rectangle in square centimetres	$\frac{\text{Square centimetres}}{\text{Square inches}}$

Verify your results by calculation from the relation 1 inch = 2·54 cm.

Errors of observation.—The student should bear in mind that the unavoidable errors in measuring a length are augmented

considerably when two such observed lengths are multiplied together, and that the *product* of the lengths may be greatly in error. Thus, in using a millimetre scale for measuring a straight line 1 inch long, it may be judged to be 2·54 cm long, but the last digit is *estimated* only, and a second reading of 2 55 cm. might appear to be equally trustworthy The *liable* error of observation would be 1 in 250, or 0·4 %. The area of a square inch, calculated from such readings, would be either 6·4516 sq cm. or 6·5025 sq. cm., and the liable error would be about 5 in 650, or 0·75 % Hence the liable error is increased very much, and it is evident that *any figures beyond the third significant digit are totally untrustworthy, and therefore they should not be written when stating the result of an experiment.*

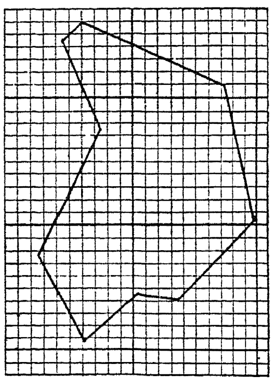

FIG. 17—Determination of the area of an irregular figure.

Measurement of irregular areas.—The following exercise will explain the method of using squared paper·

EXPT. 9—**Areas by squared paper.** Trace out an irregular figure, as shown in Fig. 17, on squared paper. Observe carefully the area of each small square: paper divided into $\frac{1}{100}$ sq. inch is used most frequently. Count the number of complete small squares enclosed within the figure; if more than half of a square is inside the boundary count it as one square, but neglect it if less than half is inside the figure

If the student is familiar with the use of a balance, the result may be verified by cutting out in thin cardboard (i) the same area, and (ii) a 2-inch square from the same material. These two areas are weighed, and the areas will be found proportional to their weights (see p. 37).

MENSURATION OF GEOMETRICAL FIGURES.

EXPT. 10.—**The parallelogram** Cut out two cardboard parallelograms ABCD, EFGH (Fig. 18) and draw a line from D perpendicular to BC, and from G perpendicular to EF. Cut off the two triangles DCL, GMF, and place them so as to convert each parallelogram into a rectangle.

Evidently the area of each complete figure is the same whether the triangle is in one position or the other. In other words, a parallelogram has the same area as a rectangle on the same base and having the same **altitude**, or perpendicular height.

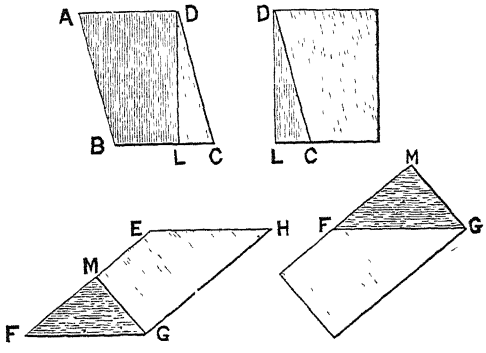

FIG 18 —Every parallelogram can be considered as a rectangle

The area of a rectangle (p. 17) is equal to the base multiplied by the height; hence,

area of a parallelogram = base × perpendicular height.

This statement may be tested by the following exercise:

EXPT. 11.—Draw any parallelogram, such as ABCD (Fig 19), on squared paper Count the number of squares, and state its area in sq. cm

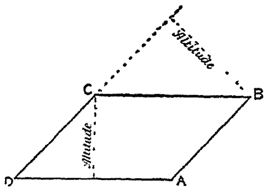

FIG 19.—Measurement of the area of a parallelogram.

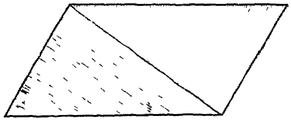

FIG 20 —A parallelogram can be cut into two equal triangles

Verify this result by calculating the area from measurements of the base and the perpendicular height, taking (i) DA as base, (ii) CD as base.

EXPT. 12.—**The triangle.** Draw a parallelogram on paper or thin card, and then cut it in two from corner to corner. You have now

two triangles, and by laying one on the other it will be found that they fit or are equal in area (Fig. 20). Repeat the exercise with a parallelogram of different form.

It has been seen that

$$\text{Area of parallelogram} = \text{base} \times \text{altitude}.$$

But a triangle is half a parallelogram

Therefore $$\textbf{area of triangle} = \frac{\textbf{base} \times \textbf{altitude}}{2}.$$

EXPT. 13.—Draw any triangle ABC on squared paper, and determine its area by counting squares.

Verify this result by calculating the area from measurements of the base and the perpendicular height, taking (i) BC as base, (ii) AC as base, (iii) AB as base

EXPT. 14—**Any four-sided figure** Draw any irregular four-sided figure on squared paper, and determine its area by counting squares.

Divide the figure into two triangles by joining either pair of opposite corners, and calculate from its dimensions the area of each triangle The sum of these areas should equal the total area as determined by squared paper.

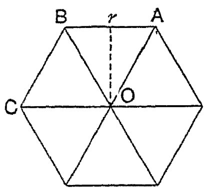

FIG 21 —Area of a hexagon

EXPT 15 —**The hexagon.** Divide up a regular hexagon into six equal triangles (Fig 21). Find the area of one of these triangles, viz OAB, it is equal to $Or \times \frac{1}{2}AB$; similarly, the area of OBC is equal to $Or \times \frac{1}{2}BC$. Hence, the area of the hexagon is equal to

$$Or \times \frac{\textbf{sum of the bases}}{2};$$

or, $$\textbf{area of hexagon} = Or \times \frac{\textbf{length of perimeter}}{2}.$$

This rule applies to any regular many-sided figure. When the number of sides is infinitely great, the figure becomes a *circle*, in which Or is the *radius*, and the perimeter is the *circumference* (Expt. 4).

Hence, $$\textbf{area of a circle} = \textbf{radius} \times \frac{\textbf{circumference}}{2}$$

This may be demonstrated in the following manner:

EXPT. 16.—**The circle** Cut out a circular disc of cardboard about four inches in diameter Divide it into small triangles as in Fig. 22 The area of the circle could be found by determining the areas of all these triangles.

Cut out the triangles and arrange them as in Fig 23.
Find the area of this figure, regarding it as a parallelogram

BD is the length of the radius of the circle, and AB is half the length of the circumference. Therefore, since the area of

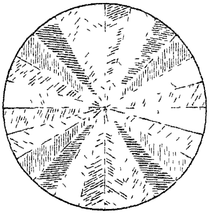

FIG 22 —Circle divided into triangles

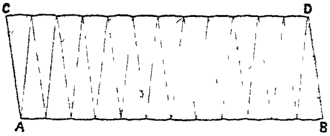

FIG 23 —Figure formed by triangles cut from a circular disc.

the figure equals that of the original circle, **the area of a circle is equal to the radius multiplied by half the circumference.** It should be understood that this rule is proved only approximately by the method adopted.

Since the circumference $= 2\pi \times$ radius, then

$$\textbf{area of circle} = \textit{radius} \times \frac{2\pi \times \textit{radius}}{2}$$
$$= \textit{radius} \times (\pi \times \textit{radius})$$
$$= \pi \times (\textbf{radius})^2$$

EXPT. 17.—**Square on radius** Draw a circle of about 5 cm. radius on squared paper, and describe a square on a radius Determine the area of the circle by counting squares in one half of the circle and multiplying the total by 2 Similarly determine the area of the square described on the radius

Repeat the measurements for a circle of larger radius. Tabulate the results thus

Area of circle.	Area of (radius)^2	$\frac{\text{Area of circle}}{\text{Area of (radius)}^2}$

The ratio, in the last column, should be a constant quantity, and be identical with that obtained in Expt. 4

Areas of Surfaces of Solids.

Pyramid.—Let A (Fig. 24) be the apex, and BCDEF .. the base, of any pyramid. It is required to find the total area of the sides ABC, ACD, ADE, .. etc. Draw AM perpendicular to BC, and calculate the area of the triangle ABC (from the product $\frac{1}{2}AM \times BC$). Proceed similarly with each of the triangular faces, and finally add together the areas thus obtained.

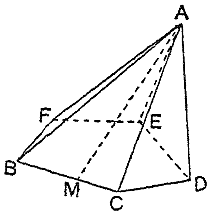

FIG 24—Lateral surface of a pyramid

The line AM is called the *slant height* of the face ABC.

In the case of a *regular* pyramid all the triangles forming the lateral surface are equal in all respects; and, therefore, all the perpendiculars similar to AM are equal. Hence, the total surface is equal to $\frac{1}{2}AM(BC+CD+ \ldots)$; or

the lateral surface of a regular pyramid $= \frac{1}{2} \times$ **slant height** $\times$ **perimeter of base.**

Curved surface of a cone.—Let DE (Fig. 25) be a small element of the circumference of the base BDEC of a cone with apex A. Let M be the middle point of DE If DE is very small, it may be considered as approximately straight, and ADE is then a plane isosceles triangle with AM as its vertical height The area of the triangle ADE is coincident, when DE is very small, with the curved surface of the cone between the lines AD and AE and the arc DE.

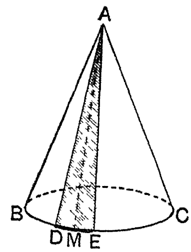

FIG 25—Curved surface of a cone

Suppose the whole of the circular base to be split up into elements like DE, and the surface into triangles corresponding to ADE; then the *height* of each triangle will be equal to AM (which is the *slant height* of the cone) Therefore,

the sum of these triangles $= \frac{1}{2}AM \times$ sum of the bases.

When each of the bases is very small, the sum of the bases is equal to the total perimeter of the base of the cone. Hence,

the lateral surface of the cone $= \frac{1}{2}AM \times$ *perimeter of the base of the cone*
$= \frac{1}{2}$ **slant height** $\times$ **circumference of base.**

Sphere.—Fig. 26 represents a sphere surrounded by a cylinder, of which both the diameter and the height are equal to the

diameter of the sphere. It can be proved that the surface of the sphere is equal to the curved surface of the cylinder. The dotted lines indicate two horizontal cross-sections, and it can be shown that the area of the zone, of which ab is a section, cut off from the surface of the sphere, is equal to the area of the

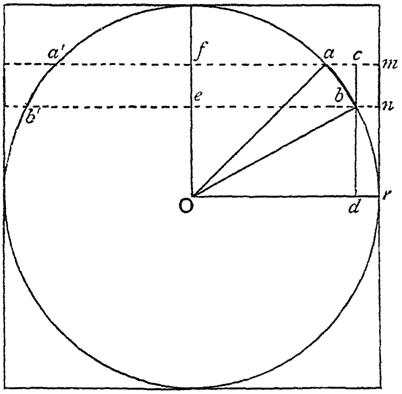

FIG 26 —Sphere and circumscribing cylinder.

corresponding band mn cut off from the cylinder's surface. If the cross-sections are *very near* together, then

ab is approximately a straight line,

therefore, ab is approximately coincident with the tangent at b,

hence, the angle Oba is a right angle;

therefore, the angle $Obe =$ angle abc

But, angle $acb =$ a right angle $=$ angle Oeb;

wherefore, the triangles abc and Oeb are equiangular, and

$$\frac{Ob}{eb} = \frac{ab}{bc},$$

or $$Ob \,.\, bc = ab \,.\, eb.$$

Since, in the limit when the sections are *very* close, $fa = eb$,

area of zone of sphere $= ab\ (2\pi\ eb)$
$= bc \,.\, (2\pi \,.\, Ob)$
$= mn \,.\, (2\pi \,.\, en)$
$=$ *area of band of cylinder.*

This result holds good for all such cross sections which may be drawn. Therefore, **the whole area of the sphere's surface is equal to that of the curved surface of the circumscribing cylinder.**

If r is the radius of the sphere, the area of the curved surface of the cylinder is equal to $2\pi r \times 2r$, or $4\pi r^2$ Hence,

the area of the surface of a sphere of radius $r = 4\pi r^2$

That the surface area of a sphere is four times the area of a circle of the same diameter as the sphere can also be demonstrated by weighing hemispherical shells of sheet brass and circular discs of sheet brass of the same thickness and of the same diameter as the sphere* If the hemispheres are fairly large, say three inches in diameter, two of them together weigh, within a fair degree of accuracy, four times as much as a disc. Here is an actual result:

Mass of disc, - - - - -	22·1 grams
Mass of two hemispheres, - - -	88·8 grams.

EXERCISES ON CHAPTER II

1. Calculate the areas of the following rectangles · 1 2 metres by 75 cm.: 1 25 ft by 10 inches ; 2 04 metres by 4·4 metres.

2. Find the length of the other side in the following rectangles: Area 3 54 sq. metres, length 59 cm.; 225 sq ft, length 5 yd.

3. Calculate the surface area of (i) a 4 cm cube, and (ii) four separate cubic centimetres

4. Describe a triangle with sides 5, 6, and 7 cm long Measure the angles with a protractor, and also calculate (without using squared paper) the area of the triangle

5. The parallel sides of a trapezium are 54 ft and 36 ft, and the perpendicular distance between them is 10 ft. Find the area

6. If the metre is equivalent to 39·371 inches, and corresponds to the ten-millionth part of the distance from the pole of the earth to the equator, find the circumference of the earth in miles.

7. A well is 100 ft. deep How many coils of rope, rolled in one layer, will be required to reach to the bottom, the roller on which they are wrapped being 8 inches in diameter?

8. How many trees at a consecutive distance of 8 yd can be planted round the edge of a circular field of radius 100 yd.?

9. If the pressure of steam in a boiler is 100 lb per sq in, find the total pressure on a circular valve 3 inches diameter

10. A circular bowling-green, of which the diameter is 125 ft, is surrounded on the outside by a gravel path $7\frac{1}{2}$ ft. wide Find the area of the path in square yards

11. Find the whole surface of a square pyramid, a side of the base being 12 ft., and the slant height 25 ft.

* Brass hemispheres and discs can be obtained from Messrs Griffin & Sons, Ltd., Kingsway, London

12. The diameter of the base of a cone is 1 ft., and the slant height is 8 in. Find the whole surface of the solid.

13. How many sq. in. of tin foil are required to cover a sphere 18 inches in diameter?

14. A round tower is 50 ft. in diameter and 120 ft. high. It is surmounted by a conical top 30 ft. high. Find the whole exterior surface.

15. Construct a square with side 2 in. in length upon paper divided into millimetre squares. Find from this figure the number of (*a*) square millimetres, (*b*) square centimetres in one square inch.

16. Construct a square with side 3 cm. upon paper divided into $\frac{1}{10}$ in. squares. Use the figure to calculate what fraction 1 sq. cm. is of 1 sq. in.

17. Construct a triangle upon squared paper. Determine its area by means of the formula area $=\frac{1}{2}$(base $\times$ altitude), and also by counting the squares.

18. Draw upon squared paper two different parallelograms upon the same base and with the same altitude. Find the area of each by counting the squares.

CHAPTER III.

MEASUREMENT OF VOLUME

Measurement of volume.—Each edge of a cubic foot (Fig 27) is measured as a *length*. Each of the faces of the cube has an *area*, which can be obtained by multiplying together the lengths of two of the edges which meet at a corner. The size of the cube, or the amount of room it takes up, or the space it occupies, is called its **volume.**

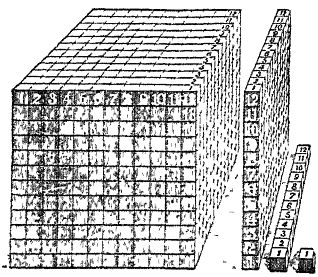

FIG. 27.—To explain why 1728 cubic inches make 1 cubic foot

A cubic inch, such as is represented upon a small scale on the right hand of Fig 27, has each edge 1 inch in length and each face 1 sq. inch in area. Twelve of these cubes placed in a row have a total length of 1 foot; and 12 of these rods laid one upon the other will build up a slab containing $12 \times 12 = 144$ cubic inches. Consider this layer of 144 cubic inches, or little cubes each edge of which is an inch, and each face of which is a square inch. Evidently there are twelve such layers in the whole cubic foot

Consequently, in the whole cube we have $144 \times 12 = 1728$ little cubes the edges of which are one inch long and the faces of which are each one square inch. Or, one cubic foot contains 1728 cubic inches.

By reasoning in the same way the number of cubic feet required

to build up a cubic yard can be found. We may write, therefore,

1728 ($=12 \times 12 \times 12$) cubic inches make 1 cubic foot.

27 ($= 3 \times 3 \times 3$) „ feet „ 1 „ yard

Units of volume and capacity.—The unit of volume in the metric system is also derived from the unit of length. A block built up with cubes representing cubic centimetres is shown in Fig 28. This cube measures 10 centimetres each way, and its volume is therefore a cubic decimetre There are 10 centimetres in a decimetre, so the edge of the decimetre cube is 10 centimetres in length; the area of one of its faces is $10 \times 10 = 100$ square centimetres; and its volume is $10 \times 10 \times 10 = 1000$ cubic centimetres

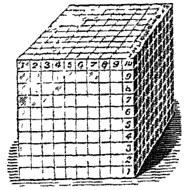

FIG 28 —Representation on a reduced scale of a cubic decimetre divided into cubic centimetres

If a *hollow* cube be made 1 decimetre long, 1 decimetre broad, and 1 decimetre deep, it will hold 1000 cubic centimetres of liquid. This capacity is called a **litre.** All liquids are measured in litres in countries where the metric system is adopted Thus in France, wine, milk, and such liquids, are sold by litres instead of by pints. A litre is equal to about one and three-quarters English pints.

In the British system an arbitrary unit, the **gallon,** is the standard unit of capacity and volume. It is defined as **the volume occupied by 10 lb. of pure water at a temperature of 62° F.** Since water, like most other substances, expands when heated, it is necessary to state the temperature in defining the unit.

A **cubic foot** is equal in volume to about $6\frac{1}{4}$ gallons One cubic foot of cold water weighs about 1000 oz. or 62·5 lb.

It is important to remember that

$$1 \text{ cubic yard}^* = 3^3 \text{ cubic feet, and}$$
$$1 \text{ cubic foot} = 12^3 \text{ cubic inches.}$$

$$1 \text{ cubic metre} = 10^3 \text{ cubic decimetres (or } \textit{litres}\text{)}$$
$$= 10^6 \text{ cubic centimetres;}$$
$$1 \text{ cubic decimetre (or litre)} = 10^3 \text{ cubic centimetres, and}$$
$$1 \text{ cubic centimetre} = 10^3 \text{ cubic millimetres}$$

* It is usual to write a small 3 on the top right-hand side of a number when the cube of a number has to be used, so that 8^3 means the cube of 8, or $8 \times 8 \times 8$ In a similar manner $4 \times 4 \times 4 \times 4$ is written 4^4, and $12 \times 12 \times 12 \times 12 \times 12$ is abbreviated to 12^5

EXPT. 18.—**British and Metric volumes.** Measure the length, breadth, and thickness of a rectangular block of wood both in inches and in centimetres. From these measurements calculate the volume of the block, both in cubic inches and in cubic centimetres. Use the results to determine how many cubic centimetres are equivalent to 1 cubic inch.

Verify your result by calculation from the relation 1 inch = 2·54 centimetres.

DETERMINATION OF VOLUMES BY GEOMETRICAL RULES.

Cylinder.—The preceding paragraphs have shown that the volume of a rectangular solid can be determined by multiplying the area of the base by the height. Fig. 29 represents a cylinder cut into horizontal slabs, each 1 cm. thick. The number of cubic centimetres (*c.c.*) in the bottom slab is given by the number of sq. cm. in the area of the base; and the total number of c.c. in the whole cylinder is obtained by multiplying the number of c.c. in the bottom slab by the number of slabs. Hence, if the radius of the base of the cylinder be r, and if the height be h, then area of base $= \pi r^2$, and the **volume of the cylinder** $= \pi r^2 h$.

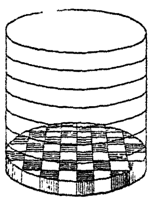
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FIG. 29.—Volume of a cylinder.

Parallelepiped.—Fig. 30 represents how two triangular blocks or prisms may be placed together, to form either a cube or a parallelepiped. The volume, the base, and the height are the same in either case. Hence, since the volume of a cube is equal to (area of base × height),

volume of parallelepiped = area of base × height.

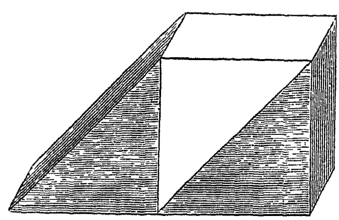


FIG. 30.—Cube and parallelepiped.

Pyramid.—A cube may be considered to be made up of six pyramids as indicated in Fig. 31. Suppose a cube of the same size be cut into six slabs as in Fig. 31. Then the volume of one of the pyramids is equal to the volume of one of the slabs, for each is one-sixth the volume of the cube. The volume of a slab is equal to the base multiplied by the height of the layer, which is one-third the height of a pyramid. Therefore, **the volume of a pyramid on a square base is equal to the base multiplied by one-third the height.** This rule applies to every pyramid. In other words,

a pyramid could be flattened down to a rectangular block having the same base and one-third the height.

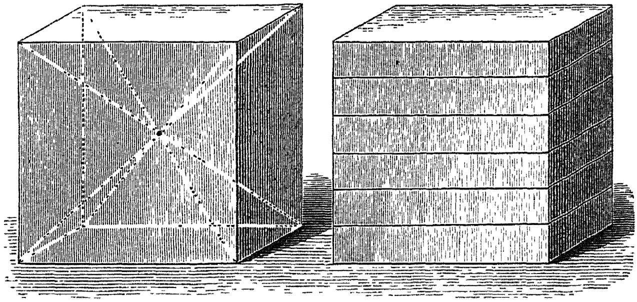


FIG 31.—A cube formed by six pyramids and a cube formed by six slabs.

EXPT. 19.—**Volume of pyramid.** Find by means of the above rule the volume of a pyramid on a square base.

Cone —A cone may be regarded as a pyramid with an infinite number of sides. The rule for obtaining its volume is, therefore, the same as for the pyramid. That is,

$$\begin{aligned}\textbf{volume of cone} &= \tfrac{1}{3} \times \textbf{area of base} \times \textbf{height}\\ &= \tfrac{1}{3} \times \pi r^2 \times h\\ &= \tfrac{1}{3}\pi r^2 h.\end{aligned}$$

EXPT. 20 —**Volume of cone.** Measure the height of a cone and also the diameter of the base, from which the radius can be determined Using the numbers thus found and the rule just given, calculate the volume of the cone.

Sphere.—The surface of a sphere can be divided into a large number of small triangles, each of which may be considered to form the base of a pyramid having a height equal to the radius of the sphere. Fig. 32 shows such a sphere with a few of the pyramids taken out. All the bases added together equal in area the surface of the sphere, and all the

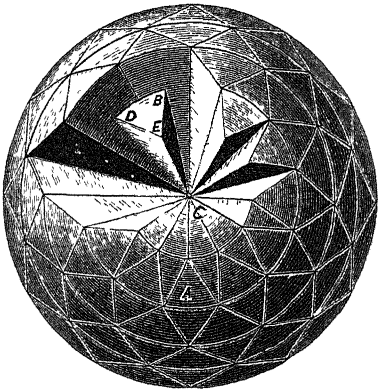


FIG 32.—Sphere formed by numerous small pyramids A few of the pyramids have been taken out

pyramids added together equal the volume of the sphere. Therefore the volume of the sphere can be found by multiplying the surface by one-third the height of the pyramids, that is, by one-third the radius. As has been previously shown:

$$\text{Surface of a sphere} = 4\pi r^2 \text{ (p. 24).}$$

But, **volume of a sphere** $= \text{surface} \times \frac{1}{3}r$

$$\text{,,} \quad \text{,,} \quad = 4\pi r^2 \times \frac{r}{3}$$

$$\text{,,} \quad \text{,,} \quad = \tfrac{4}{3}\pi r^3$$

Use of Graduated Vessels.

Vessels used as metric measures of capacity.—A number of vessels used in the measurement of capacity are shown in Fig. 33. The **flask** marked 500 c.c. has a mark upon its neck, and when filled up to this mark the number shows how many cubic centimetres of liquid are in it. The tall jar (or **cylinder**) has a mark at every 10 cubic centimetres up to 1000 c.c.; the number of cubic centimetres of a liquid may thus be found by pouring the liquid into the jar and reading the scale division which is on the same level with the surface. Since the volume indicated by each scale division varies according to the diameter of the cylinder, it is necessary, before using any cylinder, to determine by means of the numbers attached to the scale the capacity represented by consecutive scale divisions.

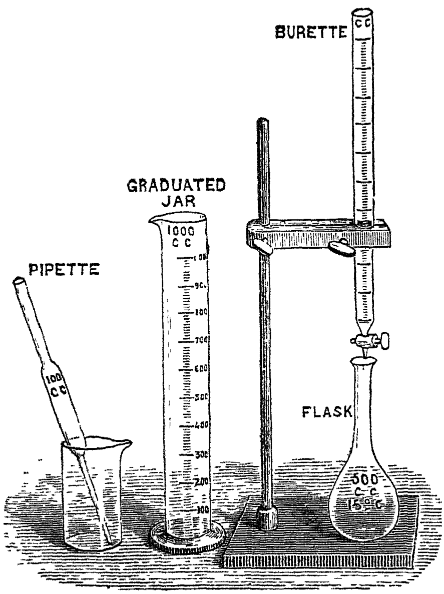


FIG. 33.—Graduated vessels for measurements of volume.

The graduated tube is a **burette**, used for measuring out exact quantities of liquid. At the bottom is a tap or clip for allowing liquid to flow out of the burette. Supposing that the burette is filled to the mark 10 c c., and that 35 c.c of liquid are required from it, the tap would be opened gradually, and when the liquid had fallen to the mark 45 c c it would be closed quickly. Burettes are graduated always from the top *downwards*. When a burette is first filled with the liquid, the tap or clip should be opened and the liquid allowed to run out until all air has been expelled from the exit tube. the instrument is then ready for use.

The narrow tube supported in a beaker is a **pipette**, by means of which small quantities of liquid may be conveyed from one vessel to another.

Fig. 34 indicates the correct method of reading the position of the liquid surface (or **meniscus**) in a measuring vessel. The surface is curved downwards (except when mercury is used), and it appears to have an upper and lower margin. the scale reading of the middle point of the lower margin should be obtained always This is observed most satisfactorily if the meniscus is illuminated by holding a piece of white paper horizontally close to the glass vessel and two or three inches below the meniscus. The eye must be on a level with the meniscus· the sloping dotted lines show how incorrect readings are obtained if this precaution is not taken

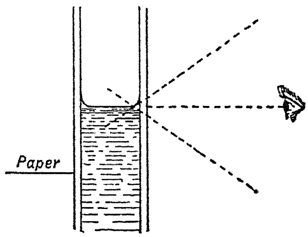


FIG. 34 – Observing the height of a liquid surface in a tube

EXPT. 20A —**Graduation of a cylinder.** Clean and dry the inside of a small glass cylinder (*e.g.* 100 c c. capacity) Gum a narrow strip of mm squared paper up the outside of the cylinder Introduce into it, from a burette, 10 c c of distilled water. Mark on the paper strip the position of the meniscus. Add another 10 c c, and mark the position. Repeat this until the cylinder is full. By means of the squared paper, the scale intervals thus obtained may be subdivided so as to read to smaller volumes than 10 c.c.

Measurement of volume by displacement.—Graduated vessels provide a convenient means of determining the volume of a solid by observing the volume of water it displaces.

EXPT 21.—**Cube.** Find the volume of a metal cube by observing the volume of water which it displaces. Select a measuring cylinder into which the cube will just pass without friction, and pour in a considerable volume of water. Read the position of the meniscus. Gently lower, by means of thin cotton, the cube into the cylinder until it is immersed *completely*, and again read the position of the meniscus. The difference between the two readings indicates the volume of the cube.

Verify this result by calculating the volume of the cube from its dimensions. Enter your results thus.

First reading of meniscus	Second reading of meniscus	Volume, by displacement	Average length of edge of cube	Volume, by calculation

EXPT 22.—**Various geometrical solids.** By the method of Expt 21, find the volume of (*a*) a cylinder, (*b*) a pyramid, (*c*) a cone, and (*d*) a sphere.

In each case verify your result by calculation from dimensions. If the solid is of a material which floats in water, it may be forced below the surface by means of a thin hat-pin.

EXPT 23.—**Large solids.** If no measuring vessel of sufficient width to admit the solid is available, the following procedure may be adopted: Attach a narrow piece of gummed paper vertically to the side of a beaker; and, at a level which will be well above the top of the immersed solid, make a fine horizontal mark by cutting through the paper with a penknife. Dry the beaker, and fill the beaker up to the mark with water measured from a burette. Again dry the beaker, place the solid inside, and fill the beaker up to the mark as before. The volume of the solid is represented by the difference in the volumes of water required.

EXPT. 24.—**Lead shot.** Find the average volume of lead shot by the following method: About half-fill a burette with water, and read the meniscus. Select a given number (*e.g.* 20 or 30) of the shot, and introduce them into the burette—which should be held in a slanting position. Again read the meniscus. From the volume of the displaced water calculate the average volume of one shot.

EXERCISES ON CHAPTER III.

1. Find the number of litres in one cubic foot

2. Find the number of gallons in 50 litres, also in one kilolitre.

3. The internal dimensions of a tank are Length 10·5 m, width 2·25 m, depth 2·75 m Find the number of litres of water in it when the tank is full.

4. The internal dimensions of a rectangular tank are 4 ft 4 in, 2 ft 8 in, and 1 ft $1\frac{1}{2}$ in. Find its volume in cubic feet, the number of gallons it will hold when full, and the weight of the water

5. Find the volume of a metal cylinder of which the diameter is 12 cm and the height 20 cm

6. How many cubic feet of water will be discharged from a pipe in 24 hours, if the diameter of the pipe is $3\frac{1}{20}$ inches, and if the velocity of the water is 2 feet per second?

7. Find the whole surface and the volume of a square pyramid, of which the side of the base is 10 ft and the height 19·36 ft.

8. The interior of a building is in the form of a cylinder of 40 ft radius and 20 ft. in height A cone surmounts it, having a radius of 40 ft and height 10 ft. How many cubic feet of air will the building contain?

9. A hollow spherical shell has an external diameter of 14 in., and the thickness of the shell is 1 inch How many cubic inches of metal does it contain?

10. A circular disc of lead, 3 in. thick and 12 in in diameter, is converted into small shot, each of radius 0·05 inch How many shot does the disc make?

11. A sphere just slips into a cubical box, one edge of which measures 5 cm. How much space is left unoccupied?

12. A cone, a hemisphere, and a cylinder stand on the same base, and are of the same height Find the ratio of their volumes

13. An ink-bottle consists of a cube of glass, out of which has been cut an exactly hemispherical hole. What measurements and what calculations would be necessary to determine the volume of the glass?

How could the previous result be verified or checked, if you had a beaker of water, a gummed paper strip, and a burette or a measuring cylinder?

14. A porcelain weight of one pound breaks into two unequal fragments How could you without using a spring balance or ordinary balance determine the weight of each fragment?

15 Measure a pint or fraction of a pint in cubic centimetres by means of a graduated vessel, and so find the number of c.c. equal to one pint.

16. A sphere just fits into a cylindrical box and is level with the top of the box 2 inches in diameter. Calculate the volume which the box will hold, and also the volume of the sphere. Find what fraction the volume of the sphere is of the internal volume of the box

CHAPTER IV.

MASS, WEIGHT, AND DENSITY.

Mass. A precise definition of mass must be deferred to a later stage (p. 100). We here limit ourselves to the statement that though mass is not weight, **masses can be compared by weighing.** When, therefore, two bodies balance one another in a pair of scales, the quantities of substance or matter in them are equal. Equal masses thus measured may, however, have very unequal volumes, as, for instance, when a small piece of lead in one pan of a pair of scales balances a large pile of cotton wool in the other.

Mass is not weight.—When the mass of a pound is dropped from the hand it falls to the ground. When the same mass is hung upon the end of a coil of wire, the coil is made longer by the downward pull of the mass fixed to its end. In the instrument called a **spring balance**, the amount by which a steel spring is lengthened, as the result of such downward pull of masses attached to its end, is used to measure their **weights.** When a delicate balance of this kind is used, the *weight* of a small piece of iron hung on to the balance may be made to appear greater by holding a strong magnet beneath it. But, though the weight may seem to have increased, the quantity of matter is, of course, the same whether the magnet is under the iron or not.

It is known by common experience that unsupported things fall to the ground; a fact which is expressed also by saying that they are pulled downwards by the earth. Even when they are supported, as in the case of things upon a table, the earth attracts them just as much, only the table prevents them from falling, as they would do if there were no table there. Exactly why there is this tendency downwards need not at present be

described. The point from which the attraction may be regarded as being exerted is the centre of the earth, and **the weight of a body may be considered as a measure of the attractive influence of the earth upon it.**

The weight of a given mass may vary from place to place.—Bearing the definition of weight in mind, it will be clear that since a mass is farther away from the earth's centre when it is up in a balloon than when at the sea-level, the weight of this mass ought to be more at the sea-level, for it is there nearer the earth's centre than when up in a balloon. This is found to be the case, but actually to demonstrate the difference, the weight must be measured by a spring balance.

Similarly, because the earth is not a perfect sphere, but is flattened in north polar regions, points at the surface of the earth in the region of the tropics are at a greater distance from the centre than points further north. Consequently, on this account, and also because the earth's rotation causes bodies near the equator to have a greater tendency to be thrown off the earth than when they are in higher latitudes, a constant mass suspended from a spring balance increases in weight when carried toward the poles As, however, equal masses balance one another when weighed in an ordinary pair of scales, the weight of an object may for all practical purposes be regarded at a given place as a measure of its mass

Measurement of mass.—Just as in measuring lengths it is necessary to have a standard with which to compare, so in measuring mass there must also be a standard, or unit. In this country the standard of mass is the amount of matter in a piece of platinum which is deposited with the Board of Trade. This lump of platinum is called the **imperial standard pound,** and from it all other imperial measures of mass are ascertained.

A mass of 1 lb. avoirdupois is kept at a weights and measures office in every city, so as to test the lb "weights" used by tradesmen, and to see whether they really have the mass of 1 lb or are too light.

The kilogram and gram.—The standard of mass adopted in France, and in other countries where the metric system is used, is called the **kilogram.** The kilogram is the amount of matter in

a lump of platinum which is kept in safety at Sèvres. It is heavier than the British pound; indeed, it is equal to about two and one-fifth of these pounds. It is interesting to know how the mass of a kilogram was derived. The mass of the piece of platinum was made equal to that of one thousand cubic centimetres of water, that is, of a litre of water, at a particular temperature. The names used for the divisions, etc., of the kilogram are as follows:

Metric Measurement of Masses.

10 milligrams	= 1 centigram.	10 grams	= 1 dekagram.
10 centigrams	= 1 decigram.	10 dekagrams	= 1 hektogram.
10 decigrams	= 1 **gram.**	10 hektograms	= 1 **kilogram.**

The relative values of some of the British and metric measures of mass are:

Metric to British.		British to Metric	
1 gram	= 15·4323 grains.	1 oz. Av. =	28·35 grams.
1 kilogram =	2 2046 lb Av.	1 lb. Av. =	453·6 grams.

The Balance and its Use.

The principle of the balance.—The principle of the balance may be explained by means of a few observations with the simple arrangement of apparatus shown in Fig 35. The apparatus consists of a boxwood half-metre scale AB; through the middle division a hole is bored (nearer to the divided edge, and not at the centre). The scale is pivoted, with its divided edge *upwards*, by means of a piece of stout iron wire bent at right angles and clamped between the boss and the upright of a retort-stand; a straight knitting-needle may be used instead of the bent iron wire Weights are hooked on to loops of cotton thread by means of thin copper wire bent so as to form a hook and twisted round the shank of the weight.

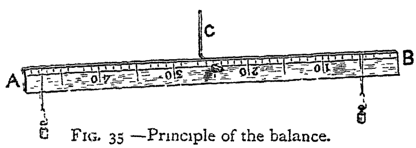


FIG. 35 —Principle of the balance.

EXPT. 25.—**Adjustment.** If the scale is not in equilibrium in a horizontal position, suspend a *small* mass from the lighter side, adjusting the position of the mass so that the scale is balanced

correctly; or, a *rider* cut from sheet lead will serve the same purpose. This mass, or rider, must remain in the same position on the scale throughout the experiment

EXPT. 26.—**Equal distances.** Hang a known mass (taken from a box of *weights*) at any convenient distance on one side of the pivot, or **fulcrum,** as it is termed, and balance it with a mass of the same amount on the other side. *The distance of the masses from the fulcrum will be found the same in each case.*

EXPT. 27.—**Unequal distances.** Suspend a mass of 50 grams from a point mid-way between the fulcrum and the left-hand end of the scale, and retain it in this position during the experiment Suspend some other mass, *e.g.* 100 grams, from the right-hand side of the scale, adjust its position until the scale is in equilibrium, and note its distance from the fulcrum. Repeat this several times, using different masses in each case, and suspending more than one mass from the cotton loop if necessary. Show that, in each case,

$$\begin{matrix}\textit{Mass on} \\ \textit{one side}\end{matrix} \times \begin{matrix}\textit{Distance} \\ \textit{from fulcrum}\end{matrix} = \begin{matrix}\textit{Mass on} \\ \textit{other side}\end{matrix} \times \begin{matrix}\textit{Distance} \\ \textit{from fulcrum.}\end{matrix}$$

From this it is evident that **the masses are equal only when the points of support are equidistant from the fulcrum.** The ordinary **balance** may be described as a beam supported at its middle point and with pans supported at points equidistant from the fulcrum, and used for the purpose of counterpoising known masses with an unknown mass.

It must be remembered carefully that in taking a so-called *weighing* of a body by means of a balance, the known mass which has the same weight as the unknown mass only is found; exactly the same result would be obtained if the balance were used at any other locality on the earth's surface, though it is known that the weight varies according to the locality. The *weight* of a mass can only be determined by means of a spring balance; in this appliance the extension of the spring is an exact measure of the weight of the mass attached to the spring.

The balance.—The balance is really another form of the supported lath in Expt. 26. All the parts are very carefully made, and every means is taken to have very delicate supports and accurate adjustments. Fig. 36 shows a simple form of balance which is suitable for easy experiments, such as are described in this book, which can all be done accurately by its means. Instead of the wooden lath used in Expt. 26

there is a brass beam supported at its middle line by a knife-edge of hard steel, which, when the balance is in use, rests on a true surface of similar steel The hooks to which the pans

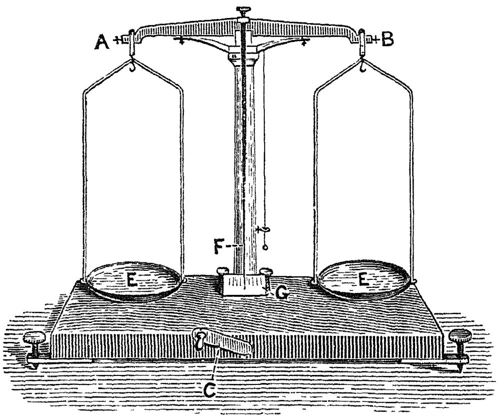


FIG. 36.—A physical balance.

are attached are provided similarly with a V-shaped depression of hard steel, which also, when the balance is in use, rests upon knife-edges on the upper parts of the beam. To the middle of the beam is attached a pointer, the end of which moves over an ivory scale fixed at the bottom of the upright which carries the beam. When not in use the beam and hooks are lifted off the knife-edges by moving the handle C.

EXPT 28.—**Parts of a balance.** Uncover the balance and identify the different parts by reference to Fig 36 Raise the beam, AB, of the balance, off the supports by turning the handle C. Notice whether the pointer F swings equally on both sides of the middle of the scale G ; if it does the balance is ready for use ; but if not, let down the beam and turn the small screw at A or B, then try again Repeat this adjustment until the swings to right and left are equal.

Set of metric weights.—Sets of metric weights are marked usually in grams and milligrams, and a complete set will include the following :

(i) (Brass weights), 100 gm., 50 gm., 20 gm., 20 gm., 10 gm., 5 gm , 2 gm., 2 gm., 1 gm.

(ii) (Aluminium weights),

500 mgm (= 0·5 gm.), 50 mgm (= 0·05 gm.),
200 mgm. (= 0·2 gm.), 20 mgm. (= 0·02 gm.),
200 mgm. (= 0·2 gm.), 20 mgm (= 0·02 gm.),
100 mgm. (= 0·1 gm.), 10 mgm (= 0·01 gm.)

By means of this set any weight which is a multiple of 10 mgm and does not exceed 211 gm. may be obtained. The box contains a pair of **forceps**, which must be used always when weights are removed from, or replaced in, the box.

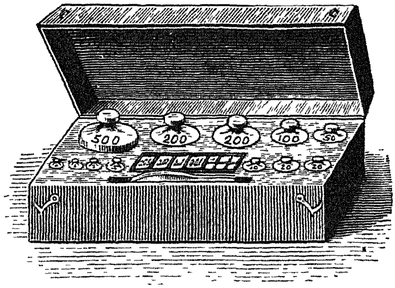


FIG 37.—Metric weights

The weights are used in the following manner: the object to be weighed is placed in the left pan, and such weights as are estimated as sufficient to counterbalance the object are placed in the right pan.

Suppose that 20 + 10 gm are used, and that, on *slightly* releasing the beam, the pointer moves towards the object: these weights are evidently too great. The 5 gm. weight is substituted for the 10 gm weight; if this is now too small, a 2 gm weight is added; and, if this is still too small, the second 2 gm. weight is added. The subsequent steps are as follows.

20 + 5 + 2 + 2, too great.
20 + 5 + 2 + 1, too small.
20 + 5 + 2 + 1 + 0·5, too small.
20 + 5 + 2 + 1 + 0·5 + 0·2, too great.
20 + 5 + 2 + 1 + 0·5 + 0·1, too great.
20 + 5 + 2 + 1 + 0·5 + 0·05, approximately correct.

The weight is thus found to be 28·55 gm.

Special precautions in weighing.—1. See, by means of the plummet, that the balance is level.

2. See that the stirrups are not displaced: see also that the pans are dry and clean.

3. Lower the arrestment to see whether the pointer swings equally on both sides of the middle point of the scale. If necessary, adjust the balance by means of the screw-nuts at either end of the beam.

4. Do not stop the swinging of the balance with a jerk, but stop it gently when the pointer is nearly at its central position.

5. Place the body to be weighed on the left-hand, and the weights on the right-hand, pan.

6. Lower the arrestment before adding or removing any weight.

7. Manipulate the arrestment with the left hand, and convey weights with the right hand On no account touch weights with the hand, but always use the forceps

8. Do not weigh a body when hot · the heat causes air currents, which affect the weighing.

9. Close the balance case when observing the swinging of the pointer, and keep the case closed when the balance is not in use.

10. Always replace each weight in its proper compartment in the box

Sensibility of a balance.—In determining the sufficiency of the weights on the balance pan it is unnecessary to wait until the beam ceases to swing; it is sufficient to observe the extreme positions on the scale to which the pointer travels, and from these the final position of rest can be calculated The scale divisions should be numbered, as shown in Fig 38 Suppose that consecutive turning points are 8 and 14; the true position of rest will not be far from $\frac{8+14}{2}=11$.*

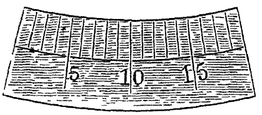


FIG. 38 —Scale divisions of a balance

The **sensibility** of a balance may be defined as *the change in the position of rest of the pointer due to a given change of weights*; it is expressed usually as the change of weights necessary to alter the position of rest through one scale division. The sensibility of the simple balances generally used in elementary experiments varies from 2 to 4 milligrams per scale division, it varies slightly, according to the load on the balance, becoming less with heavy loads; and it is usually a maximum with a load of 20-30 grams

* Strictly speaking, we ought to allow for the fact that the extent of swing is gradually becoming less, owing to friction and air resistance We should take, therefore, two consecutive readings on *one* side and the intervening reading on the other side. Thus, suppose the readings are 8 (left), 14 (right), 8.4 (left), the average of the 1st and 3rd gives the reading on the left which would have been obtained if the vigour of the swing at the moment of taking the right turning point remained unaltered. Hence, the true turning point on the left is $\frac{8+8.4}{2}=8.2$, and the true position of rest is $\frac{8.2+14}{2}=11.1$.

Example. Load, 20 gm. in each pan

$$\text{Resting point} = \frac{8+14}{2} = 11$$

Resting point, with 20·01 gm. on right pan, $= \frac{5.5+9}{2} = 7.25$.

Change of resting point, due to 10 mgm.,

$$= 11 - 7.25 = 3.75 \text{ divisions};$$

$$\text{sensibility} = \frac{10}{3.75} = 2.7 \text{ mgm. per scale division}$$

EXPT. 29.—**Sensibility with and without load.** Find the sensibility of a balance, with (i) no load, (ii) a load of 20 grams in each pan, (iii) a load of 50 grams in each pan

When recording the swings, close the front of the balance case, and allow the pointer to swing to and fro several times before taking the readings.

Weighing by vibrations.—In weighing an object it is not necessary to modify the known weights until the resting point is the same as that obtained when the balance is unloaded, providing that the sensibility of the balance is determined previously.

Example. Sensibility of balance = 3 mgm per scale div

Resting point, with no load, = 11·5

Resting point, with object on left, and 21·55 gm on right, = 9

The weight 21·55 gm is evidently *too great* by an amount equivalent to 11·5 − 9 = 2·5 div

But, 1 div. is equivalent to 3 mgm.,

or, 2·5 div. are " " 7·5 "

Hence, true weight = 21·55 − 0·0075 = 21·5425 gm.

If the sensibility is not known with sufficient certainty, the following procedure may be adopted

Example Resting point, with no load, = 11·5.

Resting point, with 21·55 gm on left, = 9

" " 21·54 " " = 12·4.

Hence, 10 mgm are equivalent to 3·4 scale div

The weight 21·54 gm. is evidently *too small* by an amount equivalent to (12·4 − 11·5) = 0·9 scale div.

But 0·9 scale div is equivalent to $\left(10 \times \frac{0.9}{3.4}\right) = 2.6$ mgm;

$\therefore$ true weight = 21·54 + 0·0026 = 21·5426 gm

EXPT. 30.—**Vibration method.** Find, by the method of vibrations, the weight, in grams, of a 1 oz. weight. Repeat the experiment, using a $\frac{1}{2}$ oz weight and a 2 oz weight

EXPT. 31.—**Graduation of a measuring flask** Select a glass flask of such a size that when 50 c c. (or 100 c c) of cold water are poured into it the meniscus is within the neck of the flask Carefully dry the flask and weigh it. Add 50 gm to the weights already on one pan of the balance, and pour cold water into the flask until the weights are *nearly* counterpoised Add more water, drop by drop, from a narrow tube until the weights are counterpoised exactly If by chance too much water is added, the excess may be removed by dipping momentarily a narrow strip of blotting paper into the water. When the adjustment is complete, remove the flask and make a horizontal file-mark with a wetted triangular file on the neck, so as to indicate exactly the position occupied by the lower edge of the meniscus The flask has now been graduated to *contain* exactly 50 c c.

If the flask were now emptied it would be found that rather less than 50 c c. of water were transferred from the flask, the remainder of the water being inside the flask and wetting its surface If the flask is to be graduated so as to *pour out* exactly 50 c c of water, the inner surface should be wetted previously to being counterpoised. The procedure should be as follows: fill the flask with water, pour out the contents, and hold the flask inverted for a definite period, *e g.* 5 seconds. Place the flask on the balance pan and weigh it. Add 50 grams to this weight, and pour water into the flask until the weights are counterpoised. Mark the position of the meniscus. If now the flask is emptied and held inverted for the same period it may be assumed that exactly 50 c c. of water have been transferred

Weighing by substitution —A balance may be, occasionally, totally out of adjustment, thus, the arms may be of unequal length, or the pans of unequal weight In such a case the following procedure may be adopted: Place some shot or sand in a box or dish, on the left-hand pan, using sufficient to have a greater weight than that of the object to be weighed Place known weights (w_1) on the right-hand pan until the balance is counterpoised, and obtain the resting point. Remove the weights, place the object on the right-hand pan, and add weights (w_2) until the balance is counterpoised again. Obtain the resting point, and calculate the correction to w_2 necessary to make the two resting points coincide. The weight of the object is then equal to ($w_1 - w_2$) grams.

EXPT. 32 —**Substitution method.** Find, by the method of substitution, the weight in grams of a 1 oz. weight.

Calibration.

It is found frequently that graduated vessels, such as the pipette, burette and cylinder, are not sufficiently correct for use in accurate experimental work. The process of determining the error, if any, in the graduations is termed **calibration.** The following experiments illustrate the principle of an approximate method of calibrating the above types of measuring vessel, and provide also exercises in weighing.

Expt. 32A—**Calibration of a pipette.** Use distilled water which has been standing in the room for several hours. Note its temperature Carefully clean the inside of a 10 c.c. pipette.* Clean and dry a small wide-necked flask. Cover the neck with a small watch-glass, and weigh the flask with the cover. Fill the pipette with the distilled water, and adjust the meniscus to the graduation mark. Transfer the water to the flask by holding the point of the pipette against the inside of the neck; when empty, blow down it once Cover the flask with the watch-glass, and again weigh it The increase in weight expressed in grams gives in c c the capacity of the pipette, when the temperature of the water is not higher than 15° C, the assumption that 1 gm of water occupies 1 c c. introduces an error of less than 1 in 1000.

If a considerable error is found in the graduation, fix a narrow strip of gummed paper along the upper stem of the pipette, and find by trial the position which the meniscus must occupy in order that the pipette may deliver exactly 10 c.c. Mark this position permanently by means of a scratch made with a file.

Expt 32B—**Calibration of a burette.** Clean the inside of the burette, and fix it vertically in a stand Fill it with distilled water at the temperature of the room. Run some of the water out at the tap or jet, so as to remove all air bubbles. Carefully run more water out until the meniscus coincides with the zero of the scale; and remove any drop adhering to the end of the jet Weigh a flask together with a small watch-glass to serve as a cover. Run water from the burette into the flask until the meniscus is at the 10 c c mark, and touch the inside of the flask with the end of the jet Replace the watch-glass, and again weigh the flask Run water into the flask until the meniscus is at the 20 c c mark, and again weigh. Continue this process of

* The best method of removing grease, and other matter, from a glass vessel is to rinse it thoroughly with strong sulphuric acid with a little potassium bichromate added If possible, leave this liquid in the vessel for an hour; then rinse several times with tap water, and finally with distilled water

weighing successive quantities, of 10 c c each, until the 50 c c. mark is reached. The total increase in weight (in gms.) of the flask at each weighing gives the approximate true volume delivered between the zero and each of the graduated marks used.

Plot on squared paper the total apparent volume of water taken from the burette, and the total increase in weight of the flask

EXPT. 32C.—**Calibration of a measuring cylinder** Clean and dry a measuring cylinder. Fix vertically in a stand a burette which has been previously calibrated, and fill it with water Transfer an observed volume of water from the burette into the cylinder 10–50 c.c may be used, according to the capacity of the cylinder Note the reading of the cylinder scale. Transfer a second volume, as before, and again read the cylinder scale. Repeat this process until the cylinder is full. Plot on squared paper the total volume introduced and the scale reading of the cylinder.

Cross-section of a narrow tube.—The following experiment represents the method usually adopted for finding the average cross-section of a narrow tube, and for finding the variation in cross-section of a considerable length of the tube.

EXPT. 32D —**Calibration of a capillary tube.** Select a length of thermometer-tubing with circular bore (1–2 mm diameter) and about 1 metre long If necessary, chemically clean and dry the inside of the tube Make a file-mark about 1 cm. from one end. Attach to one end a long piece of narrow rubber tubing ; and, by this means, suck up into the tube a column (about 10 cm long) of *pure* mercury. Note the temperature of the mercury. Lay the tube horizontally on an accurately divided metre-scale, so that the file-mark coincides with the zero of the scale By tilting the tube adjust the position of the mercury thread so that the centre of the thread is approximately over the 5 cm. mark of the scale, and carefully measure the length of the thread Move the thread along the tube until the centre is over the 15 cm. mark and again measure the length. Repeat this measurement with the centre of the thread over the 25 cm, 35 cm., etc. divisions of the scale, until the end of the tube is reached. Weigh accurately a porcelain crucible. Transfer the mercury thread into the crucible and again weigh. Having given that the density of mercury at the temperature of the room is approximately 13·56 gm. per c.c., calculate the average cross-section of the tube for each position of the thread in the tube.

Plot on squared paper the positions of the centre of the thread and the average cross-section in each position.

Example of Calibration of a Capillary Tube.

Temp. of mercury, 16° C.

Weight of crucible, 7·484 gm.

„ „ + mercury, 9 246 gm.

„ mercury alone, 1·762 gm.

Cross-section of tube = Weight ÷ (Length × Density)

= 1 762 ÷ (Length × 13·56)

= 0 130 ÷ Length

Position of centre of thread	Length of thread	Cross-section
5	10·00 cm	0 0130 cm.
15	9·95 „	0 01306 „
25	9 77 „	0·0132 „
35	9·90 „	0·01313 „
45	10·06 „	0 01292 „
55	10 02 „	0·01297 „
65	9·92 „	0 0131 „
75	9·92 „	0 0131 „
85	9 90 „	0 01313 „

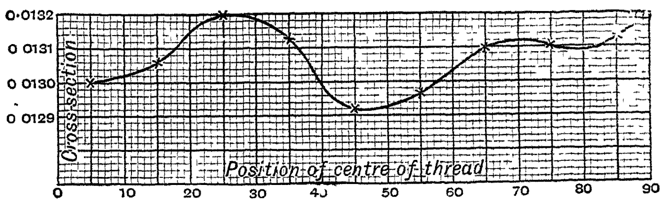


FIG 38A —Calibration curve of a capillary tube.

The principle illustrated by the foregoing experiment is used in the calibration of a thermometer tube. A thread of mercury is detached from the mercury in the bulb of the thermometer, and its length is measured, in scale-divisions, at different parts of the stem. If the bore of the tube has a uniform section throughout its length, the thread of mercury will obviously extend along the same number of scale-divisions in any part of the stem Usually, however, the mercury thread is found to have slightly different lengths in different parts of the stem, thus introducing a source of error in the readings of the thermometer.

In the case of mercurial thermometers required for the accurate determination of temperature, the instruments are calibrated in much the same way as is illustrated by Expt. 32D, and a correction curve is constructed to show the error introduced at various parts of the stem.

The spring balance.—The spring balance has been referred to already as an appliance for determining the *weight* of a body—the weight of a body being the force with which it is attracted towards the earth's centre. A spring balance (Fig. 39) consists of a spring with a hook attached to the lower end. An index is attached to the spring and travels over a graduated scale. The principle of the spring balance may be learnt by means of observations taken with a **spring dynamometer,** to which is attached an arbitrary scale (*i.e* a scale *not* graduated in grams weight).

FIG. 39—A spring balance

An efficient form of dynamometer may be obtained by suspending a spiral spring, made from thin piano wire, in a vertical position, a millimetre scale fixed to the side of the spring serves to measure the elongations. A scale pan and index are attached to the lower end of the spring. A satisfactory spring may be made by winding the thin wire in a close and uniform spiral on a round metal rod held in the chuck of a lathe; the chuck should be rotated slowly by hand, and the wire fed out under uniform and considerable tension.

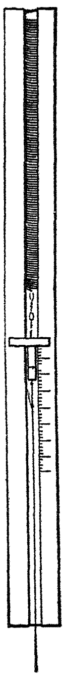


FIG 40.—Rintoul's spring dynamometer.

EXPT. 33—**Calibration of dynamometer.** Fix a dynamometer (Fig 40), with a millimetre scale attached in a vertical position, and attach to it a scale pan. Take the reading of the index Add a known weight, say 10 or 20 gm, to the pan, and again read the index. Add more weights, 10 to 20 gm. at a time, and take the index reading for each change of weight. Continue the observations until the spring is extended to nearly twice its original length Tabulate the readings in columns, under the headings *Weight* and *Index Reading.* Now gradually remove the weights, 10 or 20 gm. at a time, and take the index reading at each stage.

Plot these readings on squared paper, taking the scale of *weights* as abscissae and the scale of *index-readings* as ordinates (Fig. 41). From the form of the *graph* thus obtained, state your inference as to

the relationship between the weight applied and the resulting extension of the spring.

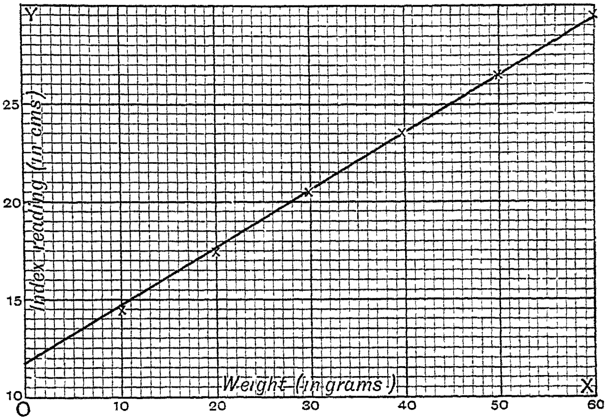


FIG. 41 —Extension of a spring dynamometer with different masses.

The following readings were obtained by means of a spiral (11 cm. long and 1 cm diameter)

Weight (in gm)	Index Reading	
	(With increasing weight)	(With diminishing weight)
0	11 75	11 75
10	14 60	14 68
20	17·50	17 50
30	20·50	20 58
40	23 52	23 52
50	26 50	26·50
60	29 50	

The results of the foregoing experiment show that, in the case of a spring, **the amount of elongation is proportional to the load.** This relationship holds good generally for the stretching of wires, rods, or similar test-pieces, provided that the limits of elasticity are not exceeded, that is, they should return to the original length when the load is removed. Using the word **strain** to signify the deformation or distortion produced by a load, and **stress** for the internal forces, which are equal to the distorting

force, tending to bring the body back to its original form, the relationship is expressed in the statement: **The strain is proportional to the stress (Hooke's Law).** This law applies to any deformation of an elastic body, such as bending or twisting, as well as to stretching.

EXPT 34.—**Use of graph.** Attach an object of unknown weight to the dynamometer or spring Note the index-reading, and deduce the weight of the object by means of the graph obtained in Expt 33.

Verify the result by weighing the object on a balance.

The rubber dynamometer.—Fig. 42 represents a dynamometer which consists of a rubber cord suspended by a hook from the top edge of a scale A pan, attached to the lower end of the rubber, serves to carry weights. Two needles, A and B, are fixed horizontally through the rubber, and adjusted so that their points traverse the fine divisions of the scale.

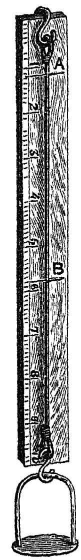


FIG 42.—Rubber dynamometer

EXPT. 35.—**Rubber cord and spiral spring.** Clamp a ½-metre wooden scale in a vertical position, and suspend the rubber cord, with pan attached, from the upper end Take the scale-reading of the two needles, and from these obtain the length of rubber between the needles. Place a 20-gram weight on the pan, and again take the reading of each needle. Take a series of readings, adding 20 grams each time to the weight, until the rubber is half as long again as it was without any weight in the pan. Now remove the weights carefully, 20 grams at a time, and read the corresponding length of rubber with each weight. When altering the weights, steady the pan with the left hand. Tabulate your readings in column, and plot on squared paper. State any inference which may be drawn from the curve obtained, when it is compared with the curve obtained when using a spiral spring dynamometer.

Density.

The meaning of density.—Different solids of the same size or volume may have different masses. Suppose, for instance, determinations are made of the mass of a cubic centimetre of wood, lead, cork, and marble, one after the other. The lead will be found to have the greatest mass, or to be heaviest, the marble will come next, and then will follow the wood and cork in this order.

By filling two bottles of the same size with different liquids it can also be shown that equal volumes of different liquids may have different masses; and when different gases are compared in the same way, equal volumes of these, too, are found to have different masses.

A pound of feathers or cotton-wool has exactly the same mass as a pound of lead, though both the feathers and cotton-wool take up much more room, or have a larger volume, than the piece of lead. The matter in the lead may be regarded as packed more closely than in the cotton-wool, which accounts for it taking up less room, or it may be said that lead is **denser** than either cotton-wool or feathers

If the relative size of a thing is small while its mass is great, then it is called a **dense** thing, or is said to have a **high density** If, on the other hand, the relative size of a thing is great and its mass small, it is said to have a **low density.** Lead is a substance with a high density, because a small piece of it has a large mass Pith and cork, on the contrary, have a low density, because a large lump of either of them has a small mass.

A cubic centimetre of marble has a mass of 2 5 gm : the same volume of iron has a mass of 7·5 gm.; of gold, 19·3 gm., of mercury, 13 6 gm ; and each of these numbers represents the density of the particular substance to which it refers, but the units of volume and mass must be expressed. In fact, **density is the mass of a unit volume of a substance.** It follows from this definition that if the volume of a body is multiplied by its density, its mass is obtained.

$$\textbf{Volume} \times \textbf{density} = \textbf{mass},$$

$$\text{or, density} = \frac{\text{mass}}{\text{volume}}.$$

In using this relation between the volume and mass, care must be taken to use the proper units. In all scientific work it is customary to adopt the cubic centimetre and gram as the units of volume and mass respectively

The ratio of the mass of *any volume* of a substance to the mass of the same volume of water is termed the **relative density** of the substance, or, as it is called frequently, the **specific gravity** (p. 66).

MEASUREMENT OF DENSITY.

EXPT. 36.—**Regular solids** Select a cube, a cylinder, a cone, and a sphere, of different materials. Calculate the volume of each solid from its dimensions, and also weigh each of them. Calculate the density of each material. Tabulate your results thus .

Material and shape	Dimensions	Volume	Mass	Density
Brass (cylinder)	Radius = Length =	$(\pi r^2 \times l) =$		

In expressing the density of any material it is essential to state the units in which the measurements are taken. Thus, the mass of 1 c.c. of water, at 4° C., is 1 gram, and that of 1 cubic foot of water is 62·5 lb. Hence, the density of water is **1 gm per c c.** (in the metric system) or **62 5 lb. per c. ft** (in the British system). It is *not* sufficient to say that the density of water is either *1*, or that it is *62·5* ; either statement is incomplete, and it is essential to state the units which have been used

On the other hand, when it is desired to express the *relative* density (p. 66) of a material—*i.e.* the number of times that the material is denser, or less dense, than a standard material—a simple numerical result is correct. Thus, the *density* of copper is 8·8 *gm. per c.c*, but the *relative density* of copper is 8·8.

EXPT. 37.—**Irregular solids.** Select a glass stopper, and carefully examine it to see that it is free from internal cavities Weigh it, and determine its volume by displacement of water. Calculate its density.

Carry out similar observations with other irregular solids.

EXPT. 38.—**Liquids** Weigh a clean, dry beaker Partly fill a pipette (20 c.c. or 25 c.c. capacity) with the liquid ; rinse it round the

inside of the pipette, and throw the liquid away. Completely fill the pipette with the liquid up to the mark on the stem, and transfer the contents to the beaker. Weigh the beaker and the liquid contained in it From these weighings, and from the known volume of the pipette, calculate the density of the liquid

EXERCISES ON CHAPTER IV.

1. The density of lead is 11 34 gm per c c. What is its density in lb per cubic foot?

2. Find the length of a lead rod 1 cm. in diameter, and weighing 1 kilogram (density of lead 11 34 gm. per c.c)

3. A block of mahogany 2 in long, $1\frac{1}{2}$ in. broad, and $\frac{7}{16}$ in thick weighs 30 35 gm. Find its density in gm. per c c.

4. Find the diameter of a cylindrical kilogram weight made of brass (density, 8·4 gm per c c), its height being 7·5 cm.

5. The radii of two spheres are 2 cm and 3 cm, and their masses are 200 gm and 250 gm respectively. Compare their densities.

6. How many grams of glycerin of density 1 26 gm per c c. can be put into a bottle which will hold 100 gm. of sulphuric acid (density 1 84 gm per c.c.)?

7. A length of glass tubing, having a narrow circular bore, weighs 14·65 gm. A thread of mercury, 10 5 cm. long, is drawn into the tube, and the tube is now found to weigh 19 13 gm If the density of mercury is 13 6 gm per c c, calculate the diameter of the bore of the tube

8. Define mass, volume, and density, and state the relation that exists between them.

Suppose you were given two irregular pieces of metal, one of which was gold and the other gilded brass How would you find out, by a physical method, which piece was gold?

9. A number of nails are driven into a rough piece of wood, one cubic centimetre of which weighs 0 5 gm It is required to find the weight of the nails without pulling them out. How could this be done by experiment?

10. By what experiments can you find (*a*) whether the arms of a balance are of the same length; (*b*) whether a balance is sensitive: (*c*) the weight of substances by means of a balance having arms of unequal length?

11. If you were provided with a 1 lb. weight and a lath balanced on a pivot, how could you determine the weight of a small bag of nails?

12. Describe the units of length, volume, and mass on the British and on the metric systems.

13. The density of a body is 7 8 on the scientific system, what is it in pounds per cubic foot? (1 lb =453 grams; 1 inch=2 54 cm)

14. A coil of brass wire consists of 55 turns of average diameter 14 inches, and the diameter of the wire is 0 017 inch Find the weight of the coil in grams if the density of the material is 8 58 grams per cubic centimetre (1 inch=2 54 cm.; $\pi=3{\cdot}14$)

PART II.

HYDROSTATICS AND MECHANICS.

CHAPTER V.

MATTER: ITS THREE STATES AND GENERAL PROPERTIES

IN the preceding chapters the fundamental measurements of length, area, volume, time, and mass have been explained. A knowledge of these principles and methods of measurement is essential in the study of physics, but before proceeding to the various parts of the science in which this knowledge is applied, it is desirable to describe the states and some of the general characteristics of matter in order that exact meanings may be attached to terms expressing particular conditions and properties.

The three states of matter.—At the present moment, the question *what is matter?* is almost unanswerable. We know much about its intimate structure and properties, but still remain ignorant of its exact nature. A definition of the term is desirable; and, of the many which have been suggested, it is perhaps sufficient to state that **matter is that which can occupy space.**

Matter is recognised generally in three different states or conditions, viz.: **solid**, **liquid**, and **gaseous**. The distinctive properties of these states of matter may be expressed in the following terms:

(1) *A solid body does not alter its size or shape readily*: it maintains its original volume and shape unless subjected to considerable force.

(ii) A *liquid readily alters its shape and adapts itself to the shape of the vessel containing it*, but it maintains its original volume unless subjected to extreme force.

(iii) A *gas readily adapts itself both to the shape and size of the vessel containing it*; a small volume of gas introduced into a large empty vessel expands and distributes itself uniformly throughout the enclosed space.

Several substances which appear to be solid are found, on closer examination, to exhibit properties characteristic of liquids. Thus, in the three series (i) *steel*, *lead*, and *jelly*, (ii) *sealing-wax*, *pitch*, *treacle*, *water*, and *ether*, (iii) *air* and *hydrogen*, the first three are solids, the following five are liquids, and the last two are gases. A rod of cold sealing-wax supported from its ends gradually becomes bent by its own weight A lump of cold pitch gradually spreads over a horizontal surface, and gradually will run down an inclined surface, also a fragment of metal will sink slowly through a slab of pitch. On the other hand, *jelly* exhibits the properties of a solid since a small solid object does not sink when placed on the surface of jelly, nor does jelly spread over a surface supporting it

Change of state.—**The same matter can exist in either of the three states**: this statement is true for nearly all substances, and it may be exemplified by the following experiments:

EXPT 39.—**Ice.** Procure a lump of ice and notice that it has a particular shape of its own, which, so long as the day is sufficiently cold, remains fixed.

EXPT 40.—**Water.** With a sharp brad-awl, or the point of a knife, break the ice into pieces, and put a convenient quantity of them into a beaker Place the beaker in a warm room, or apply heat from a laboratory burner or spirit lamp The ice disappears, and its place is taken by water. Notice the characteristics of the water. It has no definite shape, for by tilting the beaker the water can be made to flow.

EXPT. 41.—**Steam.** Replace the beaker over the burner and go on warming it. Soon the water boils and is converted into vapour, which spreads itself throughout the air in the room, and seems to disappear. The vapour, as it escapes out of the vessel, can be made visible only by blowing cold air on it, when it becomes white and visible owing to its condensation into small drops of water.

The **state**, or **physical condition**, of a substance depends largely upon its *temperature*, that is, its condition of hotness or coldness: thus, all metals can be melted by raising the temperature sufficiently, and at a still higher temperature they may be converted into gases On the other hand, by diminishing the temperature, a gas may be liquefied and, in most cases, even solidified. Iodine and camphor are typical examples of a few substances which, when heated, appear to pass suddenly from the solid condition to that of a gas, without assuming the intermediate liquid state· substances which do this are said to **sublime**

EXPT 42.—**Sublimation.** Warm a Florence flask by holding it above the flame of a Bunsen burner When it is too warm to bear the finger upon the bottom, introduce a crystal of iodine, and notice it is converted at once into a beautiful violet vapour.

Further subdivision of the states of matter.—A solid body requires a great force, or **stress**, to alter its size or shape; if its size or shape is altered, we say that the solid undergoes a **strain.**

A **perfect solid** is one which, when a stress is applied, undergoes a certain strain which remains constant so long as the stress is maintained, but the solid reassumes its original condition as soon as the stress is removed: a bent steel spring is an example of a perfect solid.

A **rigid solid** is one which does not change its shape in the least when acted upon by a stress No *perfectly rigid* solid is known, although several solids suggest this property of perfect rigidity. thus, a brick apparently is not strained when several other bricks are supported on it

A **soft solid** is one in which considerable change of shape results from a small stress applied to it. Jelly is an extreme case of a soft solid.

In the case of liquids, some flow very slowly and others flow rapidly; then we speak of the **viscosity**, or resistance to flow, as being greater or less. Thus, the viscosity of pitch is very high; that of treacle is less; it is still less in water, and least of all in ether. According as the viscosity is high or low, so the liquid is termed either **viscous** or **mobile.**

Hence, the states of matter may be classified thus :

1. **Solids** $\begin{cases} \textit{Rigid.} \\ \textit{Soft.} \end{cases}$ 2. **Liquids** $\begin{cases} \textit{Viscous.} \\ \textit{Mobile.} \end{cases}$ 3. **Gases.**

Constancy of mass in different states.—When a solid is converted into a liquid, or a liquid into a vapour, no change of mass is experienced. This has been found to be true in all cases, and the following experiments will illustrate the fact :

EXPT 43.—**Water to steam.** Boil water gently in a distilling flask, as in Fig. 43, and catch the condensed steam, taking care that none escapes, in another flask The water thus collected will be found to have the same mass as that boiled away.

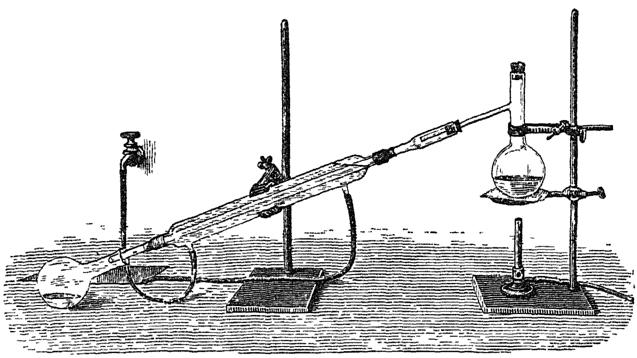


FIG 43 —Distillation of water

EXPT. 44.—**Ice to water.** Place a piece of ice in a flask suspended from one arm of a balance Counterpoise the flask with the ice in it ; then melt the ice by warming the flask, and show that the mass is unaltered.

EXPT. 45.—**Solution.** Put some warm water in a flask and some salt in a piece of paper. Counterpoise the flask of water and the paper of salt together, and then dissolve the salt in the water. The total mass remains unaltered.

Though we may change the form or condition of any portion of matter, we are not acquainted with any process by which it can be destroyed : in other words, matter is **indestructible.** Thus, when a candle burns, it ceases to exist as tallow or wax ; but, if we collect carefully the products of combustion (one is a liquid, and the other a gas), we find that these two things together weigh *more* than the part of the candle which has disappeared.

Molecular constitution of matter.—To account for many simple phenomena, *e.g* the intimate mixing of coal-gas with the air of a room into which the gas is allowed to escape, or the dissolving of salt in water, it is assumed that all forms of matter are not uniform throughout, but that they consist of very minute particles, called **molecules** It is assumed that the molecules of any substance are separated more or less from each other, and that in a mixture of two or more liquids, or in a solution of a solid in a liquid, the molecules of one substance occupy the interspaces between the molecules of the other.

Simple phenomena indicate that the molecules are in *continual and rapid movement.* The gradual evaporation of a liquid exposed to the air can be explained only by assuming that the molecules of the liquid are in more or less rapid motion, and that they escape at a definite rate from the bulk of the liquid and pass away into the surrounding space

In a gas contained within a glass flask, we have good reason to regard the molecules as moving rapidly and continuously, frequently colliding with each other, and bombarding the walls of the flask; indeed, the latter process causes the *pressure* which the gas exerts upon the surface enclosing it. We may imagine, too, that the effect of raising the temperature of the gas is to increase the rapidity of movement of the molecules and, therefore, the pressure which the gas exerts If the containing walls are elastic, the increased pressure brings about an increase in the space occupied by the gas, and this suggests the *expansion* (p 163) which the gas undergoes when its temperature is raised. The same phenomenon of expansion due to a rise of temperature is observed in the case both of liquids and of solids; we have reason, therefore, to suppose that the molecules of a solid, as well as those of a liquid and of a gas, are in a condition of movement

The difference between the molecular conditions of the three states of matter is found in the distance apart of the molecules. In the case of a gas this distance apart is great as compared with that in the case of a liquid; thus, when steam is condensed to water it shrinks to $\frac{1}{1600}$ of its original volume. To a smaller extent, the distances separating the molecules of a liquid are large in comparison with the distances in the case of a solid.

The size of the molecule of any substance is excessively small, and far smaller than can be observed by the most powerful microscope. The average diameter of a molecule is perhaps $\frac{1}{2500000}$ millimetre; and this is about six hundred times smaller than the smallest particle which can be rendered visible.

Properties of Matter.

The various forms of matter are found to exhibit some, or all, of the following properties: **divisibility, porosity, compressibility, elasticity, tenacity, malleability, hardness, cohesion.**

Divisibility.—A lump of sugar may be broken into several smaller lumps, and so possesses the property of **divisibility.** Is there no limit to this divisibility? Or, would the particles finally become so small that, with any further subdivision, they would cease to be sugar? Investigation has shown us that there *is* such a limit, and this is reached when each particle consists of *one* so-called molecule of sugar; but so fine a subdivision cannot be obtained by mechanical grinding, for the smallest speck of sugar, so obtainable, would still consist of many molecules attached together. The subdivision is, however, far more complete when the sugar is *dissolved* in water; and we may say that, in this condition, the molecules are separated from each other. We could subdivide each molecule still further by chemical means only: thus, we might *burn* the sugar; and the black carbon which is then seen proves that the molecule is complex, and that it contains carbon as one of its constituents.

Porosity.—The manner in which turbid water can be filtered through blotting-paper, and the slow passage of water through an unglazed earthenware bottle, indicate that these materials are full of **pores.** An unglazed brick is porous and permits fresh air to pass through the walls of a dwelling-house built of brick. It may be proved that metals, such as iron and lead, are porous: thus, Francis Bacon (in 1640) observed that when a sphere of lead filled with water is compressed by a great force, the water slowly oozes through the lead and appears on the outer surface.

We may even say that liquids indicate **porosity** For sugar, or salt, may be dissolved in water without appreciably increasing the bulk of the liquid. Another example is obtained in the following experiment:

Expt. 46.—**Porosity of liquids.** Half fill a barometer tube with water; then gently add alcohol until the tube is nearly full. Make

a mark on the tube at the level of the top of the liquid column, and afterwards shake the tube so as to mix the water and alcohol well together. Observe that the volume of the mixture has diminished, the reason being that some of each liquid has filled up pores between the particles of the other.

Compressibility.—This property follows as a natural consequence of porosity. If pores exist between the indivisible small particles of which matter is built up, it ought to be possible, by the adoption of suitable means, to make these particles go closer together.

This is well known to be the case in gases, and it is also true of solids and liquids. Hence, **compressibility** is not only a consequence of porosity, but actually a proof of its existence.

Elasticity.—Imagine a gas to have been made to assume one-half its size by compressing it What would happen if the pressure, which is the cause of the diminution, were removed suddenly? The gas would resume its original size or volume, and it would, so far as appearances are concerned, seem to have undergone no change. The gas is said to be perfectly **elastic**, and the property which enabled it to go back to its original state is called **elasticity**. Similar results follow with liquids, they also are perfectly elastic.

Some differences arise when solids come to be examined. Though the property can be developed in solids in at least four ways—by *pressure*, by *pulling*, by *bending*, and by *twisting*—we need consider the first only in this connection, as it is the elasticity which is developed by pressure which is most marked in all forms of matter Ivory, marble, and glass are examples of elastic solids, while putty, clays, fats, and even lead are instances of solids with scarcely any elasticity. In a scientific sense, glass is more perfectly elastic than india-rubber, because it returns to its original shape after it has been forced out of that shape, whereas india-rubber does not return to its original shape exactly.

A solid will resume its former dimensions only when the pressure is removed, provided that the pressure is within a certain limit. If the pressure be more than this minimum amount, or if it exceeds the **limit of elasticity**, as it is called, the solid will not return to the initial size; it will undergo a permanent change. As has been seen in Expt 35, this limit of elasticity is exceeded in the case of india-rubber only when the stress applied is very great.

EXPT. 47.—**Compression of a solid.** Procure a slab of polished marble or some similar material and smear it with oil. Drop a billiard ball or a large glass marble from a considerable height on the slab.

Catch it as it rebounds Notice that a blot of oil is found where the ball came into contact with the slab. Compare the size of the blot with the spot which is formed when the marble is placed in contact with the slab.

Evidently the ball underwent a compression as the result of collision with the slab, and, by virtue of its elasticity, it regained its original shape, and caused the rebound.

Tenacity.—Tenacity is measured by ascertaining what weight is necessary to break solids when in the form of wires.

EXPT. 48.—**Measure of tenacity.** Suspend a balance-pan from the lower end of a thin copper wire attached to a beam Add weights to the pan until the wire breaks. The force required to break the wire is the joint weight of the balance-pan and the weights in it. Repeat the experiment with wires of the same diameter but of different material

In making the measurement of tenacity, the area of the cross section of the wire must be estimated first carefully.*

It is found that a wire of twice the cross sectional area of another of the same material is just twice as tenacious. Evidently, then, if we wish to compare the tenacity of two wires of different materials, the experiment is made simpler if wires of the same cross section are selected Cast steel is the most tenacious of all metals, being about twice as much so as copper and forty times as tenacious as lead. But the tenacity of steel itself is exceeded by that of unspun silk, while single fibres of cotton can support millions of times their own weight without breaking.

The process of wire-drawing increases the tenacity of the wire, hence, a wire cable is stronger than a chain of the same length and weight. The tenacity of wood *along* the fibre is greater than *across* the fibre.

Ductility and malleability.—Several tenacious metals indicate a certain degree of **fluidity**, by which the molecules may alter their relative positions without losing their cohesion. A metal possesses **ductility** if a short thick cylinder can be drawn out into a long thin wire.

Malleability is a property similar to ductility, but the change of form is brought about by the application of pressure; gold, copper, and lead, for instance, can be beaten out into thin plates, and are therefore malleable substances. Lead is an example of a malleable material which is not ductile—it can be beaten out

* Area of cross section $=\frac{22}{7}\times\frac{d^2}{4}$

but cannot be drawn into wires. Platinum is the most ductile and gold the most malleable metal known. Platinum has been drawn out into wire so fine that a mile of it weighs only one and a quarter grains. Gold has been beaten into plates so thin that it would require three hundred thousand of them placed one above the other to make a layer an inch thick. Antimony is an example of a metal which possesses no malleability.

Hardness.—*Hardness is the property by virtue of which solids offer resistance to being scratched or worn by others.* This property is of great importance in the study of minerals, as it often affords a ready means of distinguishing them. The method of measuring hardness consists in selecting a series of solids, each one of the series being harder than the one above it, and softer than the one below it. At one end of the series, therefore, the hardest solid known is placed; at the other end, the softest which we may wish to measure.

Cohesion.—The term cohesion refers to the force of attraction which exists between molecules of the *same* nature. The phenomenon called into play by the attraction of molecules of different nature, *e.g.* between molecules of glass and molecules of water, is termed **adhesion.** The property of cohesion is well-marked in solids, is less so in liquids, and is absent in gases. The process of welding two pieces of red-hot iron is an application of the phenomenon of cohesion, so also, the process of making the black lead of an ordinary pencil from finely powdered graphite. The force of cohesion is seen in a rain-drop also, or in the formation of a drop of any liquid. The different degrees of tenacity, ductility, and hardness exhibited by different substances depend upon the characteristic differences in cohesion.

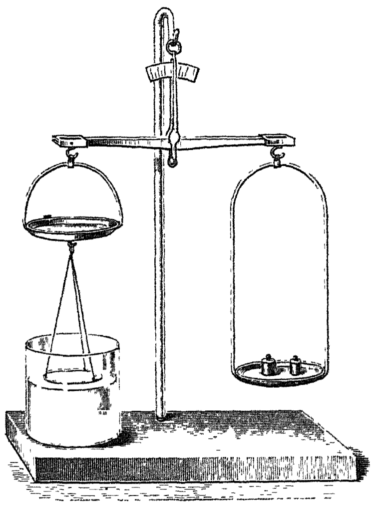


FIG 44 —Measurement of cohesion of a liquid

Fig. 44 represents a method of demonstrating the properties of cohesion and adhesion. A glass plate is suspended horizontally

from the beam of a balance, and the plate is counterpoised by means of weights placed on the pan attached to the other end of the beam. A dish containing water is placed just below the plate at such a distance that when the plate is lowered slightly below its normal position it touches the surface of the water. It is found necessary to increase considerably the weight on the pan in order to separate the plate from the water. As the lower surface of the plate is still wetted it is evident that the force of adhesion between glass and water is greater than the force of cohesion of water; and the amount by which the weight was increased is a measure of this cohesion.

Surface tension.—These molecular forces, evident in the property of cohesion, cause peculiar phenomena to appear in the layer of molecules which form the free surface of a liquid. Thus, in the case of any molecule, such as A (Fig. 45), situated well within the mass of a liquid, the forces of attraction due to neighbouring molecules are distributed uniformly in all directions, whereas any molecule, such as B, situated near the surface is acted upon by forces of attraction which are mainly acting downwards and away from the surface. This results in the surface exhibiting a kind of tension, which has the effect of making the area of free surface as small as possible.

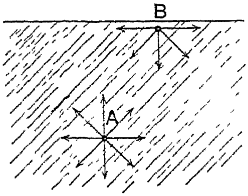


FIG 45—Forces acting on a molecule, A, within a liquid and on one, B, near the surface

Since the sphere is the geometrical figure which has the smallest surface area for a given volume, any mass of liquid would assume this form—providing that internal molecular forces only are acting upon the liquid. Since, except under special conditions, the external force of gravitation is acting upon the liquid always, the form of the free surface is seldom spherical; and the effect due to surface tension alone can be observed only when all external forces are neutralised or eliminated. In the case of very small masses of liquids the force of gravitation is negligibly small compared with the force of cohesion; and in such cases the spherical form of the mass is frequently apparent. Thus, the small globules of condensed moisture which constitute cloud, and

even falling rain-drops, are truly spherical, also, if a small quantity of mercury is dropped upon a horizontal board or sheet of paper, or if water is dropped upon a board sprinkled with powdered resin, the liquid breaks up into drops which have a more or less spherical form.

The same effect can be observed by means of olive oil and a mixture of alcohol and water, the specific gravity of the mixture being adjusted so as to be identical with that of the oil A tall glass vessel, with flat sides, is nearly filled with the alcohol mixed with water; a pipette filled with the oil is immersed into the mixture, and a small quantity of the oil is allowed to escape from the open end of the pipette. The oil will remain floating within the mixture, and will assume a truly spherical form.

The phenomenon of surface tension and the tendency to spherical form are evident in a soap-bubble. It is easy to observe that when the tube, to which the bubble is attached, is allowed to remain open the bubble slowly diminishes in size

Capillarity.—If a flat plate of solid material be partly immersed in a liquid vertically, the liquid surface seldom retains its horizontal form in the region quite near where it touches the solid. If the liquid *wets* the solid, as when glass is dipped into water, the surface in this region is concave upwards, but if the liquid does not wet the solid, as when glass is dipped into mercury, the surface in this region is convex upwards.

These effects are due simply to the relative magnitudes of the force of adhesion (between the solid and the liquid) and the force of cohesion in the liquid. When the liquid is water, the former force is greater than the latter, and *vice versâ* when the liquid is mercury.

The phenomena of capillarity are more apparent when glass tubes of narrow bore are used instead of glass plates. When such tubes are supported vertically with their lower ends immersed in water, the liquid rises up the tube to a height which depends upon the diameter of the bore—the narrower the bore the higher the water column (Fig. 46) If the diameter of one tube be one-half that of another, the total upward force exerted by the surface tension is reduced to one-half, since the circumference of the liquid surface is reduced by one-half. But the weight of the

liquid column is only one-fourth of that of a column of the same height in the wider tube, since the volume of liquid varies inversely as the square of the diameter Hence, in the narrower tube, the height of the liquid column is twice that obtained in the wider tube

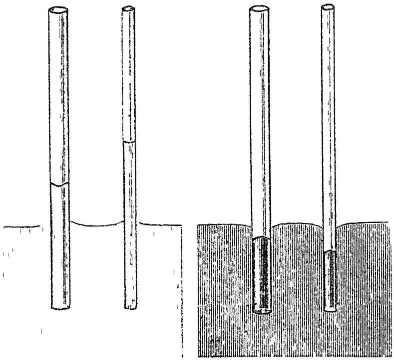


FIG 46 —Capillary elevation and depression.

When such tubes are dipped into mercury, the liquid surface within the tube assumes a convex form and is depressed below the level of the liquid outside the tube.

Tubes of small internal diameter are termed **capillary tubes**; and the phenomenon observed when such tubes dip into a liquid is termed **capillarity**. To capillarity is due the property by which oil can pass upwards between the threads of a lamp-wick. In the same way, water may be siphoned out of a vessel by allowing a towel to hang over the edge with one end dipping into the water.

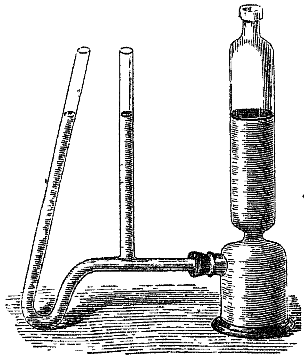


FIG. 47.—Equality of level of liquid in communicating vessels.

Distinctive characters of liquids. —Same level in communicating vessels If several vessels of varied shapes (Fig. 47) are in communication with one another, and water be poured into any one of them it is found that as soon as the water has come to rest it will stand at the same level in all the tubes, however different the form of the vessels may be.

The following simple experiments are useful in showing that the surface of a liquid at rest is horizontal and that pressure upon a liquid is communicated equally in all directions:

EXPT. 49.—**Horizontal surface.** Into a shallow glass vessel pour enough mercury to cover the bottom. Attach a ball of lead to the end of a fine string, and so construct a **plumb-line.** Hang it over the surface of the mercury, and notice that the line itself and its reflection are in one and the same line. If this were not the case, that is, if the image slanted away from the plumb-line itself, we should know the surface of the liquid was not horizontal.

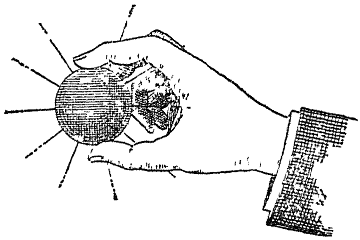


FIG. 48.—Communication of pressure by liquid.

EXPT. 50.—**Communicate pressure equally in all directions.** Bore a hole in a hollow rubber ball and fill it with water. Cover the hole with a finger, and prick several small holes in the ball with a needle. When the ball is squeezed, the water spurts out of each hole straight from the centre of the ball, thus showing that the pressure has been transmitted equally in all directions.

The transmission of pressure by liquids is made use of in the **Hydraulic Press,** which consists essentially of two cylinders in connection, with pistons fitted into them, one much larger than the other. The application of a comparatively small force to the small piston is felt on the larger, and it is as many times greater in an upward direction as the piston of B is larger in area than the piston in A. This great upward force is being used in the instance shown in Fig. 49 to compress bales of wool.

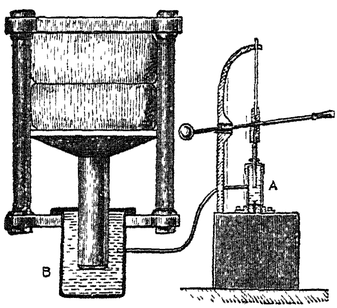


FIG. 49.—Hydraulic Press.

Distinctive characters of gases.—It appears, therefore, that solids tend to retain a definite *size* and *shape*, while liquids tend to retain a definite *size* only. **Gases show no tendency to retain either a definite size or shape.** A small quantity of any gas, when admitted into a large empty vessel, almost instantaneously distributes itself uniformly throughout the interior of a vessel.

Gases are **easily compressible**; and in this they obey a definite law, viz.: that, when the temperature remains constant, the volume increases or decreases in just the same proportion as the pressure decreases or increases (p 85).

EXERCISES ON CHAPTER V.

1. Describe experiments which prove
   (*a*) That solids are porous
   (*b*) That liquids, too, have pores

2. How would you show by experiment that the mass of the same portion of matter in different states is constant?

3. What experiment could you perform to show that a solid, say a billiard ball, is elastic? Explain as well as you can what you mean by elasticity

4. What property in particular is possessed by liquids and not by solids? And what character has a gas which neither liquids nor solids possess?

5. The same portion of matter can, under suitable conditions, assume different states Describe fully some experiment which illustrates this statement.

6. What evidence can you give that the different states of matter gradually shade into one another?

7. Give reasons for the opinion that all forms of matter are molecular in structure.

8. State some of the properties which are possessed by all kinds of matter, and explain in your own words the meaning of a property of matter.

9. Two narrow glass tubes—one twice the diameter of the other—are dipped into water and mercury in succession. Describe and sketch the effect observed in each case.

CHAPTER VI.

PRESSURE IN LIQUIDS. RELATIVE DENSITY.

Measurement of pressure.—Fig. 50 represents a glass U-tube, open at both ends, and containing mercury. The mercury surfaces are at the same level, since the atmosphere is pressing on both surfaces to an equal extent. If the mouth be applied to the open end of a rubber tube R joined to one limb of the glass tube, and if then air be forced down the tube, the pressure at A increases, the mercury surfaces are no longer at the same level—that at A being depressed, and that at B raised; and the difference in level thus produced is dependent upon the air forced into the tube. On the other hand, if some of the air be removed from the tube by suction,* the pressure at A diminishes: the mercury surface at A rises, and that at B falls. Hence *the position of the mercury in the tube indicates any difference in the pressures acting on the two surfaces.*

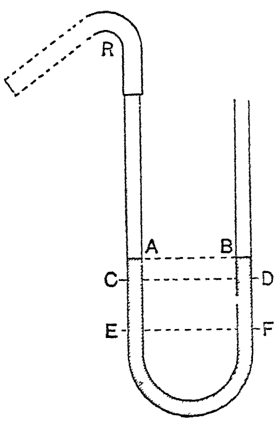


FIG. 50.—Pressure in a liquid

Let the mercury return to its original position in the tube, and consider the pressures at two points, C and D, each 1 cm. below the surface of the mercury; and, for simplicity, let the cross-section of the tube be 1 sq. cm. Neglecting the pressure of

* The U-tube should be long enough to prevent the mercury from rising up to the top and entering the mouth.

the atmosphere—which, of course, is the same on both sides of the tube—the pressure acting at both C and D is equal to the weight of 1 c.c. of mercury (*i.e.* 13·6 grams); or, *the pressure acting at either point is equal to* **13·6 grams per sq. cm.** Similarly, at any other pair of points at the same level, *e.g.* E and F, the pressures are equal. From such considerations as these we deduce the following rule: **In any U-tube containing a liquid the pressures at the same horizontal levels are the same.**

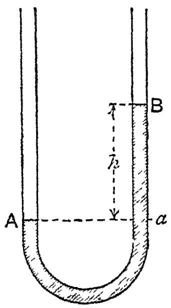


FIG. 51.—Principle of a manometer.

Suppose that the pressure applied to the surface A (Fig. 51) is increased sufficiently to depress it to the position shown. The pressure at A is equal to that at a point *a*—which is at the same horizontal level as A. If the difference of level between the two mercury surfaces be h cm., the pressure on each sq. cm. at *a* is equal to $h \times 13·6$ *gm. per sq. cm.*; and the pressure at A is equal to the same amount. This simple device for measuring the pressure of a gas is termed a **mercury manometer**. If any liquid other than mercury is used, the pressure is equal to $h \times d$ *gm. per sq. cm.*, where d is the density of the liquid.

A mercury manometer may be used for investigating the pressure beneath the free surface of a liquid. In Fig. 52 A is a long glass tube, about 1 m. long and 4 cm. in diameter, closed at the lower end with a rubber stopper, and filled with water. B is a narrow glass tube bent at the upper end, and connected by thick rubber tubing to a manometer M.

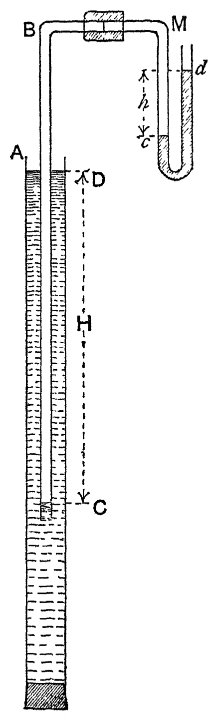


FIG. 52.—Apparatus for measuring pressure in liquids.

When the open end of the tube B is immersed in the water, the pressure exerted by the water at its surface C is transmitted

through the air in the tube to the mercury surface c, and this pressure is measured by observing the difference of level in the manometer. A series of observations will prove that **the pressure is proportional to the depth below the free surface.**

EXPT. 51 —**Pressures of liquid columns** Fit up the apparatus shown in Fig 52, and measure (i) the depth H of the surface C below the surface D, and (ii) the difference of level h in the manometer Take a series of readings for different positions of the tube B. Tabulate the readings in column, and calculate the ratio H/h for each reading

Plot the readings on squared paper, taking values of H as abscissae, and values of h as ordinates.

The magnitude of the liquid pressure at any point below the surface of a liquid may be derived in the following manner Let A (Fig. 53) be a small horizontal surface, 1 sq. cm. in area, and situated h cm. below the liquid's surface The vertical downward pressure on A is equal to the weight of a vertical column of the liquid h cm long and 1 sq cm cross-section, if the density of the liquid be d gm. per c.c this pressure is hd gm. per sq cm.

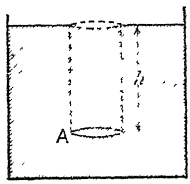


FIG 53 —Pressure of liquid column

A further point of great importance is that, at any point, there is an equal pressure *acting upwards*, as well as *downwards*. For if the pressure acting downwards was the only pressure acting on the surface A, the thin layer of liquid represented by A would not remain stationary, but would be thrust downwards in obedience to this force. Common experience tells us that such a layer of water is *not* in this state of constant movement; hence, at any point within a liquid, **there is always an upward pressure equal in magnitude to the downward pressure at the same point**

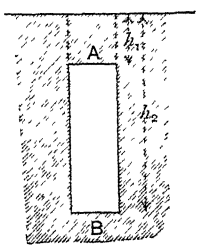


FIG 54 —Upward pressure upon an object in a liquid

Apparent loss of weight of a solid when suspended in a liquid.—Let AB (Fig. 54) represent a metal cylinder, of length l cm. and cross-section a sq. cm., immersed vertically in a liquid of density d gm. per c.c. If the upper and

lower surfaces are immersed to depths h_1 cm and h_2 cm. respectively, the downward pressure on the surface A is equal to the weight of ah_1 c c of the liquid, *i.e* to ah_1d gm ; similarly, the upward pressure on the surface B is equal to ah_2d gm Hence, the resultant *upward* pressure due to the liquid is equal to $ah_2d - ah_1d$ gm But $a(h_2 - h_1)$ is the volume of the cylinder; hence, **the upward pressure due to the liquid is equal to the weight of a quantity of the liquid equal in volume to that of the solid immersed.** This is known as the **Principle of Archimedes**· which may be stated as follows **When a solid is immersed in a liquid, the apparent loss of weight is equal to the weight of an equal volume of the liquid.**

EXPT. 52—**Upthrust.** Measure the dimensions of a metal cube, and calculate its volume Suspend the cube, by means of cotton thread, from the hook of a balance, and at such a height that the cube may afterwards hang freely in a beaker of liquid supported on a bridge placed over the pan. Use as little cotton as possible: a single strand with a loop at the top should suffice Weigh the suspended cube. Now place a beaker of cold water on the bridge so that the cube is immersed totally, taking care that it does not touch the sides of the beaker, and again weigh the cube.

Repeat the above experiment several times, using regular solids of different form and different liquids of known density Tabulate your results in the following manner, and deduce your own conclusions as to the relationship between the apparent loss of weight of the solid and the weight of an equal volume of the liquid used.

Liquid, and its density (d)	Dimensions of solid	Volume of solid	Weight of equal volume of liquid	Apparent loss of weight

Relative density (or, **specific gravity**).—It has already been proved by experiment that equal volumes of different materials have different masses, or, in other words, different materials have different densities. Such experiments show that some substances are heavier and others lighter than an equal volume of water. The **relative density** of a material is **the number of times that a fragment of the material is heavier or lighter than an equal volume of cold water.**

It is important to remember that the relative density is the ratio of one mass to another mass, and the ratio is simply a

numerical quantity. On the other hand, the absolute density of a material is not described fully unless the units of mass and volume are given. Thus, it is correct to say that the **relative density of marble is 2·8;** but the **absolute density of marble is 2·8 gm. per c.c.**

RELATIVE DENSITIES OF SOLIDS.

To determine the relative density of a solid heavier than water we require to find (i) the weight of the body of which the relative density is required, and (ii) the weight of an equal volume of water. The second quantity is obtained, by the Principle of Archimedes, by observing the apparent loss of weight when the solid is suspended in cold water. Then

$$\textbf{relative density} = \frac{\text{weight of substance}}{\text{weight of equal vol. of water}}$$

$$= \frac{\textbf{weight of substance}}{\textbf{apparent loss of weight in water}}.$$

EXPT. 53.—**Solids heavier than water** Find the relative density of two or three common solids, such as brass, sulphur, copper, lead, and glass.

In applying the Principle of Archimedes to the determination of the relative density of a solid lighter than water it is necessary to attach to the solid an insoluble sinker sufficiently heavy to keep it in the water. Three separate weighings are necessary, viz.: (*a*) the solid, in air, (*b*) the *sinker* alone, in water, and (*c*) the sinker, with the solid attached, in water. It is best perhaps to take the weighings in the order shown in Fig. 55. In Fig. 55 (i) the sinker alone is weighed in water; let this weight be w_1. In Fig. 55 (ii) the solid is supported just below the hook of the balance by looping the thread once round the solid; let this

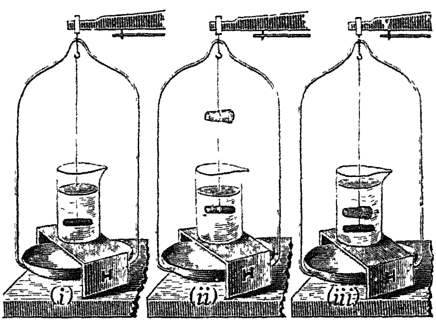


FIG 55.—Determination of the relative density of a solid lighter than water.

weight be w_2. In Fig. 55 (iii) the solid is rotated downwards until it is close to the sinker and immersed in the water, and another weighing w_3 taken. The only difference between the second and third weighings is *the weight of the water displaced by the light solid.* Hence

$$\text{relative density} = \frac{w'_2 - w'_1}{w_2 - w_3}.$$

EXPT. 54 —**Solids lighter than water.** Determine the relative density of paraffin wax, of oak, and of cork.

If a solid of less density than water be immersed totally in water, then, since the upward thrust due to the displaced water is greater than the weight of the solid, it must tend to rise to the surface. When it reaches the surface, the downward pressure on the top of the solid becomes zero, but the upward pressure on the under surface continues to act, the body continues to rise until this upward pressure is equal to the weight of the solid, *i.e.* until **the weight of the floating body is equal to the weight of the displaced water.**

Suppose a rectangular block of wood, of area A sq. cm. and height H cm., to be floating in water, and immersed to a depth h cm.;

then, total volume of wood $= AH$ c.c

and " " displaced water $= Ah$ c c.

Hence, weight of AH c c. of wood = weight of Ah c c. of water,

or, " " 1 c c " = " $\frac{h}{H}$ " "

$= \frac{h}{H} \times$ weight of 1 c c. of water.

Therefore, $\text{relative density} = \dfrac{\text{weight of 1 c.c of wood}}{\text{weight of 1 c c. of water}} = \dfrac{h}{H}.$

EXPT 55 —**Relative density by flotation** Fig. 56 represents a method of making the necessary measurements. Let a wooden strip rest across the top of the wide glass beaker in which a block of wood is floating. Fix with wax a needle to one end of a millimetre scale Take the scale reading of the wooden strip when the needle is just touching the surface of the water; and also the reading when the needle is touching the surface of the block. As the block may not be floating quite horizontally, find the average of four readings taken at opposite corners. From these observations calculate the relative density of the wood.

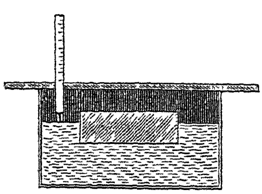


FIG 56 —Expt. 55.

RELATIVE DENSITIES OF LIQUIDS.

To determine the relative density of a liquid it is necessary to find the weight of any volume of the given liquid and the weight of an equal volume of cold water. Several methods are available.

EXPT. 56 —**By the principle of Archimedes** Weigh a glass stopper in air, then immerse it successively in water, turpentine, methylated spirit, and olive oil, and notice the apparent loss of weight in each case. The apparent loss of weight experienced by the glass stopper in each experiment is equal to the weight of a portion of liquid of the same volume as the stopper. The numbers obtained therefore represent the weights of equal volumes of water, turpentine, methylated spirit, and olive oil, and by dividing each by the number obtained in the case of water, the relative densities of the liquids are obtained.

The *relative density bottle* (Fig 57) consists of a small glass flask, holding about 50 grams of water. It is provided with a nicely-fitting ground stopper, which is in the form of a tube with a very small bore through it, or the stopper may have a groove on its side Another form of specific gravity bottle has a stopper in the form of a narrow tube or neck with a mark upon it.

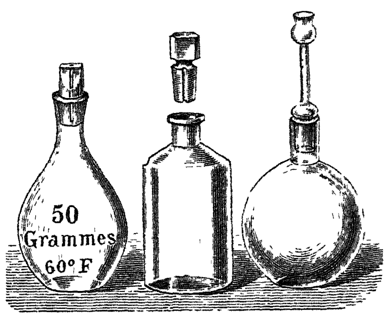


FIG 57 —Relative density bottles.

EXPT. 57.—**By the relative density bottle** Carefully dry the inside of the bottle, and weigh it. Fill the bottle with the given liquid, insert the stopper, and see that no air bubbles remain in the bottle. Dry the outside of the bottle (holding it in several folds of a duster so as not to warm the bottle by direct contact with the hand), and weigh it. Rinse out the bottle several times with water, and finally weigh it when full of cold water. In this manner you obtain the weights of *equal* volumes of the liquid and of water.

Instead of the bottle, a small flask having a file mark on the neck may be used.

An instructive method of showing the relative densities of liquids is obtained by means of a glass tube bent in the form of a U, and therefore called a U-tube.

EXPT. 58.—**By means of a U-tube.** Cut off two pieces of glass tube, each about 30 cm. long; connect the tubes with india-rubber tubing about 18 cm long, and fix them upright upon a strip of wood Or, bend a piece of tubing into the form of a U with arms about 30 cm.

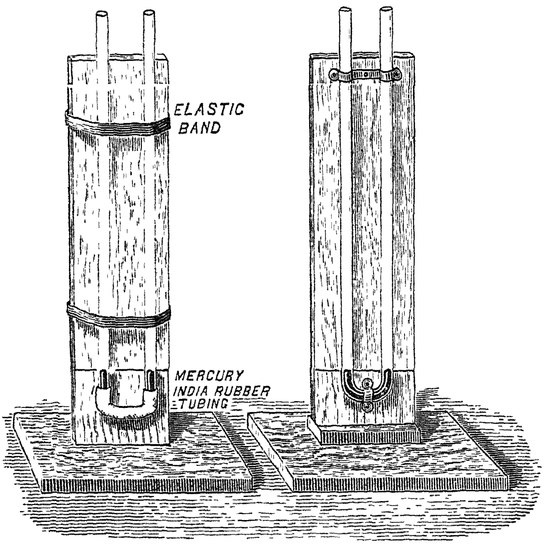


FIG. 58 —U-tubes for determining relative densities of liquids.

long. Pour mercury into one of the tubes until it reaches a horizontal line drawn upon the board (Fig. 58). Now introduce water into one of the tubes by means of a pipette, and notice that the mercury on which the water rests is pushed down; afterwards introduce sufficient water into the other tube to bring the mercury back to its original level. The length of each column of water will be found the same. Repeat the experiment with varying amounts of water.

The mercury in the bend of the U-tube evidently acts as a balance, which enables columns of different liquids in the upright arms to be balanced.

EXPT. 59.—**Spirit and water.** Nearly fill one of the tubes with methylated spirit, and balance it with water introduced into the other tube. Measure the lengths of the two columns.

As these two unequal columns balance one another it will be evident that the liquid in the shorter column, namely, the water, has a greater relative density than the liquid in the longer column.

If h_1 = height of liquid column, and h_2 = height of water column, then

Wt of h_1 cm of the liquid = wt. of h_2 cm of water, or

„ 1 cm. „ = „ h_2/h_1 „ or

= h_2/h_1 × wt of 1 cm. length of water.

Hence, any volume of the liquid is $\frac{h_2}{h_1}$ times as heavy as an equal volume of water, and

$$\text{Relative density} = \frac{h_2}{h_1}$$

To determine the relative densities of liquids which mix, an arrangement known as Hare's apparatus may be used. A simple form of this apparatus is represented in Fig 59. A wide-necked bottle is closed with a rubber stopper bored with three holes. Through two of the holes pass long glass tubes, and the third hole is fitted with a short tube terminating in rubber tubing and a clip. The lower ends of the long tubes dip into beakers containing the liquids the densities of which have to be compared. By applying suction to the rubber tubing, the liquids are drawn up the glass tubes to different heights, the *less* dense being drawn to the *greater* height. The pressure on the surface of the liquids in the beakers is the same, viz. that due to the atmosphere, hence the pressures *inside* the tubes and at these same levels must be equal. Therefore the weights of the liquid columns *measured above the level of the liquids in the beakers* must be equal; just as in the case of liquids in a U-tube, the densities of the liquids are inversely proportional to the heights of the columns.

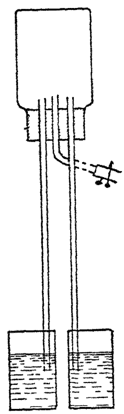


FIG. 59.—Simple form of Hare's apparatus

EXPT 60.—**By means of Hare's apparatus.** Using this apparatus, determine the relative densities of brine, solution of copper sulphate, vinegar, and milk.

HYDROMETERS.

A simple form of hydrometer may be made in the following manner: Cut off a piece of thin-walled glass tubing, about 15 cm.

long and 2 cm. diameter. Close one end with a cork; cut off the cork flush with the edge of the tubing, and make the end water-tight by means of sealing-wax. Fix inside the tube a paper millimetre scale, adjusted so as to measure distances from the extreme end of the tube. Put inside the tube sufficient small lead shot to make the tube sink vertically in water to a depth of about 12 cm, and fix the shot in position with wax or a slice of cork. Close the upper end of the tube with a short cork, and make water-tight as before.

EXPT. 61 —**Principle of a hydrometer.** Float a hydrometer, of the form described, in water Note the depth of immersion Remove the hydrometer, dry it, and weigh it Measure the diameter in several different parts, and, taking the average of these readings, calculate the volume of water displaced. Find whether there is any relationship between the weight of the hydrometer and the weight of the displaced water.

When placed in any liquid **the hydrometer will sink until the weight of the displaced liquid is equal to the weight of the hydrometer.** Hence, in a less dense liquid, the hydrometer will sink to a greater depth. If a be the cross-section of the tube, and if the tube sinks in water and in another liquid to depths H cm and h cm. respectively, the volumes of displaced water and liquid are aH c.c. and ah c.c respectively. Hence

Weight of ah c c. of liquid = wt of aH c c of water,

or " 1 " " = " H/h " "

or " 1 " " $= \frac{H}{h} \times$ wt. of 1 c.c of water.

Therefore *relative density* of liquid $= \frac{H}{h}$.

EXPT. 62.—**Brine and alcohol.** By means of a simple hydrometer find the relative density of weak brine and of dilute alcohol.

EXPT. 63.—**Verification.** If a commercial hydrometer (*e.g* a Twaddell*) is available, verify by its means the results of the previous exercise.

Nicholson's hydrometer.—The hydrometers hitherto used are known as **hydrometers of variable immersion.** In another kind of hydrometer, known as **Nicholson's hydrometer**, the instrument is

* The rule for determining a relative density by means of a Twaddell hydrometer is, "Multiply the hydrometer reading by 5, add 1000, and divide by 1000."

always immersed to a fixed mark upon it, and densities are determined by finding the weights necessary to produce this amount of immersion in different cases The densities of solids as well as of liquids can be determined with this instrument

EXPT. 64—**Solids.** Place a Nicholson's hydrometer (Fig. 60) in a tall jar of water* Load the top pan A until the mark on the stem of the hydrometer is level with the surface of the water. Record the load; then remove it and put in its place a pebble which weighs less, or some other substance of which the density is required. Find the weights which have to be added to depress the hydrometer to the mark on the stem. Then you have

Weight of substance (a) + added weight (b) = weight required to sink hydrometer to mark (c).

Therefore weight of substance $a = c - b$.

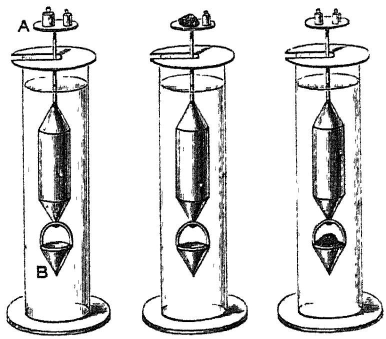


FIG 60.—Use of a Nicholson's hydrometer to determine the relative density of a solid

Now put the substance in the lower pan B, and add weights until the mark is again reached. The apparent loss of weight in water is the difference between the weights now added and those required to depress to the mark when the substance was in the top pan You have thus the weight of the substance and the apparent loss of weight in water, and can therefore determine the relative density of the substance without a balance.

Determine the relative density of sulphur and of lead by means of Nicholson's hydrometer

EXPT. 65—**Liquids** Weigh the Nicholson's hydrometer, and then place it in a jar of water. Add weights until the mark on the stem is level with the water. Then

Weight of hydrometer + weight added = weight of a certain volume of water displaced.

* If the hydrometer does not float in an upright position, place a small piece of lead on the lower pan and keep it in that position throughout the experiment. The lead must not be heavy enough to sink the hydrometer nearly to the fixed mark

Repeat the experiment with one or two other liquids The same amount of liquid is displaced in each case, so you obtain the weights of equal volumes of liquids compared with that of an equal volume of water. Determine from your observations the relative densities of the liquids.

EXERCISES ON CHAPTER VI.

1. Find the pressure due (i) to a column of water 1 metre in depth, (ii) to a column of mercury (density 13·6 gm. per c.c) 1 metre in depth.

Express your answer in grams weight per sq. cm.

2.* Calculate the total pressure (in grams weight) upon the base of a cylindrical vessel one decimetre in diameter, filled with mercury to a height of 40 cm.

3.* The specific gravity of sea-water is 1 025 Calculate the pressure (i) in grams weight per sq. cm. at a depth of 40 metres below the surface of the sea, and (ii) in pounds per sq ft., at a depth of 50 ft.

4.* To what depth must a surface be sunk in water in order that the pressure upon it may be 60 lb. per sq. inch?

5. In order to determine the pressure of the water supply at a certain tap, it is connected to a mercury manometer. On opening the tap the mercury rises in the open end of the manometer until the difference of level of the mercury surfaces is 110 cm Express the 'head of water' in feet.

6. Explain why a ship made of iron will float in water, though iron itself is heavier, bulk for bulk, than water.

7. You are given a small rectangular block of brass, and you have at your disposal a measuring rod divided into centimetres and millimetres, a balance and weights, some fine wire, and a vessel of water. How will you determine, by two perfectly independent methods, so that the results may form a check on one another, the volume of the block? Describe exactly what calculations you will have to make, and say upon what scientific principle, if any, each of your methods depends.

8. A piece of lead weighs 150 grams in air, 137 grams in water, and 138·5 grams in paraffin Find the specific gravity of lead and of paraffin—to two places of decimals.

9. In an experiment to find the specific gravity of copper sulphate by means of Hare's apparatus the following measurements were taken.

Height of water column in cm .

1st time.	2nd time.	3rd time.	4th time	5th time
10	12	14	16	20

* In those examples marked with an asterisk the pressure of the air on the liquid surface is disregarded

Height of copper sulphate column in cm :

1st time	2nd time	3rd time.	4th time	5th time.
9·5	11	13·5	15	18·8

Find the average specific gravity of copper sulphate correct to two places of decimals, and draw up a neat table of measurements to show how you obtain your result

10. A block of wood having a volume of 3 cubic feet weighs 93 lb. Find (*a*) the specific gravity of the wood ; (*b*) the weight that must be placed on the block so as just to keep the wood under water. (The weight of 1 cubic foot of water is to be taken as equal to 62 lb.)

11. (i) Why does a solid appear to weigh less in water than in air?
(ii) Describe any experiment which shows that the apparent loss of weight (expressed in grams) in water is equal numerically to the volume (expressed in c.c.) of water displaced
(iii) A piece of metal weighs 100 grams in air and 88 grams in water. What is its volume?

12. Being provided with two pieces of glass tube and a piece of india-rubber tubing, explain how you would proceed to (i) compare the relative densities of olive oil and spirits of wine, (ii) ascertain the relative density of a specimen of milk.

13. If you were provided with a burette, paraffin oil, and pieces of ice, describe how with these things you would determine the relative density of ice

A piece of ice has a volume of 1000 cubic feet. How many cubic feet would be above the water if the ice floats in (*a*) pure water, (*b*) sea water, assuming that none of the ice melts? (The relative density of sea water is 1·025 and of ice 0·918)

14. Describe a practical means of finding (*a*) the weight of one cubic inch of water, (*b*) the number of cubic inches in one pint.

A bar of wrought iron 1 inch wide, 1 inch thick, and 1 yard long, has the same weight as a gallon of water. Find the relative density of the iron. (There are 277·3 cubic inches in a gallon.)

15. If you were asked to determine the specific gravity of a small irregular piece of wood, describe exactly the method you would use.

A beam of oak measures 24 feet long, 18 inches wide, and 1 foot deep. Estimate the weight of the beam in lb, taking as the specific gravity of oak a value which you consider to be roughly correct and stating clearly the value which you take. (The weight of one cubic foot of water is 62·4 lb.)

16. How would you determine by measurement and calculation the volume in cubic centimetres of a rectangular slab of glass, such as that found in a box of weights? Describe an experiment by which you could check your result.

17. Explain how, using a U-tube, you would determine the relative density of a liquid which does not mix with water

18. Under what conditions do bodies float or sink in a liquid?

A piece of iron weighing 275 gm floats in mercury of density 13·59 with $\frac{5}{9}$ of its volume immersed. Determine the volume and density of the iron.

19. Two blocks of glass, each having a volume of 10 c c, are hung from the scale-pans of a balance by means of hooks under the pans, and balance one another Under one is brought a beaker of water, under the other a beaker of alcohol, so that the blocks are immersed in the liquids. The balance is now disturbed, and it is found that 1·82 grams have to be added to one pan to restore equilibrium. To which pan has this weight to be added, what is the explanation of the fact, and how can you determine from the figures now at your disposal the density of the alcohol?

20. A covered tin canister having a volume of 88 cubic centimetres contains just enough shot to sink it to the top of the cover when placed in cold water. Determine from this information

(i) The weight of the canister and shot.

(ii) The weight of the water displaced by the canister.

21. Explain what is meant by specific gravity A body of specific gravity 5 weighs 20 grams in air; what will the body weigh when immersed in water?

22. A bottle weighs 2 ounces When holding $3\frac{1}{2}$ ounces of shot it will just float in water, when holding 3 ounces it will just float in oil, and when holding $3\frac{3}{4}$ it will just float in brine Find the specific gravity of the oil and the brine.

23. A stone, weighing in air one kilogram, is suspended by a piece of cotton so that it is immersed entirely in water On attempting to lift the stone out of the water the cotton breaks when the stone is partly out of water. Why is this?

If when the stone is immersed completely the cotton would bear an additional pull equal to the weight of 150 grams, what volume of the stone will be out of the water when the cotton breaks?

24. A hollow stopper of glass (density, 2·6 gm. per c c.) is found to weigh 23·4 gm. in air, and 3·9 gm. when suspended in water. What is the volume of the internal cavity?

25. A solid weighs 8 lb. in air and 5 lb in water Find (i) the weight of an equal volume of water, (ii) the relative density of the body, (iii) the volume (in cubic inches) of the body, (iv) the apparent weight of the body when suspended in glycerin (relative density = 1·25).

26. The relative density of ice is 0·918 and that of sea-water is 1·03 What is the total volume of an iceberg which floats with 700 cubic yards exposed?

27. A specific-gravity bottle, filled with water, weighed 39·74 gm. Some iron nails weighing 8·5 gm. were introduced, and the bottle filled with water. The bottle and contents now weighed 47·12 gm. Find the relative density of iron.

28. In order to sink a Nicholson hydrometer to the mark in water, it was necessary to add 60·3 gm. to the upper pan. When floating in alcohol only 6·8 gm. were required. If the hydrometer weighs 200 gm., what is the relative density of the alcohol?

29. How would you determine the volume of an ordinary pen nib? State all the precautions you would adopt in order to obtain an accurate result.

30. 1 c.c. of lead (sp. gr., 11·4) and 21 c.c. of wood (sp. gr., 0·5) are fixed together. Show whether they will float or sink in water.

CHAPTER VII.

ATMOSPHERIC PRESSURE, AND BOYLE'S LAW.

Weight of the air.—Surrounding the earth in every latitude, over land and sea, is a gaseous envelope which is spoken of as the air or the atmosphere. Its presence when at rest is unperceived, though in motion it becomes apparent by its effects on trees and other bodies free to move. It is easy to prove by direct experiment that the air has weight.

EXPT. 66.—**Determination of weight of air.** Fit a one-holed india-rubber stopper into a fairly large glass flask, and fit into the stopper a short glass tube with rubber tubing and clip (Fig. 61). Put a little water in the flask, open the stop-cock; and boil the water. After boiling for some minutes, close the clip and place the flask on one side to cool. When the flask is cool, weigh it. Then open the clip; air will be heard to rush into the flask, and as it does so the balance will show an increase of weight. Carefully re-weigh the flask. The increase in weight is equal to the weight of air within the flask. Measure the water in the flask by means of a measuring cylinder; *fill* the flask with water, up to the position occupied by the bottom of the stopper, and measure its volume. The difference of these volumes gives the volume of the air which entered the flask. From these results calculate the *weight of one litre of air*.

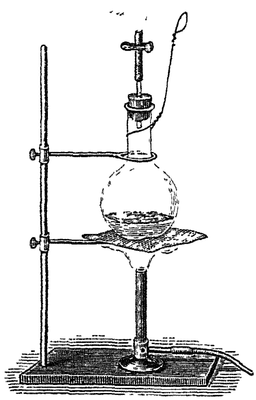


FIG. 61.—Experiment to show that air has weight.

Pressure of the atmosphere.—It is a property of all fluids that they communicate pressure in all directions, and consequently it

is a character of air. In consequence of this fact we are able to move about quite freely in spite of atmospheric pressure.

EXPT. 67.—**Effect of atmospheric pressure.** Procure a thin tin cylindrical can having a central exit-tube, fitted with rubber-tubing and clip. Remove the clip and boil a little water in the can. After the water has been boiling for some time, so that practically the can is filled with steam, remove the can from the flame, and quickly close the clip Pour a little cold water over the can, and observe how the can collapses

The explanation of the effect produced in this experiment is that as the can cools the steam inside is condensed into water, and so occupies a much smaller volume. The pressure which the steam exerts on the inside of the can is thus removed, while the pressure of the air on the outside remains practically the same, the result being that the can is crushed At the sea-level, under ordinary conditions, the pressure of the air is 15 lb. on every square inch.

The following experiments also illustrate effects of atmospheric pressure:

EXPT. 68 —**Action of a syringe.** Dip the open end of a glass syringe or squirt into a bowl of water. Pull up the piston, and notice that the water follows it, owing to the pressure of the atmosphere upon the surface of the water in the bowl. The action of a pump is very similar to this

EXPT 69.—**Upward pressure.** Take a tumbler or cylinder with ground edges and fill it completely with water Slide a piece of stout writing paper across the top and invert the vessel. If the air has been excluded from the cylinder carefully the water does not run out (Fig 62). Think what keeps the paper in its place

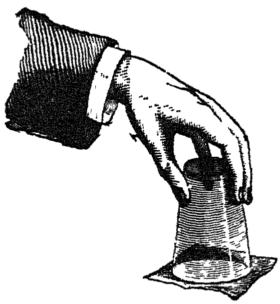


FIG 62 —Effect of atmospheric pressure.

Principle of the mercurial barometer.—It has been seen that the air has weight, and that it exerts pressure on the earth's surface; we have now to learn how this pressure is measured.

EXPT 70 —**Construction of a barometer.** Procure a barometer tube about 32 inches long, and bind a short piece of india-rubber tubing upon its open end. Bind the free end of the tubing to a glass tube about six inches long open at both ends. Rest the barometer tube with its closed end downwards and pour mercury into it (being careful to remove all air bubbles) until the liquid reaches the short tube Then fix the arrangement upright as in Fig. 63.

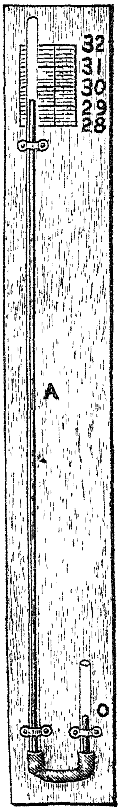


FIG 63 —To explain the principle of the barometer

The mercury in the long tube will be seen to fall so as to leave a space of a few inches between it and the closed end The distance between the top of the mercury column in the closed tube and the surface of that in the open tube will be found to be about thirty inches

The instrument used in Expt. 70 is evidently similar to a U tube. Referring to Fig. 63 it is clear that there is a column of mercury supported by some means which is not at first apparent, or else the mercury would sink to the same level in the long and the short tubes, for we know that liquids find their own level. If a hole were made in the closed end of the tube this would happen immediately. There will be no difficulty from what has been said already, in understanding that the column of mercury is kept in its position by the weight of the atmosphere pressing upon the surface of the mercury in the short open tube. The weight of the column of mercury and the weight of a column of the atmosphere with the same sectional area is exactly the same; both being measured from the level of the mercury in the short stem of the apparatus shown in Fig. 63, the mercury column to its upper limit in the long tube, the air to its upper limit, which is a great distance from the surface of the earth. When for any reason the weight of the atmosphere becomes greater, the mercury is pushed higher to preserve the balance; when it becomes less, then similarly the amount of mercury which can be

supported is less, and so the height of the column of mercury is diminished.

The height must in every case be measured above the level of the mercury in the tube or cistern open to the atmosphere. In the arrangement shown in the accompanying illustration, a line is drawn at a fixed point O, and the short tube is shifted up or down until the top of the mercury in it is on a level with the line.

The student will now understand why it is so necessary to remove all the air bubbles in Expt. 70 If this were not done, when the tube was inverted the enclosed air would rise through the mercury and take up a position in the top of the longer tube above the mercury. The reading would not then be thirty inches, for instead of measuring the whole pressure of the atmosphere, what we should be measuring really would be the difference between the pressure of the whole atmosphere and that of the air enclosed in the tube. In a properly constructed barometer, therefore, there is nothing above the mercury in the tube except a little mercury vapour.

An arrangement like that described constitutes a **barometer**, which we can define as **an instrument for measuring the pressure exerted by the atmosphere.**

EXPT. 71.—**Variation of pressure** (Expt. 70) has shown that air pressing upon the surface of the mercury in the short open arm of the U-tube will balance a long column of mercury in the closed arm. Slip a piece of india-rubber tubing upon the open end and notice what happens when you blow sharply into it. Suck air out of the tube, and observe the result.

These experiments illustrate the effect of increasing and decreasing the pressure upon the free surface of the mercury.

Weight of column of atmosphere.—The following is another form of the experiment to show atmospheric pressure by means of a barometer.

EXPT. 72.—**Simple barometer.** Procure a thick glass tube about 36 inches long and closed at one end. The tube must be quite clean and dry. Fill the tube with dry, clean mercury, leaving a small air bubble at the top. Close the tube with the thumb, and slope the tube downwards so that the bubble of air travels along the whole

length of the tube; slope the tube upwards, so that the bubble returns to the open end Thus all small air bubbles are removed from the sides of the tube Fill up the tube with mercury, place your thumb over the open end; invert the tube, place the open end in a cup of mercury and take away your thumb

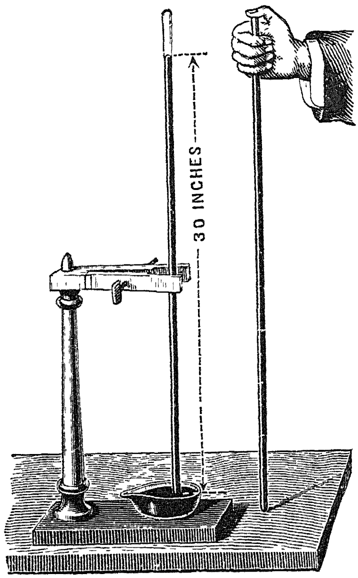


FIG 64 —A simple barometer.

A column of mercury will be supported in the tube by the pressure of the atmosphere. The distance between the top of the column and the surface of the mercury in the cup will be about 30 inches, or 76 cm., when the tube is vertical. When the tube is inclined so that the closed end of it is less than this height above the mercury in the cup the mercury fills it completely; and when the tube is less than 30 inches long, it is filled by the mercury also. On an average, the atmosphere at sea-level will balance a column of mercury 30 inches in length. No matter if the closed tube be 30 feet long, the top of the mercury column will be about 30 inches only above the level of the mercury in the cistern.

If the tube had a bore with a sectional area of exactly one sq. cm., there would be 76 c.c. of mercury in a column 76 cm. long; and since 1 c.c. of mercury weighs 13·6 gm., the whole column would weigh 1033·6 gm. This column balances a column of air of the same area, so that we find that the weight of a column of air upon an area of one sq cm. is 1·03 kgm. when the barometer stands at 76 cm Converting these numbers into British units, it will be seen that the pressure on one square inch of a mercury column 30 in. high is 15 lb

Mercury a convenient liquid for barometers.—The use of mercury for barometers is a matter of convenience. Since the column of mercury which the atmosphere is able to support is

30 inches high, it is clear that, as water, *e.g.*, is 13.6 times less dense than mercury, the column of water which could be supported would be $30 \times 13.6 = 408$ inches $= 34$ feet, which would not be a convenient length for a barometer. The length of the column of glycerin which can be supported similarly is 27 feet. But in the case of lighter liquids like these, any small variation in the weight of the atmosphere is accompanied with a much greater alteration in the level of the column of liquid, and in consequence it is possible to measure such variations with much greater accuracy. For this reason barometers are sometimes made of glycerin.

Pressure of the atmosphere at different altitudes.—The atmosphere being a material substance, the shorter the column of it there is above the barometer, the less is the weight of that column, and the less the pressure it exerts upon the mercury in the barometer. Hence, as we ascend through the atmosphere with a barometer, we reduce the amount of air above it pressing down upon the mercury in it, and in consequence the column of mercury the air is able to support becomes less and less as we ascend. On the contrary, if we can descend from any position, *e.g.* down the shaft of a mine, the mercury column is pushed higher and higher as we gradually increase the length of the column of air above it. Since the height of the column of mercury varies thus with the position of the barometer, it is clear that the variation in its readings supplies a means of ascertaining the height of the place of observation above the sea-level, provided we know the rate at which the height of the barometer varies with an alteration in the altitude of the place. The rule which expresses this relation is not a simple one, but for small elevations it is said that a rise or fall of one inch in the height of the barometer corresponds to an alteration of 900 feet in the altitude of the barometer.

Boyle's Law.

Relation between volume and pressure of a gas.—To understand how and why the density of the atmosphere varies, it is necessary to become acquainted with the rule expressing the relation between the volume and pressure of a gas. This can

be done satisfactorily by one of the forms of apparatus employed in the following experiments, which provide a means of subjecting an enclosed quantity of air to varying pressures, by the addition of smaller or larger quantities of mercury.

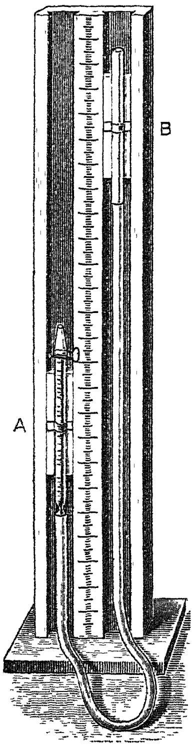


FIG 65.—Boyle's law apparatus

In Fig. 65, two glass tubes are supported on blocks which slide in grooves on a vertical stand and are joined at their lower ends by stout rubber tubing; one of the tubes, B, is open at its upper end, and the other, A, is either sealed off with its upper end nearly flat or else it terminates in a glass tap. Before proceeding to take observations, the apparatus is arranged so that, when mercury contained in the apparatus is at the same level in both tubes, the closed tube is about half full of dry air. Under these conditions the enclosed air is acted upon by a pressure equal to that of the atmosphere, which is obtained by reading the height of the barometer at the time of the experiment. If the closed tube be graduated the volume of the enclosed air is obtained at once by taking the scale-reading of the mercury surface in the tube; if the tube be ungraduated but uniform in bore, the *length* of the air column may be taken as numerically equivalent to the volume of the enclosed air. Having taken these readings of pressure and volume, the open tube is raised until the mercury surface in the open tube is several centimetres higher than that in the closed tube. The enclosed air is now under a total pressure equal to the sum of that due to the atmosphere and that due to a column of mercury equal in length to the difference of levels of the mercury in the two tubes. The volume of the enclosed air is now less than before. A series of such readings of pressure and volume are taken, the open tube being raised finally to the greatest extent permitted by the apparatus. Similar readings are taken with the open tube *lower* than the closed tube; in this case the pressure acting on the enclosed air is *less* than that of the atmosphere, and its magnitude is obtained

by *subtracting* the difference of level of the mercury surfaces from the height of the barometer.

EXPT. 73.—**Relation between pressure and volume.** Take a series of readings of the *pressure* acting on the enclosed air and of the *volume* of the air by means of apparatus similar to Fig. 65. The difference of level of the mercury surfaces is obtained readily by measuring the heights of the surfaces above the base-board by means of a metre scale Record your results thus ·

Height of Barometer	Difference of Level of Mercury	Total Pressure on the Air, P	Volume of Air, V	Total Pressure × Volume, P × V
Cm	Cm			

From these observations it will be found that the volume regularly diminishes as the pressure is increased, and in the same proportion The converse is also found to be true, viz., that as the volume of a gas increases the pressure upon it diminishes, and exactly in the same proportion. But, in both these cases, it is understood that the *temperature of the gas remains the same*, that is, the temperature of the gas under the different pressures must not alter.

The tabulated results of the experiments reveal another important relation, which is, however, another way of expressing those already noticed. It is found that, when there is no alteration of temperature, the product obtained by multiplying the volume of a given mass of gas by the pressure to which it is subjected is always the same, or remains constant.

These facts were discovered by Robert Boyle in 1662, and are included in what is known as **Boyle's Law.** It can be expressed by saying that **when the temperature remains the same, the volume of a given mass of gas varies inversely as its pressure.** Or, what is the same thing, **the temperature remaining the same, the product of the pressure into the volume of a given mass of gas is constant.**

But it has been learnt that if the volume occupied by a given mass of a substance be increased, its density is decreased, and if the volume be decreased, its density is increased Therefore, by decreasing the volume of the enclosed air in the above experiment, its density is increased The increase of density and the increase of pressure are proportional to one another. It is not

difficult to apply these facts to the case of the atmosphere. It has been stated that the pressure of the atmosphere decreases as we ascend, and we are now able to add that its density decreases also and at the same rate. Therefore the densest atmosphere is that at the surface of the earth, leaving out, of course, the air of mines and other cavities below the surface, where the air is denser still The air gets less dense, or rarer, as we leave the earth's surface, until eventually it becomes so rare that its existence is practically not discernible.

The following experiment provides an alternative method of illustrating Boyle's Law·

EXPT 74—**Simple method for Boyle's Law.** Take a length of thermometer tubing, (Fig 66), about 75 cm long and 1 mm bore. Seal it at one end and expand the open end somewhat. Clamp the tube in a vertical position, with the closed end below, by the side of a metre scale, and connect a small funnel to the top by means of a short piece of rubber-tubing. Pour a little pure, clean mercury into the funnel and induce it to run down the bore of the tube by inserting a thin, clean, steel wire In this way any desired volume of air can be enclosed

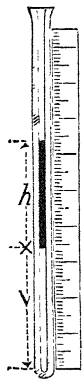


FIG. 66 —Simple experiment on Boyle's law.

The length of the column of enclosed air may be taken to represent its *volume* (V) If H = the height of the barometer, and h = the length of the mercury thread (both expressed in the same units), then the total pressure on the enclosed air $=(H+h)$.

Introduce more mercury in the same manner, and in this way alter the values of V and h The volume of the air under the pressure of the atmosphere alone can be observed by laying the glass tube flat on the table, and the volume under pressures less than that of the atmosphere can be observed by inverting the tube with its open end downwards

Perform several experiments and record the results in the following way:

Volume (V)	Pressure $(H \pm h)$	Volume $\times$ Pressure

SOME INSTRUMENTS DEPENDING UPON ATMOSPHERIC AND FLUID PRESSURE.

The air-pump.—Several forms of air-pumps are in use, but in this place it will be sufficient to describe one of the simplest, that designed by Hawksbee, the essential parts of which are shown in Fig. 67. V is the receiver, from which it is required to remove air. V is connected with a cylinder *cv* by means of a tube, shown on the base in the illustration, bent twice at right angles. At the end of this tube remote from the receiver, and just at the bottom of the cylinder *cv*, is a valve *v* opening upwards. In the cylinder works, in an air-tight manner, a piston provided with a valve *v′* opening upwards, and a handle for pulling the piston up and pushing it down is provided. The action is very simple. Imagine the piston to be at the bottom of the cylinder to begin with, and then that it is pulled up gradually. As this takes place, the air in the receiver and below the valve *v* is subjected to a diminished pressure, and consequently expands, filling the space which is formed as the piston moves upwards. This expansion continues until the piston arrives at the end of its stroke. The piston is now pushed down. This movement compresses the air between *v′* and *v* and increases its pressure, causing the valve *v* to shut. But as the piston descends the pressure on the under surface of the valve *v′* becomes greater than that of the atmosphere upon its upper surface, with the result that the valve *v′* opens upwards and the air in the space *vv′* rushes through the open valve into the outside air. The final result, when the piston reaches the bottom of the cylinder, is that there is less air in the receiver and tube connecting therewith than there was originally. As the piston is worked up and down the same opening and shutting of valves is repeated, with the result, that by and by, nearly all the air is removed from the receiver.

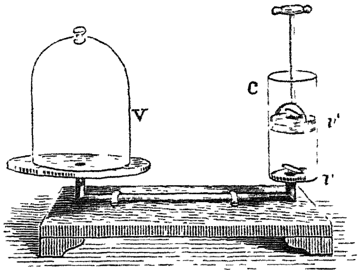


FIG. 67.—Simple form of air pump.

Sprengel's air-pump.—More perfect vacua can be obtained by a simple form of air-pump, due to Sprengel, in which the piston of Hawksbee's instrument is replaced by drops of mercury and

in which valves are dispensed with The essential parts of this pump are shown in Fig. 68 The flask, or other vessel, which it is desired to exhaust of air is connected with the tube at D.

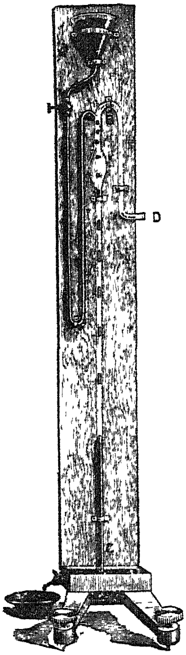


Fig 68.—The principle of Sprengel's air-pump.

Mercury is poured into the funnel and falls continuously down the long vertical tube. As each drop of mercury passes the opening at the top of the tube connected with the vessel to be exhausted, it carries with it a little air, until eventually, after the stream of mercury has been running for some time, practically the whole of the air in the vessel is removed.

The common pump.—After examining a glass model like that shown in Fig. 69, there is no difficulty in understanding the action of a common pump To begin with, suppose that the pump is full of air and that the end of the tube below the valve *b* is dipped into a basin of water. The piston *ec* is, to start with, at the bottom of its stroke near the valve *b*. As the piston is raised the air in the cylinder above *b* expands; its pressure consequently decreases; the pressure on the lower surface of the valve *b* is, therefore, soon greater than that on its upper surface, and the valve is pushed upwards by the air below it, the air flowing into the cylinder *acb*. The result is that the air in the cylinder below the piston is at a lower pressure than that of the outside air, and as a consequence water is pushed up the tube *b* This action continues until the piston reaches the end of its stroke towards the top of the pump.

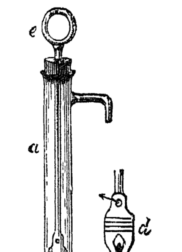


Fig 69.—Model of a common pump

As the piston descends, the air in the cylinder below the piston *c* is compressed and its pressure becomes gradually greater. The valve *b* closes and that in *c* opens, through which latter, of course, the air in the cylinder escapes. On raising the piston

again the same effects are repeated until all the air in the pump is removed and the outside air pushes the water up until it reaches the spout and escapes.

As the air is able to support a column of mercury 30 inches in height, and as mercury is about $13\frac{1}{2}$ times heavier than water, the air can support a column of water of a height equal to

$$30 \text{ inches} \times 13\tfrac{1}{2} = 2\tfrac{1}{2} \times 13\tfrac{1}{2} \text{ ft.} = 33\tfrac{3}{4} \text{ ft}$$

It will be understood at once, therefore, since the efficacy of the common pump depends wholly upon the pressure of the air, that the spout of the pump must never be more than $33\frac{3}{4}$ feet, or, roughly, 33 feet from the level of the water. On account of leakage and friction against the pipe, water cannot be made to rise much more than 30 feet with an ordinary pump.

The siphon —The siphon is a simple instrument which depends upon atmospheric pressure for its action It consists usually of a bent tube, one leg of which is longer than the other. It is filled with the liquid to be transferred from one vessel to another, and while both ends of the tube are kept closed, the shorter limb is placed into the vessel of liquid. The result is that the liquid flows until the level of the liquids is the same in both vessels, or the higher liquid has been siphoned to the lower level.

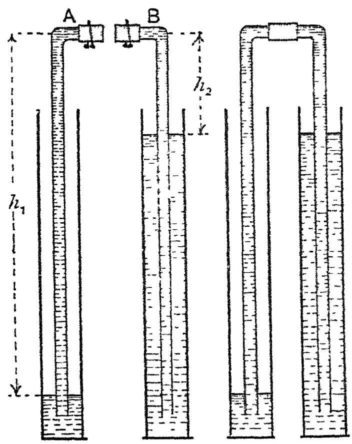


FIG 70 —Principle of the action of the syphon

Fig 70 represents two narrow glass tubes, closed at the upper ends with rubber tubing and clips, filled with mercury, and with their lower ends immersed in mercury contained in tall cylinders The cylinder on the left is nearly empty, and that on the right is nearly full. If the atmospheric pressure, as observed with a barometer, is H cm of mercury, and if the vertical distance between the top of tube A and the mercury surface in the cylinder is h_1 cm., then the pressure at the top of tube A is $(H - h_1)$ cm. of mercury. If the corresponding height of the top of tube B is h_2 cm., then the pressure at the top of tube B is $(H - h_2)$ cm. of mercury. Since h_1 is greater than h_2, the pressure at the top of B is greater than the pressure at the top of A, by an amount equal to

$(h_1 - h_2)$ cm. of mercury. Suppose that the two tubes are now joined together, as shown in Fig 70 · the greater pressure at the top of the right-hand tube will push the liquid from right to left. The flow of liquid continues until the level of liquid is the same in each vessel.

EXERCISES ON CHAPTER VII.

1. About what height does the mercury column of the barometer generally stand? How is the reading affected if the tube is not in a vertical position? Calculate the length of the column of a barometer which is inclined at an angle of 30° to the vertical, the true barometric height being 30 inches.

2. What is the height of a water barometer when a mercurial barometer reads 30 inches? If the mercury barometer falls to 29 inches, through what distance would a water barometer fall under the same conditions?

3. If the atmospheric pressure be 15 pounds per sq. inch, what is the pressure in kilograms per sq centimetre?

4. What is the atmospheric pressure, in pounds per sq. inch, when the height of the barometer is 28 5 inches?

5. A quantity of gas occupies 323 c c when the barometer reads 72 cm What volume will it occupy at 76 cm if there be no change of temperature?

6. A litre of air weighs 1 293 gm. at normal pressure, and when the temperature is 0° C What weight of air at the same temperature will a litre flask hold when the barometer stands at 78 cm.?

7. A 20-cm cube weighs 23 5 gm in air at 0° C. and normal pressure. What will it weigh (i) in hydrogen, and (ii) in carbonic acid gas, under the same conditions of temperature and pressure? [At normal pressure and 0° C, 1 litre of air weighs 1 293 gm, 1 litre of hydrogen weighs 0 09 gm., and 1 litre of carbonic acid gas weighs 1·98 gm.]

8. When the pressure of the air is normal, what is the greatest height over which you could siphon mercury? At the same atmospheric pressure, what is the greatest height over which you could siphon sulphuric acid (relative density, 1·84)?

9. Describe carefully any experiment you have seen or done to show that the air exerts pressure, and sketch carefully the apparatus used.

One arm of a bent tube containing water is attached by rubber tubing to the gas supply and a difference of level of 6 5 cm. is obtained when the gas is turned on. Find the pressure of the gas in grams per sq. cm., and in lb. per sq. inch. (2·54 cm. = 1 inch 453 gm. = 1 lb.)

10. Why is mercury generally used in making a barometer? Could a barometer be made using water as the liquid? State and explain what would happen if a hole were bored through the glass of the barometer above the mercury.

11. Describe exactly what you would need to make a rough barometer, and how you would proceed to do it. If you made successively two such barometers, and found on comparing them that the mercury did not stand at the same height in the two, which would more nearly indicate the true pressure of the atmosphere, and what would almost certainly be the cause of the difference?

12. Would the difference in vertical height between the two barometers in the last question be greater when the tubes were vertical, or when they were both sloped away from the vertical to the same extent? Would the difference between them alter if the atmospheric pressure were to change? Why, in each case?

13. Water cannot be raised to a height much greater than 30 feet by means of a common pump State the reason of this and describe a laboratory experiment by which you could prove your explanation to be correct

14. State Boyle's Law Describe an experiment you have performed to verify the law, and mention any precautions you took to ensure accuracy.

15. (*a*) Why does the mercury stand higher in the tube than in the cup of a barometer? (*b*) What is the average height of the mercury in a barometer tube at the sea-level?

16. Describe how to make a simple barometer, and explain its working. What will be seen if (*a*) the tube be sloped away from the vertical, (*b*) the cistern and the lower part of the tube be immersed in a deep vessel of water, (*c*) the whole instrument be taken in a lift to the top of the Eiffel tower? Give explanations in each case.

17. Describe a simple experiment to show that the volume of a quantity of gas changes with the pressure upon it What is the exact relation between the volume and the pressure? A small india-rubber balloon is partly filled with air, tied at the mouth, and loaded so as to sink in a vessel of water in which it is placed; and the whole is now placed under the receiver of an air pump and the air exhausted from the receiver. What will happen, and why?

18. If I take a barometer tube, the internal sectional area of which is one-fourth of a square inch, calculate, from the known pressure of the atmosphere, the weight of mercury which would be supported in the tube. When the mercury stands at the height of 30 inches in the barometer, what is the weight of air pressing on an acre of ground? (There are 5,280 feet in a mile (linear) and 640 acres to the square mile.)

CHAPTER VIII.

MOTION, FORCE, AND INERTIA.

Definition of motion.—The word *motion* is meant to convey the idea of *change of place*. The simplest forms of motion are changes in the positions of bodies with regard to one another. When a boy runs down the street he is in motion; as regards the houses and lamp-posts he moves. To describe fully the boy's motion it would be necessary to know his **velocity**, that is, the *direction* in which he is moving or the *line* along which he runs, and the *speed* with which he travels. If during every second through which he moves he travels over a distance of five yards, he has a **uniform speed** of five yards a second.

But suppose he does not move regularly over five yards in every second; he sometimes dawdles, sometimes stops to look at a shop, at other times he puts on a spurt to make up for lost time. How should his motion be described now? His rate varies from time to time, or he may be said to have a **variable speed,** and to describe such a variable speed it is usual to speak of the speed *at any instant* as being a certain number of yards per second. Suppose the boy moving with a variable speed had at a given instant a speed of eight yards per second. If he continued to move at the same rate he would travel over eight yards in the succeeding second.

A distinction in meaning between the terms **speed** and **velocity** should be observed always. The former term refers simply to the distance traversed in a given time and irrespective of the direction; whereas the latter term implies that the space is traversed in a known direction. Thus, in the case of a train travelling round a curve with a speed of 60 miles per hour, the

speed may be constant, but the velocity is changing at every instant.

Average speed.—For many problems it is necessary to know the **average speed** of the moving body. Returning to the boy, suppose he travelled 800 yards in 400 seconds; if the first number be divided by the second the boy's average speed is obtained, namely, two yards in a second; this then is the speed with which he would have had to travel, if he moved uniformly, in order to complete his journey in the same time.

The **unit of speed** is generally taken as being a speed of one foot per second. Thus a speed of six means a speed of six feet per second.

Measurement of uniform linear velocity.—It is a very simple matter to calculate the velocity of a body moving uniformly in a straight line when the distance it has travelled, measured in units of length, and the time it has taken to perform the journey, measured in units of time, are known. All that has to be done in order to find its uniform velocity (v) is to divide the number of units of length (s) passed over by the number of units of time (t) taken to complete the distance. Thus:

$$v = \frac{s}{t} \quad \text{or} \quad s = vt.$$

Velocities can be represented completely by straight lines.—To determine a velocity completely its magnitude (or the distance travelled in a given time) and its direction must be known. But, a straight line can be drawn in any direction and of any length; and it can be arranged that the length shall contain as many inches or feet, whichever is more convenient, as there are feet or yards per second of velocity, depending on the way in which we decide to measure our velocities. Velocities can therefore be represented completely by straight lines.

EXPT. 75.—**Same direction.** Taking a line an inch long to represent a velocity of one foot per second, draw lines representing velocities of $3\frac{1}{2}$, $2\frac{3}{4}$, 4, and $1\frac{1}{4}$ feet per second, making the lengths of the lines proportional to the rates of motion.

EXPT. 76—**Parallel directions** Draw a line to represent the velocity of a river flowing at the rate of 2 miles an hour. Suppose

a man who can row 6 miles an hour in still water to be rowing in this river. Draw lines to represent his velocity with reference to the bank when rowing (1) with the stream, (2) against the stream.

EXPT. 77.—**Inclined directions.** Draw a large circle upon a sheet of paper. Draw two diameters at right angles to each other. Taking $\frac{1}{2}$ inch to represent a velocity of 1 foot per second, and starting from the centre of the circle, represent by graphic construction the path of a body moving with the following velocities. 2 ft. per sec N.E.; 2 feet per sec. N.; 3 feet per sec. W.; 4 feet per sec S.E

Composition of velocities.—Consider the case of a marble moving along a tube with a uniform velocity, when the tube itself is all the time being moved uniformly across a table (Fig 71). It is evident that since the marble is in the tube it must have the same velocity *across* the table that the tube has; and at the same time it moves along the tube, that is, in a direction at right angles to its former velocity. There are thus two independent velocities to be considered—one the velocity of the tube and the other the velocity of the marble relative to the tube. Similarly, we can think of a ship sailing across the ocean with a man on deck walking from one side of the ship to the other. The man is moving onwards with the ship at a certain velocity, and at the same time he is moving across the ship with another velocity.

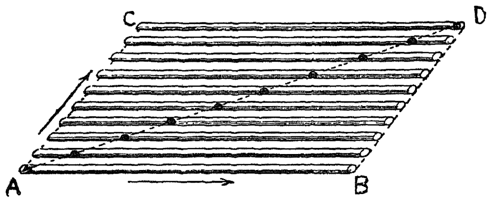


FIG 71 —To illustrate the combination of two velocities, and the principle of the parallelogram of velocities

Really, a body can only move at any instant in one direction with one definite velocity. The best way to consider the composition of velocities is to think of the velocities as existing successively rather than at the same instant, in order to find the actual change of position in any time in the case of the marble or of the man referred to above. The velocity to be found is called the **resultant** of the two independent velocities, which are themselves spoken of as **components.** If the two

velocities have the same direction, the resultant is their sum; and if they are in opposite directions along the same straight line the resultant is their difference. If they have directions which make an angle with one another, it is clear that the resultant must lie somewhere between the components.

Referring to the case of the marble, let OA (Fig. 72) represent by its length the number of inches the marble moves along the tube in a second, and OB the distance moved by the tube, and consequently by the marble, in the same time across the table. Draw BR parallel to OA and AR parallel to OB, thus completing the parallelogram, then the line OR represents the actual change of position which would be produced if the two velocities in question existed successively for equal intervals of time In the same way in the second example OA could stand for the ship's velocity and OB for the man's, then OR would represent the resultant of the two velocities. When therefore, the velocity of a body A (such as the marble or the man) relative to a body B (such as the tube or the ship) is represented by one side of a parallelogram and the velocity of B relative to C (such as the table or the ocean) is represented by the other side, the velocity of A relative to C is represented by the diagonal. This principle is called the **Parallelogram of Velocities.**

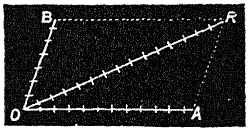


FIG. 72.—Parallelogram of velocities

Meaning of acceleration.—An express train starting from a terminus begins to move slowly, and, as the journey proceeds, the rate of motion goes on increasing until the train gets its full speed. A stone let fall from a height similarly starts from rest, and as it moves it travels faster and faster until brought to rest again on reaching the ground. Or, we may imagine a cyclist starting for a ride, and regularly increasing his speed until he could not go any faster. In all these examples the velocity of the moving body has increased regularly, and the rate at which the change has taken place is spoken of as **acceleration.**

Acceleration is the rate of change of velocity.—But acceleration may be of an opposite kind to the instances given above. Reverse each of the examples and consider what happens. An

express train going at full speed approaches a station, and its velocity is diminished regularly until it is brought to rest at the platform. A stone is thrown upwards with a certain velocity, it moves more slowly and more slowly until it comes to rest, and then starts falling. A cyclist travelling at full speed slackens his rate regularly until he comes to a standstill. In all these cases we have examples of an acceleration of an exactly opposite kind to the previous instances, but yet an acceleration. In ordinary language this kind of acceleration is given a name of its own, viz. *retardation*.

EXPT. 78.—**Motion down an inclined board.** Obtain a smooth board about six feet long, having a slight groove cut in it from one end to the other. Incline the board slightly at one end. Place a marble or other small sphere near the raised end and let it roll down the board. Notice that as it moves its velocity increases. To show that the space traversed increases every second, fit up a seconds' pendulum and set it in motion. Let the bob of the pendulum strike against a sheet of paper or some other light object at the end of each swing, so that you can hear when the seconds commence. Now start the marble from a mark upon the board exactly when the pendulum taps the paper on one side. Notice how far the marble has rolled by the time the pendulum taps the paper on the other side. Make a mark at the place reached, and do the same for succeeding seconds until the marble rolls off the board Measure the length of board traversed by the marble in each second. The distances will be found to increase in proceeding down the board from the starting place.

Measurement of uniform acceleration.—In measuring a regular or **uniform acceleration**, it is necessary to know what addition to, or subtraction from, the velocity of the moving body there has been during each second of its journey. Suppose there is an addition of one foot per second to the velocity of a moving body, and that it has taken one second to bring about this change, we should refer to this as an acceleration of one foot per second in a second, or **one foot per second per second.** An acceleration which increases the velocity is referred to as *positive*, while that which diminishes it is *negative*. The first examples given above are instances of positive acceleration, while when we reverse them they afford cases of negative acceleration.

Unit of acceleration.—As in every other measurement, so, when accelerations have to be measured, it is necessary to have a unit in terms of which the quantity under consideration can be expressed. **The unit of acceleration is an increase of unit velocity in a unit of time**; it is generally taken as equal to **an increase of velocity of one foot per second per second** An acceleration of *two units* would thus be an increase of velocity of *two* feet per second per second; similarly, an acceleration of three units equals an increase of velocity of three feet per second per second, and so on for any number of units

Acceleration, like velocities, can be represented graphically by straight lines.

Equations of motion.—If we take the unit of acceleration as equal to an increasing velocity of one foot per second in one second, an acceleration of f means an increase of velocity of f feet per second in one second. Suppose a body starts from rest, at the end of the first second it has a velocity of f feet per second, at the end of the next second $2f$, at the end of t seconds ft feet per second. Or if v = change of velocity in t seconds we can write

$$v = ft. \quad \ldots \quad (1)$$

The space travelled over by a body in one second is equal to its average velocity, and that travelled over in t seconds is equal to its average velocity multiplied by t. If it starts from rest and travels for t seconds, finishing with a velocity of v feet per second, its average velocity is $\frac{1}{2}v$ during this time, and

$$\therefore \; s = \tfrac{1}{2}vt. \quad \ldots \quad (2)$$

Substituting the value of v from equation (1) we get

$$s = \tfrac{1}{2}ft \times t = \tfrac{1}{2}ft^2,$$

or since from (1) $t = \dfrac{v}{f}$ we can write equation (2) thus:

$$s = \tfrac{1}{2}v \times \frac{v}{f} = \tfrac{1}{2}\frac{v^2}{f},$$

from which

$$v^2 = 2fs. \quad \ldots \quad (3)$$

If the body be moving freely towards the earth its acceleration is g (see p. 102), and equation (3) becomes

$$v^2 = 2gs. \quad \ldots \quad (4)$$

Acceleration due to gravity.—It has already been shown (Expt. 5) that the time of vibration of a pendulum varies as the square root of the length. The time of oscillation can, by a simple application of dynamical principles, be shown to be given by the expression:

$$\text{Time of oscillation} = 2 \times 3\tfrac{1}{7}\sqrt{\frac{\text{length}}{\text{gravity}}}, \quad \text{or} \quad t = 2\pi\sqrt{\frac{l}{g}},$$

where t stands for the time of oscillation, l for the length of the pendulum, g for the value of the acceleration due to gravitation,* and π for the ratio between the circumference and diameter of a circle.

From this equation it is easy to obtain an expression for the value of g. Thus, squaring both sides, we have

$$t^2 = \frac{4\pi^2 l}{g},$$

so that

$$g = \frac{4\pi^2 l}{t^2}.$$

EXPT 79.—**Determination of "g"** Using the observations you have made as to the time of a double swing of your pendulum, and taking as the length of the pendulum the distance in feet from the bottom of the support holding the thread to the centre of the ball, determine by means of this equation the value of g.

EXPT. 80.—**Second method.** Calculate the value of g as in the preceding experiment, but express the measurement in centimetres.

Equality of masses.—Motion and mass have hitherto been considered separately, but now their relations to one another will be described. It has been shown that equality of mass can be tested by weighing. The balance thus provides a convenient practical method of comparing masses, but it does not give a fundamental conception of what mass means. To obtain a clear idea of the subject, consider first of all that we are dealing with two variable quantities, namely, mass, or quantity of matter, and motion. Suppose two bodies moving in opposite directions with *equal* velocities to collide with one another and stick together. If the two bodies stopped dead after the impact we could conclude that their masses were equal, and that each exactly destroyed the motion of the other; but if the combined bodies

* The quantities l and g must be expressed in the same units.

moved after the collision, the masses could evidently not have been equal With this in mind, it will be conceded readily that the following definition of equality of mass holds good: **Two masses are equal, if when they are made to impinge on one another in opposite directions with equal velocities and stick together, they come to rest** *

The effect of the impact of two moving bodies thus depends upon the masses of the bodies and the velocities before the collision If both the velocities and masses are equal, the bodies come to rest, if the velocities are equal but the masses are unequal, the greater mass predominates after collision, and if the masses are equal while the velocities are unequal the greater velocity will predominate.

Momentum.—The preceding paragraph has introduced the idea of a condition involving both motion and mass. This condition is known as momentum, which is defined as follows·

The momentum of a body is the quantity of motion it has, and is equal to the product of its mass and its velocity.

Expressed as an equation we have

$$\text{Momentum} = \text{mass} \times \text{velocity},$$

or if momentum is represented by M, mass by m, and velocity by v, all expressed in corresponding units, we can write

$$M = mv$$

The unit of momentum is consequently that of a unit of mass moving with a unit of velocity, or if the unit mass be that of the imperial standard pound and the unit velocity a velocity of one foot per second, **the unit of momentum is the quantity of motion in a mass of one pound moving with a velocity of one foot per second.** The meaning of momentum will be better grasped after a concrete example.

When a shot is fired from a cannon, the same momentum is generated in both the cannon and the shot, but since the mass of the cannon is immensely greater than that of the shot, it will be evident that the velocity of the shot must be correspondingly

* This definition, and the general treatment here adopted of matter in relation to motion are based upon Prof W. M. Hicks's work on *Elementary Dynamics of Particles and Solids*

greater than that of the cannon in order that the product of the two quantities may be the same. This we know is the case, the velocity of the "kick" or "recoil" of the cannon being very much less than the velocity with which the shot is sent on its journey.

From the point of view of momentum, however, the action and the reaction are equal and opposite. Thus a shot weighing 28 lb. is fired from a cannon weighing 10 tons, that is 22,400 lb., and the shot leaves the gun with a velocity of 200 feet per second. The velocity of the recoil of the cannon is therefore one-quarter of a foot per second (for $28 \times 200 = 22{,}400 \times \frac{1}{4}$).

Third law of motion.—Newton expressed the fundamental principles of the relationship between matter and motion in three laws, known as his Laws of Motion. The third law (which may be considered before the others) states that **action and reaction are equal and opposite.**

Consider the case of a heavy magnet and an iron nail suspended by threads so that they can move toward each other. The nail will move faster than the magnet, and its acceleration will be greater than that of the magnet by exactly the same amount that the mass of the magnet exceeds the mass of the nail. The action and reaction are equal and opposite, so the momentum of the nail is the same as that of the magnet. We can say, therefore, that **the masses of two bodies are inversely proportional to the accelerations which they acquire in virtue of their mutual action and reaction.** This definition of mass involves no assumption, and is true whether we think of the mutual action, or **stress**, as between a nail and a magnet, between the sun and the earth, or between any two bodies or particles.

Force.—Suppose a body to possess a certain momentum; then for the momentum to change or tend to change, something must act upon the body, and that something is termed **force.** In other words: **When a gradual change of momentum is either produced or tends to be produced in a body, that body is acted on by force**

It must be understood clearly that by thus defining force we do not get to know anything more about it. Nobody can tell what force is. All we can know are the effects produced by a something we call force.

Since a change of momentum is produced by force, the rate at which the momentum changes may be used as a measure of

force, and we can say, therefore, that **equal forces are those which produce equal momentums in equal times**

Acceleration produced by a force.—The momentum of any particular body is determined by the body's mass and velocity. Since the mass of the body may be regarded as constant, change of momentum can be produced only by changing the velocity. But rate of change of velocity is acceleration, hence when a body is moving with accelerated velocity, the momentum is altered, and an alteration of momentum signifies, as has been explained, that the body is being acted upon by a force. If the acceleration be uniform, the body must be acted upon by a uniform force.

Hence we come to the very important fact that **the number of units of force in any force is equal to the product of the number of units of mass in any body on which it may act and the number of units of acceleration produced in that mass by the force in question.**

The relation between force, mass, and acceleration may be expressed algebraically as follows:—Let F represent the number of units of force in a given force, **m** the number of units of mass on which it acts producing **a** units of acceleration, then the definition can be written,

$$F = m \times a,$$

from which equation the third quantity can be obtained whenever we know the other two.

$$\begin{array}{c}\text{Number of}\\\text{units of force}\end{array} = \begin{array}{c}\text{Number of}\\\text{units of mass}\end{array} \times \begin{array}{c}\text{Number of units}\\\text{of acceleration.}\end{array}$$

$$F = ma; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\therefore \quad a = \frac{F}{m}, \qquad (2) \qquad \text{or} \qquad m = \frac{F}{a}. \quad \ldots\ldots\ldots\ldots (3)$$

The second equation can be expressed in words by saying that the number of units of acceleration produced in the velocity of a moving body is equal to the number of units of force acting upon it, divided by the number of units of mass on which it acts

Similarly, the third expression means that the number of units of mass in a moving body can be calculated by dividing the number of units of force acting upon it by the number of units of acceleration produced in it.

The second equation tells us, moreover, that if the acceleration produced in a moving body remains the same, or is uniform, that

the value of the force, or the number of units of force it contains, must be the same throughout, or what is the same thing, the force is uniform.

Second law of motion.—This law is stated generally by saying that **change of motion is proportional to the impressed force, and takes place in the direction in which that force acts** This expression, 'change of motion,' implies something more than the conception of motion as a mere change of place. By change of motion is meant **rate of change of momentum.**

The second law of motion states that the momentum generated in unit time by a force of two units is twice as great as that produced by one unit; and it implies, moreover, that a force of one unit acting for two seconds produces twice the momentum which it would do if it only acted for one second.

Absolute units of force.—It has been shown that the unit of force is the force which, acting on unit mass, produces in it unit acceleration

In the **British** (or F P.S.) **system** the absolute unit of force is that which, acting on a mass of one pound, gives to it an acceleration of 1 ft. per sec per sec It is called the **poundal.**

In the **metric** (or C G.S.) **system** the absolute unit of force is that force which, acting on a mass of 1 gram, gives to it an acceleration of 1 cm. per sec per sec It is called the **dyne**

Gravitational units of force.—The **weight** of a body is another name for the **force** exerted by gravity on the mass of a body. It varies slightly at different places on the earth's surface, and it also depends upon the distance above sea-level (p. 35). At any one locality it remains constant, and produces a uniform acceleration of g units. The value of g may be taken as approximately 32·2 ft. (or 981 cm) per sec. per sec.

For every-day purposes the *weight of* 1 *lb.* and the *weight of* 1 *gm.* are used frequently as the units of force in the British and Metric systems respectively. These are termed the **gravitational units of force.**

Hence. in the formula $F = ma$, if $m = 1$ lb., and $a = 32{\cdot}2$ ft. per sec per sec., then

$$F = (1 \times 32{\cdot}2) \text{ poundals,}$$

or

$$\text{the weight of 1 lb.} = 32{\cdot}2 \text{ poundals,}$$

or

$$1 \text{ poundal} = \frac{\text{weight of 1 lb.}}{32{\cdot}2} = \text{wt. of } 0{\cdot}5 \text{ oz. approximately.}$$

Similarly, if $\mathbf{m} = 1$ gram, and $\mathbf{a} = 981$ cm. per sec. per sec.,

$$\text{the weight of 1 gram} = (1 \times 981) \text{ dynes},$$

or
$$1 \text{ dyne} = \frac{\text{the weight of 1 gram}}{981} = \text{wt. of } 0{\cdot}001 \text{ gm. approx.}$$

Inertia.—Common experience tells everyone that things do not move of themselves. An object at rest remains at rest until it is forced to move. Moreover, if it is moving it tends to go on moving in the same direction and with the same velocity until either is made to change by the application of force. In a word, dead matter is helpless and conservative. **The inability shown by a material body to change by itself its condition of rest or of uniform motion in a straight line is called its inertia.** It is exemplified when a cyclist is stopped suddenly, for the tendency to continue moving is so great that the cyclist, if he be travelling quickly, falls over the handle-bar of his machine This law of inertia is often referred to as Newton's First Law of Motion.

First law of motion.—**Every object remains at rest or moves with uniform velocity in a straight line until compelled by force to act otherwise**

This law, which Newton first stated as being obeyed always by bodies in nature, means, first, that if a body is at rest, it will remain still until there is some reason for its moving—until some outside influence or force acts upon it. Consider a mass at rest somewhere in infinite space, evidently it will remain at rest so long as it is not acted upon by an external force. Consider, again, that another body is moving in infinite space in a straight line; it will continue to move in this direction until compelled to deviate from this path by some external force acting upon it.

The force of gravitation.—Experiments and observations made by Newton led him to the conclusion that it was the rule of nature for every material object to attract every other object, and that this force of attraction is proportional to the masses of the bodies; a large mass exerts a greater force of attraction than a small mass. But the farther these bodies are apart the less is the attraction between them, though it is not less in the

proportion of this distance, but in that of the square of the distance. This law may be stated thus:

Every body in nature attracts every other body with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between the bodies; and the direction of the force is in the line joining the centres of the bodies.

Consider the case of a cricket ball on the top of a house The earth attracts the ball, and, by Newton's law, the ball attracts the earth. The ball, if free to move, falls to the earth; to be correct, however, we must think of the ball and the earth moving to meet one another along the line joining their centres. But the ball moves as much farther than the earth as the earth's mass is greater than that of the ball; and for practical purposes this is the same as saying that only the ball moves and that the earth remains still

This force of attraction between all material bodies is called the force of gravitation, but we must again point out that this is only a name Calling this force 'gravitation,' and the rule according to which it acts the 'law of gravitation,' does not teach anything about the nature of the force itself.

Graphic representation of forces.—Every force has a certain magnitude, and acts in a certain direction. It is, therefore, possible to represent a force completely by a line, the length of which is proportional to the magnitude of the force and the direction of which represents the direction in which the force is exerted. If the length of an inch be taken to represent a unit force, then a force of 5 units would be represented by a line 5 inches long, and two forces of 5 and 3 units acting together in the same direction would be represented by a line 8 inches long. If, however, a body were acted upon by a force of 5 units in one direction, and 3 units in the opposite direction, then the effect would be that of a force of 2 units acting in the direction of the force of 5 units; for 3 of the units of this force would be rendered inoperative by the three units acting in the opposite direction

Parallelogram of forces.—A body can only move in one direction at any given instant, though it may be acted upon by any number of forces. Each force has a certain magnitude and acts in a certain direction, and, in consequence of their joint

action, the body moves with a certain velocity, if it be free to do so. The same velocity could be given to the body by a single force instead of the separate forces, and *the single force which would produce the same effect as the separate forces* is called the **resultant** of the forces. Any system of forces acting upon a particle is equivalent to a single resultant force. When two forces act upon a body at the same time, their resultant usually can be found by means of the parallelogram of forces, which may be expressed thus: **If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram, the resultant of these two forces will be represented in magnitude and direction by that diagonal of the parallelogram which passes through this point**

Let O represent a material body acted upon by two forces, represented both in magnitude and direction by the lines OB, OA (Fig. 73). To find the *resultant* of these two forces, both as regards its amount and direction, complete the parallelogram OBRA and join OR, which will be the resultant required

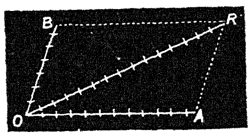


FIG. 73.—Graphic representation of the parallelogram of forces

The **equilibrant** of two or more forces acting on a body is the single force which, acting with them, maintains the body at rest. It is evident, therefore, that the equilibrant is equal in magnitude, but opposite in direction, to the resultant.

EXPT. 81 —**Demonstration of the principle of the parallelogram of forces.** Lay a large sheet of paper on a flat table (Fig 74) Connect three threads together at a point O. Pass one of the threads over a pulley wheel P, clamped firmly near to the edge of the table, and fasten a known weight to the end of the thread. Join the other threads to spring dynamometers, D_1 and D_2, to which graduated scales are attached Allow the known weight to hang freely. The point O is now in equilibrium under the action of the *two forces* (exerted by the dynamometers) and their *equilibrant* (which, in this case, is the weight hanging over the pulley). Mark on the paper, by means of a needle point, the direction of the three threads; and write down the magnitudes of the three forces. Remove the paper, and mark off the lengths OE, OF_1, and OF_2 proportional to the three forces. Complete the parallelogram, constructed with OF_1 and OF_2

as adjacent sides; and draw the diagonal **Or**. According to the principle of the parallelogram of forces, this diagonal should be equal in magnitude and opposite in direction to the equilibrant OE. Produce OE backwards, and mark off a length OR equal to OE. If the experiment is carried out with care the lines OR and **Or** should be identical in length and in direction.

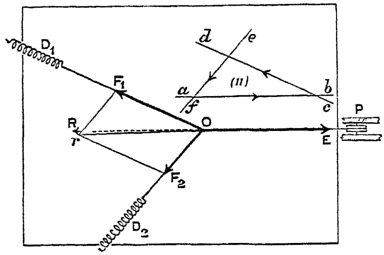


FIG 74.—Experiment on the parallelogram of forces

(ii) Alter the directions of the forces OF_1 and OF_2, and again observe whether the true resultant coincides with that deduced by geometrical construction.

EXPT 82.—**The triangle of forces** On the same sheet of paper used in Expt. 81 draw three lines *ab*, *cd*, and *ef* parallel to the forces OE, OF_1, and OF_2 respectively (Fig. 74) These lines enclose a triangular area. Measure the lengths of the sides of the triangle thus obtained. These three lengths will be found to be proportional to the three forces respectively. Mark, by means of arrow-heads, the directions of the forces. It is evident, therefore, that **if three forces acting at a point are in equilibrium they can be represented in magnitude and direction by the sides of a triangle taken in order.** This is known as the principle of **the Triangle of Forces.**

Geometrical determination of the resultant of two forces.—When the two forces of which the resultant is required act at right angles to one another, the calculation is a simple application of a proposition in the first book of Euclid (I. 47). In these circumstances the triangle ORA (Fig. 73) is right-angled, and it is easily proved that $(OA)^2+(AR)^2=(OR)^2$;

consequently $(OA^2)+(OB)^2=(OR)^2$,

from which when OA and OB are known we can calculate OR.

When the directions of the two forces OB and OA are inclined to each other at an angle which is not a right angle, the calculation involves an elementary knowledge of trigonometry. This can be obviated, however, by the simple expedient of what is called the graphical method. This consists in drawing two lines inclined at the angle at which the directions of the forces

are inclined, and making them of such lengths that they contain as many units of length as the forces do units of force (Fig. 73). The parallelogram is then completed by drawing AR and BR parallel respectively to OB and OA and joining the diagonal OR, the direction of which will be that of the resultant, and its length will contain as many units of length as there are units of force in the resultant force. It is immaterial what lengths are used to represent the units of force so long as the components and the resultant are measured in the same units

Resolution of forces.—A single force can be replaced by other forces which together will produce the same effect. Such a substitution is called **resolving** the force, or a **resolution of the force.** The parts into which it is resolved are spoken of as **components.** When this has been done it is clear that we have made the original force become the resultant of certain other forces which have replaced it. Referring back to what has been said about the parallelogram of forces, it will be seen that any single force can have any two components in any directions we like, for by trying, the student will be able to make any straight line become the diagonal of any number of different parallelograms. The most convenient components into which a force can be resolved are those the directions of which are at right angles to each other. In this method of resolution, neither component has any part in the other.

A kite at rest in the air affords an example of the principle of the parallelogram of forces (Fig. 75). There are two downward forces—one represented by AB, due to the weight of the kite, and the other represented by AD, due to the pull of the string. The pressure of the air on the face of the kite can be resolved into two forces, one acting along the face and the other at right angles to it. The latter force is an upward one, and if the kite is at rest it is equal

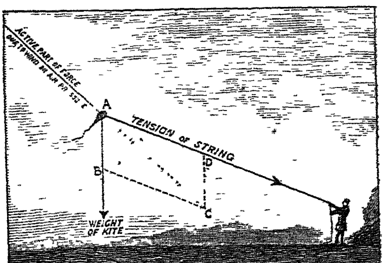


FIG. 75.—Forces acting on a kite at rest in the air.

to the resultant AC of the two downward forces. If it is greater than the resultant AC, the kite rises; if it is less, the kite falls.

EXPT. 82A.—**The polygon of forces.** Place a sheet of paper on a table or bench. Tie four threads together at a point O (Fig. 75A), and attach the free ends to four separate spring dynamometers. Fasten the upper ends of the latter to different points of the table, adjusting the tension of each so that a suitable elongation of the springs is obtained. Let the forces exerted by the springs be denoted by F_1, F_2, F_3, F_4. Mark on the paper the directions of these forces; and mark off lengths OF_1, OF_2, etc., proportional to their magnitudes. The point O is now in equilibrium under the action of the four forces represented.

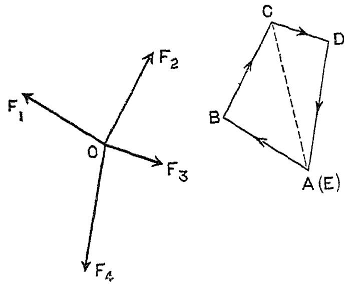


FIG 75A —The polygon of forces

As in Expt. 82, construct a *force-diagram* by drawing lines AB, BC, CD and DE parallel to, and proportional to the magnitudes of, the forces F_1, F_2, etc. Mark with arrowheads the direction of the forces. Notice whether the point E coincides with the point A. If the measurements are accurate, these points will coincide, and the force-diagram will completely enclose a space.

This experiment is an application of the principle of **the polygon of forces,** which may be stated thus:—**If any number of forces acting at a point are in equilibrium, they may be represented in magnitude and direction by the sides of a polygon taken in order.** It is seen readily that this is simply an extension of the principle of the triangle of forces. For, in Fig. 75A, if the points A and C are joined, CA represents the equilibrant, and AC represents the resultant of AB and BC. Thus the force-diagram is reduced to the triangle ACD, and DA is the equilibrant, and AD is the resultant, of the forces AC and CD.

The principle can be used for finding the resultant of any number of forces acting at a point. For, if a force-diagram be constructed from the given forces, *taken in order*, then the line joining the last point to the first point represents the equilibrant of all the forces. The same line, with the direction reversed, represents the resultant of all the forces.

EXERCISES ON CHAPTER VIII.

1. A line is drawn upon the floor of a railway carriage from door to door. When the carriage is at rest a ball is dropped from the roof and falls upon this line. What difference would be observed:

(*a*) If the train is moving when the ball is dropped?

(*b*) If the train starts when the ball is half way down?

(*c*) If the ball is dropped when the train is in motion, but the train stops suddenly when it is half way down?

2. A man walks backwards and forwards on the deck of a steamer along a line parallel to the direction in which the steamer is moving. If the man walks at the rate of 3·5 miles an hour, and the steamer goes through the water at the rate of 8·4 miles an hour, what is the velocity of the man with reference to the water (1) when he is walking towards the bow of the boat, and (2) when he is walking towards the stern?

3. What is meant by acceleration? Give examples of uniformly accelerated velocities (*a*) where the acceleration is positive, (*b*) where the acceleration is negative.

4. Four forces act at a point. The first of 10 lb. acts due north, the second of 15 lb. due east, the third of 20 lb. due south, and the fourth of 25 lb. due west. Find the magnitude and direction of the resultant.

5. As two ships pass in opposite directions a person in one of them throws a ball to a person on the other. How must he aim? Draw a diagram to explain your answer.

6. Explain what is meant by the inertia of a material body. Give as many of the results of the possession of this property by a material body as you can.

7. The horizontal and vertical components of a certain force are equal to the weights of 60 lb. and 114 lb. respectively. What is the magnitude of the force?

8. Describe an experiment for demonstrating the principle of the parallelogram of forces to a class.

A nail is driven into a wall and two strings are tied to its head. When the two strings are pulled horizontally and at right angles to one another with forces equal to 6 and 8 lb. respectively, the nail is dislodged. What force would be needed if the strings were brought together and the nail pulled straight out? Illustrate your answer with a diagram.

9. Two forces, the magnitudes of which are proportional to the numbers 3 and 4, act on a point at right angles to each other. Draw a parallelogram as nearly to scale as you can to show the direction and magnitude of the resultant, and deduce by measuring your diagram, or in any other way, the magnitude of the resultant.

10. Two forces, P and Q, act upon a body. If P acted alone it would, in two seconds, produce in the body a velocity of 10 feet per second, while if Q acted alone it would in three seconds produce in the body a velocity of 18 feet per second. What velocities will P and Q produce in one second when acting together if the directions in which they tend to move the body are—(1) in the same direction; (2) directly opposed?

11. A weight which is hung by means of a piece of elastic from a nail in the ceiling is pulled some way to one side by a thread which is always kept horizontal. Explain why this operation will increase the stretching of the elastic. Illustrate your answer by a diagram.

12. Forces of 3 gm., 4 gm., and 5 gm. act at a point, and are in equilibrium. What are the angles between their lines of action?

13. Forces of 2 gm., 4 gm., and 5 gm. act at a point, and are in equilibrium. What are the angles between their lines of action?

14. A picture of mass 3 lb. hangs vertically from a nail by a cord attached to rings at the two upper corners of the frame. If the string and the upper edge of the frame form an equilateral triangle, find the tension in the string.

15. A mass of 30 gm. is suspended by a string. What horizontal force is required to displace it until the string makes an angle of 30° with the vertical?

16. A heavy weight is suspended from the end of a piece of string. A piece of the same string is attached to the lower surface of the weight. If the latter string is pulled with a sudden jerk it will snap; but if it is pulled gradually the upper string will be the first to break. Explain this.

17. A projectile weighing 560 lb. is fired from a gun weighing 40 tons with a velocity of 1600 ft. per second. Find the velocity of recoil.

18. A hammer head of $2\frac{1}{2}$ lb. moving with a velocity of 50 ft. per second is stopped in 0·001 second. What is the average force of the blow?

19. What do you understand by

(*a*) velocity, (*b*) acceleration, (*c*) average velocity?

A body starting from rest moves with an acceleration of 3 centimetre second units; in what time will it acquire a velocity of 30 cm. per second, and what distance does it traverse in that time?

20. A body falling freely under gravity drops s ft. in t sec. from the time of starting. If corresponding values of s and t at intervals of half a second are as follows:

t	0·5,	1,	1·5,	2,	2·5,	3,	3·5,	4,
s	4,	16,	36,	64,	100,	114,	196,	256,

draw a curve connecting s and t, and find from it (i) the distance through which the body has fallen after 1 min. 48 sec.; (ii) the distance through which it drops in the 4th second.

CHAPTER IX.

WORK. FRICTION. ENERGY.

Work.—When a man endeavours to raise a heavy mass from the floor, he applies a force, acting vertically upwards, by means of the muscles. At first the mass does not move, because the force of gravitation pulling the mass downwards is greater than the force exerted by the muscles upwards; so far no work has been done by the muscles. When the force applied is exactly equal to the force of gravitation, the mass no longer exerts any pressure on the floor, but it does not move: in such a case it is only necessary to increase the upward pull to the slightest degree in order to make the mass commence to move upwards; then the muscles begin to do work.

We thus obtain the following definition of work: **Work is done when the point of application of a force moves.**

The work done is proportional to the force overcome and to the distance through which the force has been overcome; or,

Work = force overcome × distance.

Unit work is done when unit force is overcome through unit distance.

For practical purposes the **unit of work** which is adopted is **the work done in raising the mass of one pound through a vertical distance of one foot**, and it is called the **foot-pound.** This is not a strictly constant unit, for it will be evident, in the light of what has been said about the weight of a body, that where the weight is greater the amount of work done will be greater. The unit of work will vary slightly in different latitudes in a precisely similar manner to that in which the weight of a mass varies.

Another unit adopted frequently in practical work is the **Kilogram-metre,** which is equivalent to **the work done in raising a mass of one kilogram through a vertical distance of one metre.**

In the metric system, the **absolute unit of work,** called the **erg,** is **the quantity of work done when a force of one dyne is overcome through a distance of one centimetre.** Thus, nearly 1 erg of work is done when 1 mgm. is raised vertically through 1 cm In practice the erg is often found to be inconveniently small, and a unit called the **Joule,** which is equal to 10^7 ergs, is then used.

EXAMPLE.—How much work is done when an engine weighing 12 tons moves a mile on a horizontal road, when the total resistance is equal to a retarding force of 10 lb. weight per ton?

The total resistance equals $12 \times 10 = 120$ lb weight, the distance traversed is 5280 feet.

$$\text{work done} = (120 \times 5280) \text{ foot-pounds.}$$

It is important to bear in mind that the *distance* through which a force is overcome must be measured *in the direction in which the force is acting*: thus, in conveying a weight to the top of a building, the work done on the weight is the same when it is lifted vertically by means of a pulley as when it is carried to the same height by means of a sloping ladder or a spiral stair-case.

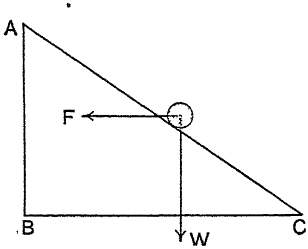


FIG. 76.—Work done by a force

Similarly, if a weight W be raised up an inclined plane (Fig. 76) from B to A by means of a force F acting horizontally, the work done by the force is measured by the product $F \times BC$, since BC is the distance through which the point of application of the force moves *in the direction of* the force. On the other hand, the work done against gravity is equal to the product $W \times AB$. Hence, if the whole of the work done is spent in raising the weight, and none of it is absorbed in overcoming resistance due to friction,

$$F \times BC = W \times AB,$$

or

$$F/W = AB/BC.$$

This result introduces one of the fundamental principles of the *inclined plane*, which is discussed more fully in Chap. XI.

EXAMPLE.—How much work is done against the force of gravity

by a horse in drawing a load of $\frac{1}{2}$ ton along a road 1 mile long which rises 1 in 30?

The distance through which the vertical force of gravity is overcome is equal to $(\frac{1}{30} \times 5280)$ ft., and the force overcome is equal to 1120 lb. Hence,

$$\text{Work done} = 1120 \times (\tfrac{1}{30} \times 5280) = 197,120 \text{ ft. lb.}$$

Power.—It will have been noticed that the question of time does not enter into an estimation of the amount of work done. It is manifest that the same quantity of work is accomplished whether a day is spent in raising a weight to a given height from the ground or only a minute. If we introduce the time taken to perform the work we begin to consider what is called the **power** of the agent. We should measure this power by the quantity of work the agent can perform in a given time; or **power is the rate of doing work** and is measured by the work done in a second. Thus, engineers use the expression **horse-power,** by which they mean the rate at which a good horse works. James Watt estimated this at 33,000 foot-pounds per minute, or 550 foot-pounds a second.

EXAMPLE.—A man, weighing 10 stone, runs up a flight of stairs to a height of 30 ft. in 5 seconds. Express, in horse-power, the rate at which he does work against the force of gravity.

$$\text{Work done in 5 seconds is } (140 \times 30) \text{ ft. lb.}$$

$$\text{,, \quad ,, \quad 1 second is } \frac{140 \times 30}{5} \text{ ,,}$$

$$\text{rate of doing work} = \frac{1}{550} \times \frac{140 \times 30}{5} = 1 \cdot 70 \text{ horse-power.}$$

FRICTION.

Friction.—When a rectangular wooden block is resting on a horizontal table, a small force may be applied horizontally to the block without causing it to move. The reason for the block remaining at rest is that the force applied to it is neutralised by an equal and opposite force which tends to keep the block at rest and is located between the two surfaces in contact: this latter force may be expressed more accurately as a *stress*, and it is called into play by **friction** between the two surfaces.

When the force is increased gradually, the opposing stress due to friction increases at the same rate until a certain maximum is

reached, which the stress cannot exceed; if the applied force slightly exceeds this maximum the block begins to move The magnitude of this maximum force measures what is termed the **limiting friction** When motion has commenced it will be found that a smaller force is sufficient to maintain the body in motion. Hence, the stress due to friction between two surfaces in relative motion—termed the **sliding friction**—is less than the limiting friction

EXPT 83.—**Friction between wood and wood.** Fix a small staple into a middle point a (Fig. 77) of one of the smaller faces of a rectangular block of wood. Weigh the block, and lay it on a clean horizontal wooden surface. Attach a spring dynamometer to the staple, and gradually apply an increasing horizontal force P to the block Note the reading of the dynamometer (i) at the instant when motion is just beginning, and (ii) when motion is just maintained. Repeat the observation several times, being careful to use in each case the same portion of the table top.

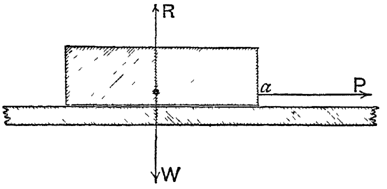


FIG 77.—Experiment on friction

Increase the load W by placing weights of 100, 200, 300, . grams on the top of the block; and measure for each load the limiting and sliding friction. Tabulate the observations, and plot them on squared paper, taking loads as abscissae and the limiting friction as ordinates. What conclusion may be derived from the curve obtained?

Turn the block so that it rests on one of its smaller faces. Repeat the previous observations, and determine whether the limiting friction, when the load is constant, depends upon the area of the surfaces in contact.

EXPT. 84—**Friction between glass and glass.** Repeat Expt 83 using a glass slab resting on a clean horizontal glass surface Plot on squared paper, to the same scale as that adopted in the previous experiment, the readings obtained for the load and the limiting friction.

Smear a few spots of oil over the glass surfaces and determine whether the limiting friction and the sliding friction are influenced thereby.

Laws of friction.—By means of these experiments it is possible to verify, though perhaps roughly, the main laws of friction. These may be stated as follows:

(i) The stress due to friction is greater between two surfaces at rest than when in relative motion

(ii) The friction is proportional to the load.

(iii) The friction is independent of the area of the surfaces in contact.

The coefficient of friction.—The experiments will have shown also that the limiting friction depends upon the nature of the surfaces in contact: thus, it is evidently greater between wood and wood than between glass and glass. Also, it will be evident from the curves obtained that the ratio

$$\frac{\text{limiting friction (P)}}{\text{pressure between the surfaces (W)}}$$

is a constant quantity for any given pair of surfaces. This ratio is termed the **coefficient of friction** (μ).

ENERGY.

Energy.—All moving bodies possess energy. Moving air or wind drives round the sails of a windmill and so works the machinery to which the sails are attached, it drives along a ship, thus overcoming the resistance of the water. The running stream works the mill-wheel and the energy it possessed is expended in grinding corn. The bullet fired from a rifle can pierce a sheet of metal by overcoming the cohesion between its particles It may therefore be said that **the energy of a body is the power of overcoming resistance or doing work.**

EXPT. 85.—Stretch a piece of tissue-paper over the top of an empty jam-pot. Carefully place a bullet on the paper and notice the paper will support it. Now lift the bullet and allow it to drop on to the paper. It is seen that the bullet pierces the paper.

EXPT. 86.—Support a weight by a thin thread. Show that though the thread will support the weight at rest it will be broken if the weight is allowed to fall.

EXPT. 87.—Show that a falling weight attached by a string to a spring balance extends the balance beyond the point which it indicates when the weight is at rest.

All these examples are cases of the energy of moving bodies, or the energy of motion, or *Kinetic Energy*. **Kinetic Energy is the energy of matter in motion All energy which is not kinetic is known as Potential Energy.** Potential energy is capable of becoming kinetic or active when the conditions become suitable Imagine a mass raised from the ground and placed upon a high shelf. We know that to place it in this position we must expend a certain amount of work, which is measured by multiplying its weight by the height through which it is raised. Further, we know that just so soon as we release it from its position of rest, making it free to move, it will travel with an ever-increasing velocity until it reaches the ground. On the shelf the mass, *by virtue of its position*, possessed a certain amount of *potential energy* exactly equal to the work expended in placing it there.

Similarly, an ordinary dining-room clock, which is worked by a spring, affords us an example of potential energy. The wound-up spring possesses potential energy exactly equal to the amount of work done in winding it up. This potential energy is being converted into kinetic energy continually as the spring becomes unwound in working the clock.

The kinetic energy of a mass **m** moving with a velocity **v** is expressed numerically by half the product of the mass and the square of the velocity. For, suppose the mass to be initially at rest, and to be acted upon by a constant force; then, after any interval of time, **the kinetic energy of the mass is equal to the work done by the force.** If, at any moment, the body has acquired a velocity **v**, then

$$
\begin{aligned}
\text{Kinetic energy} &= \text{force} \times \text{distance through which it has acted} \\
&= (\text{mass} \times \text{acceleration}) \times (\text{average velocity} \times \text{time}) \\
&= \left(\text{mass} \times \frac{v}{\text{time}}\right) \times \left(\frac{0+v}{2} \times \text{time}\right) \\
&= \frac{mv^2}{2}.
\end{aligned}
$$

If **m** be given in lb. and **v** in ft. per second, then the kinetic energy is in *foot-poundals*. This can be reduced to foot-pounds by dividing by 32

The motion of a pendulum affords an interesting example of the two forms of energy. At the end of its swing, in the position

A (Fig. 78), the bob of the pendulum possesses potential energy enough to carry it through half a single vibration, that is, until it reaches its lowest position N, when the whole of the energy of position which it possessed at A is expended, as it can reach no lower position. But though it lacks potential energy, since it is a mass moving with the velocity it has gained in its passage from A to N, it possesses energy of motion or kinetic energy enough to carry it up to its next position of rest at A′—where the only energy it will have will be again potential. Through the next vibration from A′ to A it will pass through just the same transformations again.

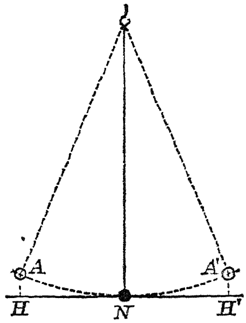


FIG 78.—Pendulum in oscillation

At any point in the swing the pendulum will possess a certain amount of energy due to position and a certain amount due to motion, but the total amount of energy—the sum of the potential and the kinetic energy—is always the same, the loss of one form of energy being equalised exactly by the gain of the other.

Forms of energy.—A body may possess energy due to other causes than that of the actual motion of the body as a whole. When it is in rapid vibration, or when it is heated, or when it is electrified, it is endowed with energy in consequence of these conditions. But when a body is in rapid vibration it gives out sound or becomes a sounding body, hence we may regard sound as a form of energy. We shall see that work may be done by the passage of heat from a hot body to a cold one, and, in consequence, heat is regarded properly as another form of energy. An intensely hot body emits light, hence it would seem that light and heat have a common cause and that we must also regard light, like heat, as a manifestation of energy. When a body is electrified it has the power of attracting unelectrified and certain electrified bodies also, and when such bodies are attracted as a result of this electrification we see that electrification must be looked upon similarly as still another kind of energy. But it must be borne in mind that electrification is not electricity. Then, too, there is the attraction of magnetism, which is capable

of accomplishing work, and hence must be looked upon as a form of energy. Chemical combinations, again, are accompanied always by the development of heat, and resulting as they do from the chemical attraction of two more or less unlike forms of matter, we shall be right in saying chemical combinations are accompanied always by energy changes, and so in regarding chemical attraction as another kind of energy. Therefore, **in addition to the energy of moving bodies we have energy manifested as sound, heat, light, electrification, magnetism, and chemical action.**

Transformation of energy.—Energy can cease to exist in one particular form and can assume another form. Thus, the energy of moving bodies can give rise to sound and heat; heat can be changed into the energy of moving bodies, electric currents, chemical action, and so on The general tendency of all forms of energy is gradually to get converted into heat. In the course of ages the change may become complete, the universe would then exist at one temperature and no further transformation would be possible. When this imaginary condition is realised by the degradation of all energy to one heat-level the universe will be dead and no movements of any kind will take place.

Conservation of energy.—**Energy, like matter, is indestructible.** The total amount of energy in the universe remains the same. One form may be changed into another, but we can create no new energy. We may be unable to trace and account for some of it in the numerous transformations which it undergoes, but we are sure, from many considerations, that if the methods of experiment were only refined enough, we should be able to account for the whole amount

Work done in overcoming friction.—When two surfaces are rubbed together—as, for example, when a wooden block is pulled slowly along a table—work is done; yet neither of the bodies rubbed together acquires either kinetic energy or an increase of potential energy. The work is spent in overcoming friction (p. 113), and the energy equivalent in amount to this work done appears in the form of heat generated between the two surfaces.

Since the total amount of available energy is constant, a certain quantity of mechanical work must be capable of conversion into a certain quantity of heat—or, in other words, they

must be equivalent to each other This relationship has been subjected to most rigorous experimental proof, and the quantity of mechanical work which corresponds to unit quantity of heat is termed the **mechanical equivalent of heat** (p. 225).

EXERCISES ON CHAPTER IX.

1. What is the meaning of the terms velocity, mass, force, work, energy, inertia?

2. Explain the terms 'foot-pound' and 'horse-power.' How much work is done in raising 25 weights of 56 lb. each from the ground to a height of 4 feet?

3. A man can pump 25 gallons of water per minute to a height of 16 feet How many foot-pounds of work does he do in an hour?

4. A ladder 20 ft. long rests against a vertical wall and is inclined at 30° to it. How much work is done by a man weighing 10 stone in ascending it?

5. What should be the indicated horse-power of an engine which is intended to pump 250 gallons of water per minute to a height of 40 yards?

6. The mass of a train is 250 tons, and the resistances to its motion on a level line amount to 15 lb per ton. Find the horse-power of the locomotive which can maintain a speed of 40 miles per hour on the level

7. What is the difference between kinetic energy and potential energy?

8. Define work, and describe an experiment to prove that a falling ball is capable of doing work

9. What is meant by a foot-pound of work? What is the value of a horse-power in terms of this unit?

10. How is kinetic energy measured? If we wish to express the result in foot-pounds, how do we proceed?

11. A man weighing 140 lb. puts a load of 100 lb on his back and carries it up a ladder to a height of 50 feet How many foot-pounds of work does he do altogether and what part of his work is done usefully?

12. A body weighing 10 lb is placed on a horizontal plane and is made to slide over a distance of 50 feet by a force of 4 lb What number of units of work is done by the force?

13. If a man can work at the rate of 210,000 foot-pounds an hour, how long would it take him to raise a weight of 10 tons through 150 feet, supposing him to be provided with a suitable machine?

14. A horse pulling a horizontal trace with a force equal to the weight of 72 lb., draws a cart along a level road at the rate of $3\frac{3}{4}$ miles per hour. What amount of work is done by the horse in 5 minutes?

15. A cannon-ball the mass of which is 60 lb. falls through a vertical height of 400 feet. What is its energy at the end of its fall?

16. What is the kinetic energy of a mass of 5 lb. moving with a velocity of 10 feet per second? State clearly what the unit is in terms of which your answer is expressed.

17. A body having a mass of 10 lb. is carried up to the top of a house 30 feet high. By how many foot-pounds has the change of position increased its potential energy? If it be allowed to fall, what number of foot-pounds of kinetic energy will it have when it reaches the ground?

18. Describe an experiment to prove that energy due to visible motion can be transferred from one body to another.

19. What proof can you adduce that the energy of visible motion can be transformed into heat?

CHAPTER X.

THE LEVER. PARALLEL FORCES. CENTRE OF GRAVITY.

The lever.—A lever is a rigid bar which can be turned freely about a fixed point. The **fulcrum** of a lever is the fixed point about which the lever can be turned. The force exerted when using a lever is often described as the **Power** and the body lifted or resistance overcome as the **Weight.** These words are convenient, but they are not used correctly in connection with levers, as their true meanings are confused by so doing. It is better to substitute the word **effort** for power, and **resistance** or **load** for weight. It should be borne in mind, that, so far as mechanical principles are concerned, there is no difference between the power and the weight, both represent forces, and as such they must be considered in the action of levers.

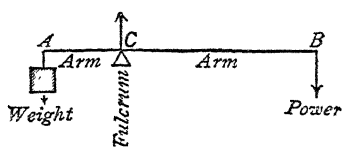


FIG 79—Terms used in connection with levers

The perpendicular distances from the fulcrum to the lines of actions of forces acting upon a lever, are known as the **arms** of the lever. In Fig. 79 the distance AC is the arm of the end at which the load acts, and BC is the arm of the end at which the effort acts

The principle of the simple lever has been learnt previously, in Expt. 27. It was proved that the

$$\begin{matrix}\text{Weight on one}\\ \text{side of fulcrum}\end{matrix} \times \begin{matrix}\text{Perp. distance}\\ \text{from fulcrum}\end{matrix} = \begin{matrix}\text{Weight on}\\ \text{other side}\end{matrix} \times \begin{matrix}\text{Perp. distance}\\ \text{from fulcrum.}\end{matrix}$$

Each of these products is termed the **moment of the force** about the fulcrum: the following definition is important.

The moment of a force about any point is the product obtained by multiplying the force by the perpendicular distance between the point and the line of action of the force.

The law of moments.—Refer to the diagram (Fig. 80), where a lever AB is in equilibrium under the action of two forces, M_1 and M_2. These forces *tend* to rotate the lever in an anti-clockwise and a clockwise direction respectively. It is agreed generally that these directions of rotation shall be denoted by the algebraic symbols *plus* and *minus*. Hence,

FIG 80

The moment of M_1 about F is written $+(M_1 \times AF)$, and
" " M_2 " " $-(M_2 \times BF)$.

Since these moments are numerically equal, it follows that the algebraic sum of the moments is zero. Hence, **when a lever is in equilibrium under the action of two (or more) forces the algebraic sum of the moments of the forces about the fulcrum is zero.** This deduction is termed the **Law of Moments.**

EXPT. 88 —**Sum of moments.** A slight modification in Expt 27 will afford a proof that the Law of Moments is true when three or more forces are acting upon the lever. Thus, three weights may be suspended from the lever, and their position adjusted so that the lever is in equilibrium. The *algebraic* sum of the moments will be found to be equal to zero

Classes of levers.—For convenience, levers are sometimes divided into three orders or classes, according to the relative positions of the fulcrum, resistance, and effort. This classification is, however, of no real consequence; for the principle underlying the action of all levers is the same. The following experiments illustrate the relation between the fulcrum, resistance, and effort in each of the three cases:

EXPT. 89.—**Fulcrum between resistance and effort.** Using the lever described on page 36, show that a weight placed at a short distance

from the fulcrum on one side can be moved by a much smaller weight placed at a proportionally greater distance on the other side of the fulcrum. A lever of this kind is represented by a see-saw, a pump-handle, a balance, and a spade used in digging.

Find the weight of a piece of metal by means of a simple lever and a box of weights

EXPT 90.—**Resistance between effort and fulcrum.** Pivot a metre scale at its centre (Fig. 81) Suspend a weight R from any point A Tie one end of a long thread round the lever at any point B, pass the thread over a pulley, and fasten a scale pan (of known weight) to the other end. Adjust the weights in the scale pan until the lever is in equilibrium. The effort P is then equal to the sum of the weights of the scale pan and of the weights supported on the pan. Show that the algebraic sum of the moments, $+(P \times FB) - (R \times FA)$, is equal to zero.

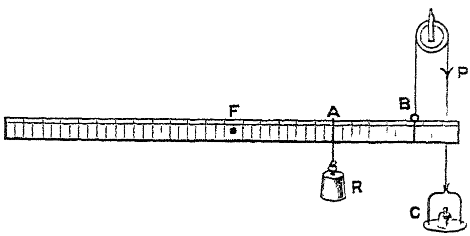


FIG 81 —Experiment with a lever

Modify the magnitudes of the weights and of the arms several times; and in each case find whether the same deduction is obtained A common kind of nutcrackers and a wheelbarrow are examples of levers of this type.

EXPT 91 —**Effort between fulcrum and resistance** Use the same apparatus as in the previous experiment, but interchange the points of application of the effort and of the resistance. Take a series of observations, and determine whether the Law of Moments still holds good

Examples of this class of lever are furnished by sugar tongs, ordinary fire-tongs, and the pedal of a grindstone.

Parallel forces.—It has been seen that the earth exerts a downward pull upon all objects on its surface, and that in consequence of this all things fall to the ground if unsupported It follows, therefore, that everything which is supported above the earth's surface is being pulled downwards constantly, even though it does not fall. When a beam, for instance, is supported

horizontally by resting the ends upon two posts, each particle of it may be regarded as being pulled earthwards by an attractive force. The direction of the pull is everywhere towards the centre of the earth, so, in view of the distance of the earth's centre from its surface, for any one spot on the earth's surface we may consider the attractive forces due to gravity to be parallel to one another.

When a stiff lath or rod of uniform thickness rests upon two letter balances, or is supported by hanging each end from a spring balance, the experiment represents on a small scale the case of the beam referred to before; and by using spring balances it can be proved that the weight supported at its ends is equally divided between the two supports. In other words, the two upward forces exerted by the balances are together equal to the downward force represented by the weight of the beam.

If a load be placed anywhere upon the lath, the balances still show that when the lath is in equilibrium the sum of the upward forces is equal to the sum of the downward forces.

In the experiments with a lever having the fulcrum between the resistance and effort, the lever remains at rest although acted upon by two forces tending to pull it downwards: evidently the fulcrum must be exerting a force exactly equal, and opposite in direction to the combined forces acting downwards; or, in other words, the force exerted by the fulcrum must be equal and opposite to the resultant of the downward parallel forces. The following experiment will serve to demonstrate, (i) the magnitude, and (ii) the point of application, of the resultant of two or more parallel forces.

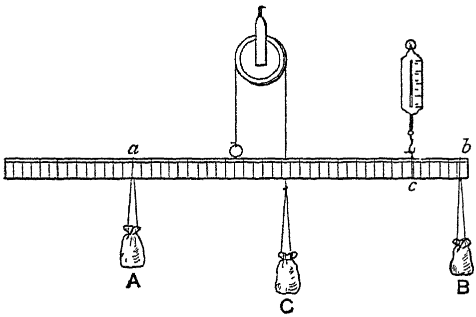


FIG 82 —Parallel forces in equilibrium.

EXPT. 92.—**Parallel forces.** Suspend the lath by a string which passes over a pulley and has attached to the other end a weight equal

to the weight of the lath (Fig. 82). The rod can then move as if it had no weight

An upward force can be applied to it by a spring balance, or by a weight attached to a string passing over a pulley, and downward forces by hanging weights from it.

Attach a spring balance to any convenient point on the lath by passing the hook of the balance through a hole in the lever or by means of thread. Suspend weights A and B from the lath so that they counterpoise one another (Fig. 82) Notice the reading of the spring balance, and the weights used.

Since the lath is in equilibrium, the experiment would not be affected if the lath were pivoted *at any point along its length.* Suppose that the lath were pivoted at the point *c*; then the algebraic sum of the moments of the weights of A and B about the point *c* should be equal to zero. Calculate these moments, and verify this deduction. Record your observations thus

Sum of Weights (A+B)	Reading of Spring Balance	Moment of A about *c*	Moment of B about *c*

The force exerted by the spring balance must, necessarily, be equal and opposite to the resultant of the downward parallel forces, and the point of application of the resultant is determined by the fact that the algebraic sum of the moments of the component parallel forces round that point must be equal to zero.

Centre of gravity.—Consider a large number of weights, some heavier than others, suspended from a horizontal rod A certain position can be found at which a spring balance has to be attached in order to keep the rod in equilibrium. When the rod is hung from this point the tendency to turn in one direction is counteracted by the tendency to turn in the other, so the rod remains horizontal The weights may be regarded as parallel forces, and the pull of the spring balance as equal to their resultant. Now consider a stone, or any other object, suspended by a string. Every particle of the stone is being pulled downwards by the force of gravity, as indicated in Fig. 83. The resultant of these parallel forces is represented by the line GF, and the centre of the forces is the point G. The point G, through

which the resultant (GF) of the parallel forces due to the weights of the individual particles of the stone passes, is known as the **centre of gravity.** For the stone to be in equilibrium, the string must be attached to a point in the line GF, produced upwards.

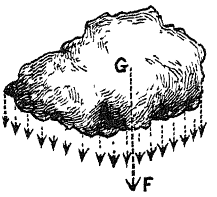


FIG. 83.—Parallel forces due to gravity.

Every material object has a centre of gravity, and the position of this point for a particular object is the same so long as the object retains the same form. The centre of gravity need not be, however, a point on the actual object.

Geometrical determination of centres of gravity.—It has been explained sufficiently that the centres of gravity of straight lines, circles, squares, and other regular figures are at their geometrical centres. Hence, the geometrical constructions for determining these central points also locate the position of their centres of gravity.

The centre of gravity of a parallelogram is at the intersection of its diagonals.

The centre of gravity of a triangle is determined by bisecting any two sides and joining the middle points so obtained to the opposite angles. The intersection of the lines so drawn gives the centre of gravity. The centre of gravity is found, by measuring, to be one-third the whole length of the line drawn from the middle point of the side to the opposite angle, away from the side bisected.

To find the centre of gravity of a quadrilateral by construction, divide the figure into two triangles by drawing a diagonal. By the method just described the centre of gravity of each triangle is found, and the points so obtained are joined. The centre of gravity of the quadrilateral lies on this line. Repeat the process by drawing the other diagonal. Join the centres of gravity of the second pair of triangles, the centre of gravity of the quadrilateral lies on this line. Hence, it is situated at the point of intersection of this line and the first one obtained in the same way.

Experimental methods of determining centres of gravity.—In the case of unsymmetrical figures the centre of gravity cannot be found easily by geometry, and is best determined by experiment.

EXPT 93.—**Thin uniform sheet.** Procure a disc or an irregular piece of sheet cardboard and find by trial the point on which it may be balanced, that is, the centre of gravity of the card. Make a hole in the card near the edge, and make a plumb-line consisting of a thread with a piece of lead tied at one end and a hook of thin wire at the other. Hang the card from the hook, and then suspend both as shown in Fig. 84, so that the card and lead are both suspended and the thread passes over the point of suspension. The thread also passes through the centre of gravity. Do this for various holes in the edge of the card, and see that in all cases the *vertical line through the point of suspension passes through the centre of gravity.*

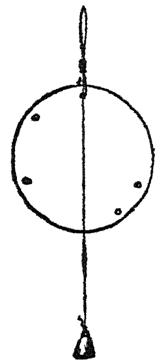


FIG. 84.—A method of determining centre of gravity.

After the centre of gravity of a sheet of metal, or other stiff material, has been determined by hanging it from a support in the manner described in Expt. 93, it will be found that if this sheet be so arranged that a pointed upright is immediately under the centre of gravity, the plate will be supported in a horizontal position. This affords a convenient means of checking the correctness of the experiment performed.

Relation of centre of gravity to base of support.—A circular disc, in which the centre of gravity coincides with the geometrical centre, will not rest upon a table if the centre is beyond the edge of the table, but will topple over. In a similar way, if any plane figure lies flat upon a table the centre of gravity of the figure must be within the edge of the table. The same conditions apply to any object resting upon a support. For an object resting upon a base to be in equilibrium, a vertical line drawn from the centre of gravity downward must fall within the base. When this vertical line falls outside the base, the body topples over.

Consider the case of an omnibus on level ground. The centre of gravity of an omnibus ought to be kept as low and central as possible, so that a vertical line drawn from it downwards will fall well within a line traced around the omnibus upon the ground. But when the outside of the omnibus is filled with people and

the inside is empty, the centre of gravity is much higher, and if the vehicle happens to be running across a sloping road, it may cause so great a change of position of the centre of gravity as to make the vertical line from the centre fall outside the base of support; in such a case the omnibus must topple over (Fig. 85).

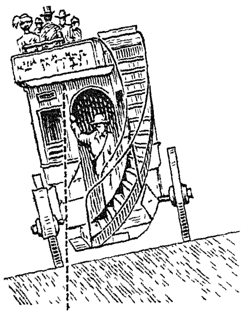


FIG 85.—Conditions of stability of an omnibus

Equilibrium.—When a body is at rest, all the forces acting upon it balance one another (or, what is the same thing, any force is equal and opposite to the resultant of the remaining forces), and it is said to be in equilibrium. It is in **stable equilibrium** when any turning motion to which it is subjected raises the centre of gravity; in **unstable equilibrium** when a similar movement lowers the centre of gravity, and in **neutral equilibrium** when the height of the centre of gravity is unaffected by such movement. Consequently, if a body in stable equilibrium be disturbed, it returns to its original position; if in unstable equilibrium, it will, if disturbed, fall away from its original position; while if the condition of equilibrium be neutral it will, in similar circumstances, stay where it is placed.

EXPT. 94.—**Base of support.** Place upon a square-edged table or board one of the cardboard figures of which you have found the centre of gravity. Gradually slide the figure near the edge until it would just topple over; keeping it in this position, draw a line along the under side of the cardboard where the edge of the table touches it. Then place the cardboard in another position and again mark where the edge of the table touches it when it would just topple over. The intersection of these lines is the centre of gravity, and it will be noticed that the cardboard will just topple over when the centre of gravity falls outside the edge of the table.

EXPT 95.—**Suspension and equilibrium.** Procure an oblong strip of wood or cardboard (Fig. 86). Support the strip as at A by a long pin pushed through it; it is then in stable equilibrium, for the slightest turn either to right or left raises the centre of gravity. When supported as at B, the strip is in neutral equilibrium; and

when supported as at C, it is in unstable equilibrium, for the slightest movement lowers the centre of gravity.

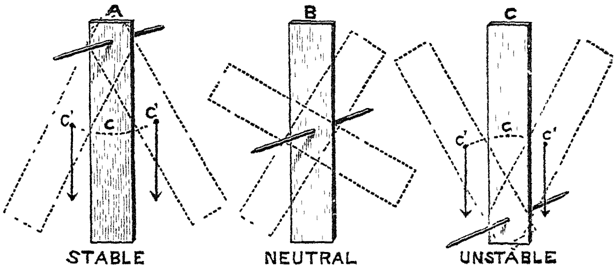


FIG 86 —Relative positions of centre of gravity and point of support for stable, neutral, and unstable equilibrium

Conditions of stability.—The centre of gravity must in every case be below the point of support for a suspended object to be in equilibrium. The greater the distance between the point of support and the centre of gravity the greater is the tendency to return to the position of equilibrium.

When the centre of gravity and the point of support of a suspended object are close together the equilibrium of the object is disturbed easily. A good balance partly owes its sensitiveness to this condition, the centre of gravity and point of support being brought close together designedly.

It has been shown that in the case of a freely suspended object the centre of gravity is at its lowest point when the object is in equilibrium Let us see how this applies to a body supported upon a surface below the centre of gravity. A body is least liable to be upset when the centre of gravity is at a considerable distance from all parts of the edge of the base; for when this is the case the body has to be tilted through a large arc before the centre of gravity falls outside the base.

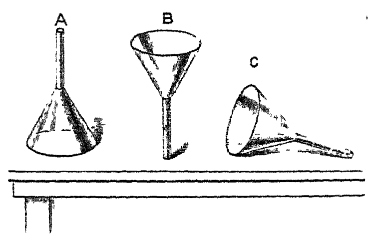


FIG 87 —A funnel in (A) stable equilibrium, (B) unstable equilibrium, (C) neutral equilibrium

A funnel standing upon its mouth is an example of a body which cannot be overturned easily on account of the low centre

of gravity and its distance from the edge of the base (Fig. 87). It is then in stable equilibrium. If the funnel be stood upon the end of the neck it can be overturned easily, because very little movement is required to bring the centre of gravity outside the base It is then in unstable equilibrium. When the funnel lies upon the table it is in neutral equilibrium, for its centre of gravity cannot then get outside the points of support

The steelyard.—The steelyard is a modification of the simple lever. In weighing an object by means of it, the object is suspended from one arm of the lever, at a fixed distance from the fulcrum, and its weight is determined by varying the distance from the fulcrum of a constant mass which hangs from the other arm of the lever. A scale marked on the variable arm enables the weight of the object to be read directly.

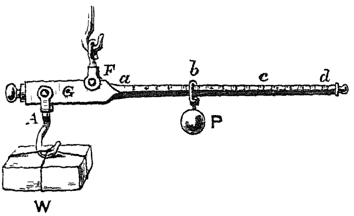


FIG 87A —A steelyard

The instrument consists of a steel bar block supported by knife-edges at F (Fig 87A). The object (W) to be weighed is attached to a hook suspended from the point A; and the constant mass P is moved along the other arm until the lever remains in a horizontal position, when the scale-reading (*b*) of its point of support gives the weight of the object.

If the centre of gravity (G) of the lever coincided with F the scale would commence at F. In actual practice, however, the centre of gravity is usually between A and F. The weight P, therefore, has to be suspended from some point *a* in order to balance the weight of the lever; and the scale must start from this point.

By making the distances *ab*, *bc*, *cd*, ... each equal to AF, the weight W is equal to that of P, 2P, or 3P, . , according as P is suspended from the 1st, 2nd, or 3rd, ... division of the scale. These divisions on the scale are sub-divided so as to give intermediate fractions of the weight P.

EXERCISES ON CHAPTER X.

1. Describe the principle of the action of a simple lever.

A stiff wooden rod, six feet long, and so light that its weight may be neglected, lies upon a table with one end projecting four feet over

the edge. Upon the end of the rod lying on the table a weight of 8 lb. is placed. What weight must be placed upon the other end so as just to tip the rod?

2. What is a lever? What is the "fulcrum" of a lever?

Name four or five levers in common use, and say where the fulcrum of each may be?

3. What is meant by the resultant of two forces?

Describe an experiment to prove that the resultant of two parallel forces is equal to the algebraic sum of the forces

4. A uniform rod is pivoted at its middle point, and a weight of 20 grams is attached at a point 25 centimetres from the fulcrum To what point on the rod must a weight of 15 grams be attached in order that the rod may balance in a horizontal position?

5. A lever, two feet long, has a force equal to the weight of 10 lb acting at one end, 18 inches from the fulcrum. What is the greatest weight it will support at the other end?

6. How do you define the moment of a force about a point, and how can you apply the principle of moments to find the resultant of parallel forces?

Two men A and B support the ends of a wooden beam six feet long and weighing 1 cwt A weight of $2\frac{1}{4}$ cwt hangs from the beam at a distance of 2 ft. from A. What are the total weights supported by A and B respectively?

7. A piece of cardboard, nine inches long and six inches broad, is divided into six equal squares by means of a ruler and pencil One of the two squares that are not corner squares is cut away with a penknife Find the centre of gravity of the remaining piece of cardboard.

8. A man with a bucket in one hand, stands with his feet close together Why is it that in order to preserve his balance the man has to stand with his body leaning to one side? Illustrate your answer by a sketch

9. A square sheet of cardboard weighing 8 oz. is suspended by a thread fastened to one corner, and a weight of 4 oz. is fastened to one of the corners adjacent to the corner of suspension Draw a diagram to show the position in which the sheet will hang, and say what is the total weight that the thread supports.

10. How would you determine the centre of gravity of an iron hoop made by joining together two semicircles, one thicker than the other? Explain how the observations could be used to find out which was the thicker half of the hoop.

11. A solid hemisphere made of uniform material is placed with any part of its curved surface upon a horizontal plane Show that, however thus placed, it will tend always to a position of stable equilibrium with its flat surface horizontal and uppermost What other positions of equilibrium are there? Which of them are stable and which unstable?

CHAPTER XI.

THE PULLEY, INCLINED PLANE, AND SCREW.

Machines.—The term **machine** is applied to **any contrivance by which a force acting at a given point and in a given direction may be rendered available at some other point and in some other direction.** In some machines a *small* applied force may give rise to a *greater* force acting at another point and in another direction; in such cases, the ratio of the resulting force to the applied force is termed the **mechanical advantage** of the machine. Also, in any machine, the total work given out is never greater than the work put into it: as a general rule, owing to work absorbed in overcoming friction of the working parts, the available work is *less* than the work put in, and the ratio of the former to the latter is termed the **efficiency** of the machine.

Use of a single fixed pulley.—With a single *fixed* pulley, no mechanical advantage is obtained. All that the pulley does is *to change the direction of the pull*; if one of the loads, for instance, be pulled down, the other rises. The pulley thus acts in the same way as a lever balanced at its centre; the distance from the centre to the circumference, in other words, the radius of the pulley, being regarded as one arm of the lever. A pulley having a radius of three inches has therefore an equivalent lever-arm three times as great as one with a radius of one inch.

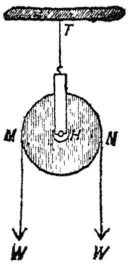


FIG. 88 —Parallel forces acting on a pulley.

A pulley supported as in Fig. 88 may be considered as an example of the action of parallel forces. The forces W + W + the weight of the pulley, acting downwards, are

kept in equilibrium by the single force T acting upwards If a single mass, the weight of which is equal to W + W + **pulley**, were hung from the cord, it would produce the same tension as the three forces, that is, it would be their resultant.

Use of a single movable pulley.—The fixed pulley is of no advantage in reducing the force required to raise a mass, the advantage gained is derived from the use of a movable pulley. Thus, one half of the mass W (Fig. 89) is supported by the part of the string hooked to the beam, and the other half is supported by the part of the string which goes to the spring balance. There are several different combinations of pulleys, but the principle exemplified by the following experiments, namely, that every movable pulley reduces by one-half the effort required to support or raise the mass below it, is utilised in them all.

EXPT. 96—**One movable pulley.** Place a weight in one of the pans previously used (p. 132), and weigh the pan and a pulley together by suspending them from a spring balance Record the reading of the balance. Now arrange the pulley and balance as shown in Fig. 89, and again record the reading. Increase the total load by adding other weights to the pan, and repeat the observations.

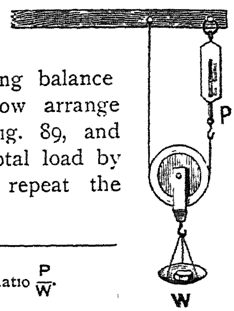


FIG 89
Expt 96

Total load, W.	Reading of Spring Balance, P.	Ratio $\frac{P}{W}$.

It will be noticed that P is only $\frac{1}{2}$W (roughly) in every case; in other words, the tension in a cord supporting a movable pulley having a mass hung from it is equal to one-half the total mass supported

EXPT. 97—**Two movable pulleys.** Arrange two pulleys in connection with the spring balance as shown in Fig. 90, and observe the reading Add a weight W to the pan, and observe the *increase* (P) of the tension in the spring balance. Take a series of readings, using heavier weights. Tabulate your observations as in Expt. 96.

In this case, that is, with two movable pulleys, it will be noticed that the force required to support the pulleys and load is less than before. Consider this experiment from a theoretical point of view. The upper pulley is supported by two cords, in each of which the tension is P. If the pulley had no weight the single force required to counteract this tension would be equal to 2P, but taking the pulley into account, the force is $2P - W_1$, where W_1 represents the weight of the pulley. The tension in the string around the lower pulley is thus equal to $2P - W_1$, and if the pulley had no weight the load W would be equal to $2(2P - W_1)$ But taking the weight (W_2) of the second pulley into account it is found that W is equal to $2(2P - W_1) - W_2$. In the same way the relation between the load and sustaining force for any number of pulleys can be determined.

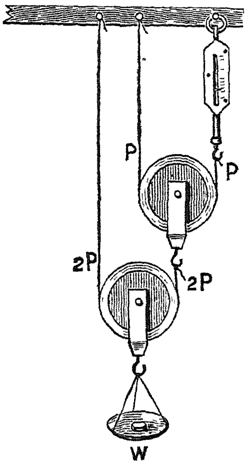


FIG 90—Expt 97.

The principle of work applied to pulleys.—With pulleys, as with levers, there is neither loss nor gain of work. If, in any combination of pulleys, a force of 10 lb. wt balances a force of 120 lb. wt.—the mechanical advantage or resistance ÷ effort, thus being 12—the effort will have to be exerted through twelve feet in order to move the resistance through one foot. For it is an invariable rule that

$$\text{Effort} \times \begin{matrix}\text{the distance through}\\ \text{which it acts}\end{matrix} = \text{Resistance} \times \text{Distance moved.}$$

EXPT. 98.—**Principle of work.** Remove the spring balance used in Expt 97, pass the string over a fixed pulley supported from the beam, and attach a scale pan to the free-end of the string. Add weights to this pan until the system is in equilibrium Place a weight (W) of 100 gm. in the lower pan and add weights (P) to the upper pan until the system is again in equilibrium. Measure the height of the bottom of each tray from the table. Then move the weight P through a certain distance, and observe how much the weight W is moved. It will be found that.

$$P \times \text{distance moved} = W \times \text{distance moved.}$$

The inclined plane.—A plane in mechanics is a rigid flat surface, and an inclined plane is one that makes an angle with the horizon.

The reason for the decrease of tension in an elastic cord attached to a mass resting on an inclined board, compared with the tension when the mass hangs freely, will be best understood by applying the principle of the triangle of forces to the inclined plane. Suppose an object O (Fig. 91) is kept in position upon a smooth inclined plane by a force acting up the plane. The object is acted upon by three forces, namely, W due to its weight, acting vertically downwards, P the force exerted up the plane, R the reaction of the plane.

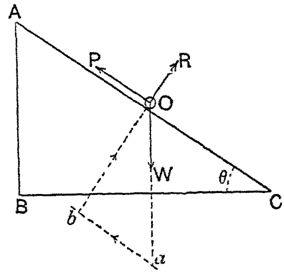


FIG. 91.—Equilibrium upon an inclined plane.

By the principle of the **triangle of forces** (p. 106) these three forces can be represented in magnitude and direction by the sides of a triangle taken in order. Then, supposing the weight (W) of the object to be known, if a vertical line O*a* be drawn to scale so that its length is proportional to W, and the triangle O*ba* is completed so that the sides *ab* and *b*O are parallel to the forces P and R respectively, this triangle may be regarded as a **force-diagram**, and the lengths of its sides are proportional to the forces to which the sides are parallel. Hence,

$$\frac{P}{W}=\frac{ab}{aO}.$$

But the triangles *ab*O and ABC are similar, since their sides are mutually perpendicular; therefore,

$$\frac{P}{W}=\frac{ab}{aO}=\frac{AB^*}{AC}, \quad \text{or,} \quad P=W\times\frac{AB}{AC}.$$

* It is more convenient to express this ratio in terms of the inclination (θ) of the plane. The ratio $\frac{\text{perpendicular}}{\text{hypotenuse}}$ is termed the *sine* of the angle (θ); hence, the above relationship may be written $P/W=\sin\theta$.

The ratio $\frac{\text{perpendicular}}{\text{base}}$ and $\frac{\text{base}}{\text{hypotenuse}}$ are termed respectively the *tangent* and the *cosine* of the angle; hence, in Fig. 91, AB/BC$=\tan\theta$, and BC/AC$=\cos\theta$.

The same result may be deduced from the **principle of work.** Thus, if the object O starts from C, and is moved up the plane to A, it is lifted through a vertical height AB. For this to take place, the effort has to be exerted through a distance equal to AC, the length of the plane. Therefore,

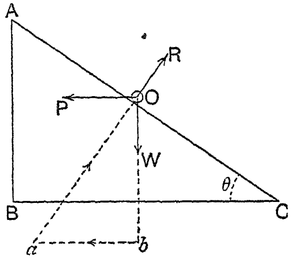


FIG 92.—Equilibrium on an inclined plane

$$P \times AC = W \times AB,$$

or

$$P = W \times \frac{AB}{AC}.$$

The magnitude of the reaction R which the plane exerts on the object may be determined from the equation

$$\frac{R}{W} = \frac{Ob}{Oa} = \frac{BC}{AC},$$

or,

$$R = W \times \frac{BC}{AC}.$$

Fig. 92 represents the relationship of the forces P, W and R when the effort P acts horizontally. The triangle abO is the force-diagram, and it is similar to the triangle ABC. Hence,

$$\frac{P}{W} = \frac{ab}{bO} = \frac{AB}{BC},$$

or,

$$P = W \times \frac{AB}{BC}.$$

EXPT 99.—**Inclined plane with effort acting up the plane** Fig 93 represents a suitable form of apparatus for demonstrating the principle of the inclined plane when the effort is parallel to the inclined surface. Clamp the inclined plane firmly in position, weigh the roller, and attach it to the thread by which the effort is applied Determine the weights which must be hung from the end of the thread and are just sufficient (i) to prevent the object from rolling down the plane, and (ii) to cause the roller to start rolling up the plane The average of these weights represents the effort P. Read by means of the plumb-line the angle of inclination of the plane

Draw a horizontal line of any convenient length to represent the base of the plane and construct upon it an angle equal to the inclination of the plane. At any convenient point drop a perpendicular to the horizontal line from the inclined line, so as to construct a right-angled triangle ABC, the angle at C representing the inclination of the plane.

Measure AB and AC and calculate the ratio AB/AC. Calculate also the ratio P/W. The two results will be found approximately the same.* Test further this relationship by repeating the experiment with the plane incline at different angles.

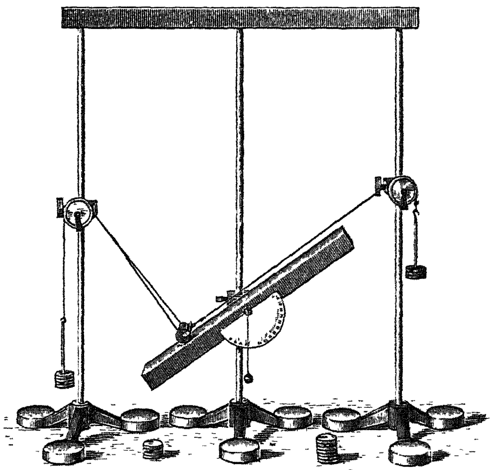


FIG 93.—"Capstan" apparatus for inclined plane experiment (G Cussons, Manchester)

EXPT. 100.—**Measurement of the reaction of an inclined plane.** Attach to the roller the thread which passes over the pulley on the left of the apparatus; and adjust the position of the roller so that this thread is perpendicular to the surface of the plane Adjust the weight attached to this thread so that the roller is just on the point of being raised from the plane This weight represents the magnitude of the reaction R Calculate the ratio R/W Construct a force-diagram as in the preceding experiment, and calculate the ratio BC/AC Notice that the result is the same as that of R/W. Repeat the observations with the plane inclined at other angles.

*The values of the ratio AB/AC, that is, of sines of angles, are given in trigonometrical tables, and can be used instead of determining them by constructing a triangle as described and measuring the lengths of the perpendicular and hypotenuse In the same way, the values of cosines and tangents required for Experiments 100 and 101 can be found by referring to trigonometrical tables instead of by construction.

EXPT. 101.—**Inclined plane with effort horizontal.** Fig. 94 represents a similar form of apparatus, but the inclined plane has a slot cut along

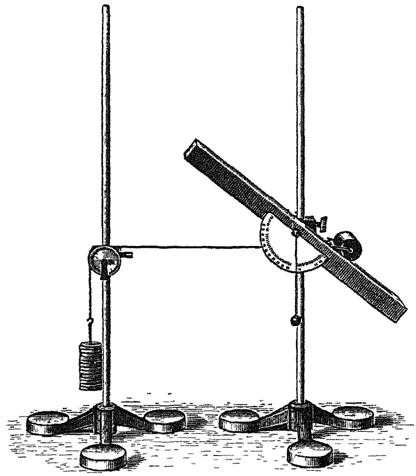


FIG 94.—"Capstan" apparatus for inclined plane experiment (G Cussons, Manchester)

its axis so that the effort may be applied horizontally Take a series of observations in order to demonstrate that the ratio P/W is equal to the ratio AB/AC in a force-diagram constructed as in the preceding experiments.

Principle of the screw.—The principle of the action of a screw is similar to that of an inclined plane. Thus comparing a screw with an inclined plane, it will be seen that

Height of inclined plane represents distance between threads,
Base of " " " circumference of screw.

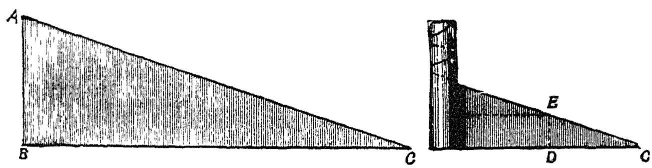


FIG 95 —Formation of a screw thread by the slope side of an inclined plane

The angle of inclination of the inclined plane is represented by the angle ECD (Fig. 95), and this determines the *pitch* of the screw.

EXPT. 102 —Cut out of paper a right-angled triangle such as ABC (Fig. 95) and wind it round a pencil. The slant side of the triangle forms a spiral upon the pencil, similar in appearance to the thread of a screw. If the inclination of the triangle be small, the threads appear close together, and if it be large they occur farther apart. Mark where the end C of the paper touches the base of the triangle and draw a line DE, perpendicular to the base, from this point to the slant side. The small triangle CDE thus formed is similar to the large one, and it represents one turn of the screw thread.

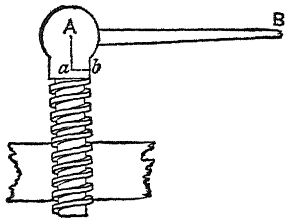


FIG. 96 —A screw turned by a lever.

In considering the use of a screw, the resistance to be overcome can be regarded as a load upon an inclined plane. With a screw such as is shown in Fig. 96 the effort is exerted in a direction parallel to the base of the plane. Under this condition

$$\frac{\text{Load}}{\text{Effort}}=\frac{\text{Base of plane}}{\text{Height of plane}}.$$

Or, expressing the equality in terms which apply to screws:

$$\frac{\text{Resistance}}{\text{Effort}}=\frac{\text{Circumference of screw}}{\text{Distance between successive threads}}.$$

When force is applied at B (Fig. 96), leverage is gained in the ratio of AB to *ab*, and so further mechanical advantage is obtained on this account. But, in order to advance the screw by a distance equal to that between two successive threads, the end of the handle B has to be turned through a complete circumference. This fact can be used to deduce the mechanical advantage of a screw from the principle of work. For

$$\text{Effort} \times \begin{array}{c}\text{Circumference of}\\ \text{circle described}\\ \text{by it}\end{array} = \text{Resistance} \times \begin{array}{c}\text{Distance between}\\ \text{two successive}\\ \text{threads.}\end{array}$$

Hence the mechanical advantage, or resistance ÷ effort, can be found from the relation:

$$\text{Mech. adv.}=\frac{\text{Circumference of circle described by effort}}{\text{Distance between two successive threads}}.$$

EXERCISES ON CHAPTER XI.

1. State the principle of the action of a simple pulley.
How would you show that the strain or tension in a cord supporting a pulley is equal to half the weight hanging from the pulley?

2. Explain by reference to an inclined plane what you understand by the "mechanical advantage" of a machine.

3. Give in a few words the principle of the screw. On what does the ratio of the resistance overcome to the effort exerted depend?

4. Explain how a weight, a pulley, and a chain can be arranged so as to make a door shut when it is let go Draw a diagram. What forces has the weight to overcome as it shuts the door?

5. A penny lies at rest on a sloping desk What forces are acting on it? Draw a diagram showing clearly the direction of each.

6. What must be the inclination of an inclined plane so that a given force, whether it acts horizontally or parallel to the length of the plane, will support the same mass? (This question should be answered by means of diagrams)

7. Part of a chain rests on an inclined plane, and the remainder hangs over the back. Draw a diagram to show in what position the chain will rest if the plane is so smooth that friction need not be taken into account. Give any explanation you think necessary.

8. What is the mechanical advantage of a lever, the load arm of which is 30 cm. long and the effort arm 135 cm. long?

9. By means of a lever, an effort equal to a weight of 100 gm moving through 16 cm lifts a weight of 290 gm through a vertical distance of 5 cm. What is the efficiency of the lever?

10. What is the inclination of a plane when a force equal to the weight of 120 gm. can move a weight of 170 gm. with uniform velocity up a smooth plane, the force being parallel to the plane? If the length of the plane is 70 cm, how much work, expressed in kilogram-metres, is done in moving the weight up the entire length of the plane?

11. What work is done by a horse against the action of gravity in drawing a carriage with its load, all weighing 1000 lb., 100 yd. up a slope of 1 in 25?

12. A cyclist and his machine weigh 180 lb. Assuming the absence of frictional resistance, at what horse-power must he work in order to ascend a slope of 1 in 30 at 5 miles an hour?

PART III.

HEAT.

CHAPTER XII.

EXPANSION. THERMOMETERS.

Change of size.—As a rule all bodies, whether solid, liquid, or gaseous get larger when heated, and smaller when cooled. The change of size which a body undergoes is referred to as the amount it expands or contracts; or, heat is said to cause expansion in the body. This expansion is regarded in three ways. When dealing with solids, expansion may take place in length (linear expansion), in area (superficial expansion), and in volume (cubical expansion). In the case of liquids and gases we have only cubical expansion. Similar terms can be used with reference to contraction.

The expansion which substances undergo when heated has to be allowed for in many cases. Railway rails, for instance, are usually not placed with their ends in actual contact, but a little space is allowed between the separate rails, so that they can expand in summer without meeting Steam pipes used for heating rooms are also not firmly fixed to the walls at both ends, but are left slightly loose or are loose-jointed, so that they can expand or contract without doing any damage. For the same reason the ends of iron bridges are not fixed to the supports upon which they rest. Iron tyres are put on carriage wheels by first heating the tyre and, while it is hot, slipping it over the wheel. As the tyre cools it contracts and clasps the wheel very tightly.

The common occurrence in domestic life of the cracking of thick glasses when boiling water is poured on them, is explained by this expansion of solids by heating. The part of the glass with which the hot water comes in contact is heated and expands; but the effect is quite local; the heating is confined to one spot, because glass does not allow heat to pass through it readily. It is this local expansion of the glass which results in the cracking of the vessel. On the contrary, as silica expands only very slightly when heated, a flask constructed of this substance may be made red-hot in a flame and then plunged into cold water without cracking.

EXPT. 103.—**Expansion of metal.** Select a rod of copper, iron, or brass, about 30 cm. long. See that the ends are planed truly at right angles to the length of the rod. Lay the rod on a steel millimetre scale, and note its length Place the rod on a tripod stand, and heat it with a Bunsen flame. Transfer it, by means of tongs, to the scale again, and note its length while hot.

EXPT. 104.—**Expansion apparatus.** Fit up the apparatus shown in Fig. 97. W is a wooden box with a sheet of paper pinned upon one of its sides. AB is a long knitting-needle fixed vertically with its upper end A held firmly in a clamp CD is a strip of wood, about 12-15 cm. long and with square cross-section (like a very long household match). A shallow notch is made in the strip near its end D, in which the lower end of the needle can rest The strip is pivoted at B, by burning a small hole through the strip with a red-hot needle; and a loosely fitting pin is passed through the hole and driven into the box Mark the position of the end C of the strip by a pencil mark on the paper. Raise the temperature of the needle by passing a Bunsen flame up and down it. Note the change in the position of the pointer, and mark the final position of the end C. Allow

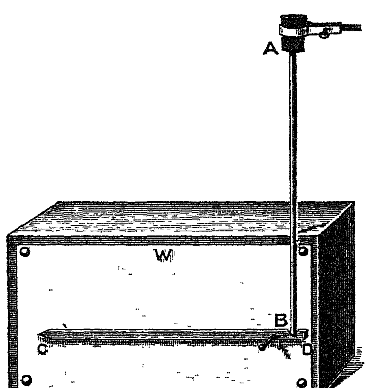


FIG. 97.—Simple apparatus to show expansion by heat.

the needle to cool, and notice how the wooden strip returns to its original position.

EXPT. 105.—**Compound bar.** Make a compound strip (Fig 98) of brass and of soft iron, by soldering or riveting the strips together If necessary, straighten the strip by hammering, then heat it Notice that the strip bends, because the brass expands more than the iron

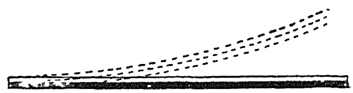


FIG 98.—Effect of heat on a compound bar.

EXPT 106.—**Water.** Procure a 4-oz flask and fit it with a cork. Bore a hole through the cork and pass through it a long glass tube which fits tightly. *Fill* the flask with water to which red ink has been added Push the cork into the neck of the flask and so cause the coloured water to rise up the tube. There should be no air between the cork and the water. Now dip the flask in warm water, and notice that the liquid soon gets larger and rises up the tube (Fig. 99). Take the flask out of the warm water, and see that the coloured water gets smaller as it cools, and that it sinks in the tube.

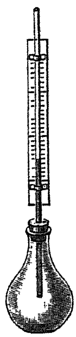


FIG. 99.—To show the expansion of a liquid when heated

EXPT. 107.—**Other liquids.** Arrange two other flasks as in the last experiment, but filled respectively with alcohol and turpentine. Push in the corks till the liquid stands in each tube at the same height Put all three flasks to the same depth into a vessel of warm water. Notice that the expansion of the glass causes a momentary sinking of the liquids; and that ultimately the expansions of the three liquids are very different

EXPT 108.—**Air thermometer.** Tightly fit a cork, through which a straight tube passes, into the neck of a 2-oz. flask. Support the flask so that the tube dips into a beaker containing coloured water (Fig 100) Warm the flask with the hand or a flame so as to expel some of the air, and let the liquid rise in the stem. This instrument constitutes an *air thermometer*.

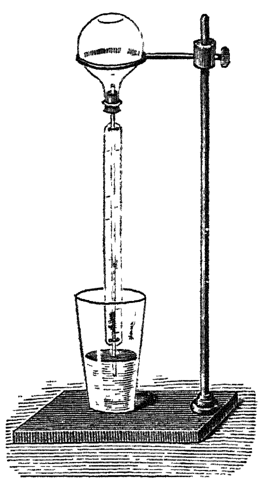


FIG 100.—An air thermometer.

EXPT. 109.—**Differential thermometer.** Fasten two bulbs or flasks together (air-tight) by a tube bent six times at right angles, and containing some coloured liquid in the middle bends (Fig 101). Show that the liquid moves if one flask is warmed more than the other. (This instrument is known as a *differential thermometer*)

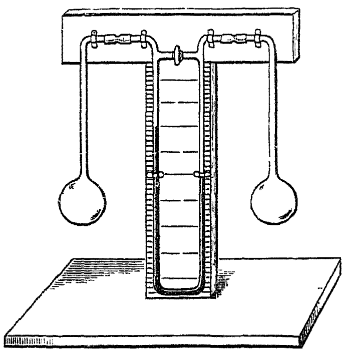


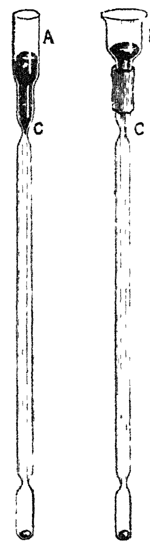
FIG 101 —A differential thermometer.

Measurement of change of temperature.—Change of temperature means change in the state of hotness of a body. The change of size which takes place when a thing is heated provides a means of measuring the change of temperature which it undergoes. Consider the experiment with the coloured water in the flask with a long tube attached to it. Suppose the coloured water in the tube rises through a certain number of inches after the water has been heated somewhat, and that when the flask is placed into some other liquid, or some more water, the coloured water is found to rise up the tube to just the same place, we should have every right to say that the second liquid was exactly as hot as the first This is measuring its temperature. The flask and tube with the water have become a "temperature measurer," that is, a **thermometer.**

A flask filled with water, and having a stopper through which a glass tube passes, could thus be used to show the expansion produced by warming and the contraction by cooling But this flask and tube make only a very rough temperature measurer The water does not get larger by the same amount for every equal addition of heat. Neither is it very sensitive, that is to say, it does not show very small increases in the degree of hotness; in other words, it does not record very small differences of temperature, and for a thermometer to be any good it must do this. Then, too, as every one knows, when water is made very cold it becomes ice, which, being larger than the water from which it is made, might crack the flask. For many

reasons, therefore, water is not a good substance to use in a thermometer.

Mercury and spirit thermometers.—There are many reasons for selecting mercury as the liquid for an ordinary thermometer. It is a liquid the level of which can be seen easily; it does not wet the vessel in which it is contained; it expands a considerable amount for a small increment of temperature; it is a good conductor of heat, and consequently it assumes very quickly the temperature of the body with which it is placed in contact. Very little heat is required to raise its temperature, and there is therefore little loss of heat due to warming the thermometer. Another liquid frequently used is spirits of wine, which is particularly valuable for measuring temperatures below that at which mercury would be frozen.



(i) (ii)

FIG. 102.—Thermometers in course of construction. (i) tube enlarged at top, (ii) tube with small funnel attached

EXPT. 110.—**Construction of a thermometer.** Procure or make a thermometer tube, with a bulb at one end. With a little practice it is easy to blow a bulb upon a piece of thermometer tubing. One end of the tubing is held in a blow-pipe flame and twirled round until the glass melts and runs together so as to seal up the tube. A small blob of glass is then allowed to form, and while the glass is molten the tube is taken out of the flame and blown into steadily. Fit a small funnel to the open end, as shown in Fig. 102; pour clean dry mercury into the funnel, and fill the bulb by heating and cooling it several times in succession. Arrange the quantity of mercury so that, when cool, the bulb and *part* of the stem are filled with mercury.

EXPT. 111.—**Use of a thermometer.** Place in hot water the bulb of the instrument just constructed, and make a mark at the level of the mercury in the tube. Now place the instrument in cold water, and notice that the mercury sinks in the tube. The mercury is thus seen to expand when heated and contract when cooled.

EXPT 112—**Degrees of temperature.** Examine a thermometer. Notice that it is similar to the simple instrument already described, but the top is sealed up, and divisions or graduations are marked upon it, so that the height of the mercury in the tube can be recorded easily These divisions are called *degrees* (Fig 103).

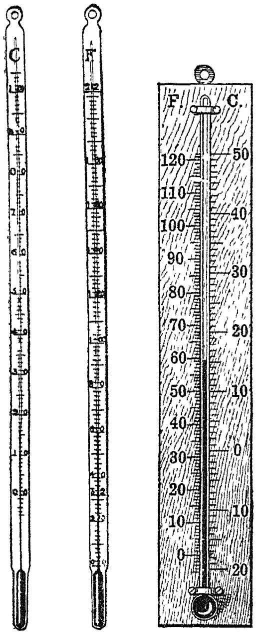


FIG. 103—Forms of mounted and unmounted thermometers

The fixed points on a thermometer—In the graduation of a thermometer the plan always adopted is to choose two 'fixed points' from which to number the degrees of temperature. A convenient lower fixed point is the temperature at which ice melts, or water freezes, for this is always the same if the ice be pure, and remains the same so long as there is any ice left unmelted. Whenever the thermometer is put into melting ice the mercury in it always stands at the same level, or melting ice is always at the same temperature, and thus may be used to give one fixed point. The higher 'fixed point' chosen is that at which pure water boils at normal atmospheric pressure (viz. 760 mm. of mercury). This condition is necessary because the boiling point of a liquid is altered when the pressure upon it is changed, being raised when the pressure is greater, and lowered when the pressure is less When the water boils, the temperature of the steam is the same as that of the water, only providing that the water is perfectly pure, but the temperature of the steam is dependent upon the pressure alone; hence, in determining the higher fixed point, it is usual to suspend the thermometer so that its bulb is just *above* the surface of the boiling water.

Thermometer scales.—It is necessary to give definite values to the fixed points, and to divide the interval between the two points according to some accepted plan, in order to be able

to compare observations made with different thermometers. The thermometers used in this country are graduated in two ways—(1) the Centigrade scale, (2) the Fahrenheit scale. A third scale—the Réaumur scale—is used extensively in Germany.

The Centigrade scale.—Here the freezing point is called *zero* or *no degrees Centigrade*, written 0° C. The boiling point is called *one hundred degrees Centigrade*, and is written 100° C. The space between these two limits is divided into 100 parts, and each division is called a *degree Centigrade*. This scale was devised by Celsius, who, however, proposed to call the boiling point of water 0° and the freezing point 100°.

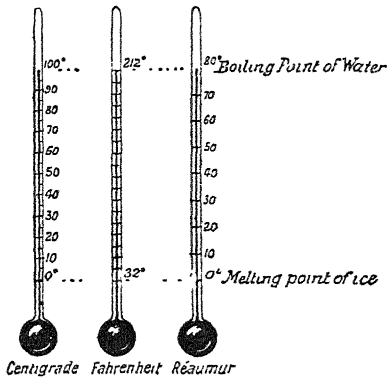


FIG. 104.—Thermometric scales

The Fahrenheit scale.—On thermometers marked in this way the freezing point is called *thirty-two degrees Fahrenheit*, written 32° F., and the boiling point *two hundred and twelve degrees Fahrenheit*, written 212° F. The space between the two limits is divided into 180 parts, and each division is called a *degree Fahrenheit*. The reason of this difference is interesting.

Newton made a thermometer before Fahrenheit and divided into twelve equal parts or degrees the space through which the liquid in it expanded when the instrument was placed (1) in ice-water, (2) in the mouth of a healthy human being. Fahrenheit obtained a lower temperature by means of a mixture of ice-water and sal-ammoniac or sea-salt. and he used this temperature to mark the starting point or zero of his thermometer. He first divided the interval between this temperature and the temperature of the body into twice the number of parts used by Newton, namely 24; and he found that when the thermometer was placed in ice and water the liquid stood at the mark 8. The spaces between the degrees were however found to be too large, so later he divided each into four, thus making the freezing point of

water 32° and the body-temperature 96°. When the scale was extended by dividing the tube of the thermometer into equal spaces the mark 212° was found to coincide with the point at which the top of the liquid stood when the thermometer was placed in boiling water There is no evidence that Fahrenheit used the boiling point of water as one of his fixed points, so that the existence of 180° between freezing point and boiling point may be accidental.

The Réaumur scale.—Upon thermometers graduated according to this scale the freezing point is marked 0° and the boiling point 80°. Réaumur adopted the number 80° as the temperature of steam, because he found that alcohol, diluted with one-fifth water, expanded in volume from 1000 to 1080 when raised from the freezing to the boiling point. The relation between the three scales is shown in Fig 104.

Conversion of scales.—It should be clear from what has been said that the interval between the boiling and freezing points, that is, the same temperature difference, is divided into 100 parts on the Centigrade scale and 180 parts on the Fahrenheit, and consequently 100 Centigrade degrees are equal to 180 Fahrenheit degrees, which is the same as saying one degree Centigrade is equal to nine-fifths of a Fahrenheit degree, or one degree Fahrenheit is equal to five-ninths of a degree Centigrade.

$$100 \text{ C. degs.} = 180 \text{ F. degs.}; \quad \therefore \quad 5 \text{ C.} = 9 \text{ F.}$$

$$\therefore \quad 1^\circ \text{ C.} = \tfrac{9}{5}^\circ \text{ F. or } 1^\circ \text{ F.} = \tfrac{5}{9}^\circ \text{ C.}$$

In converting Fahrenheit readings into Centigrade degrees, we must subtract 32 (because of what has been said of the freezing point on the former scale) and multiply the number thus obtained by 5 and divide by 9. To change from Centigrade to Fahrenheit, multiply the former reading by 9 and divide by 5 and add 32 to the result.

EXAMPLE.—What temperature on the Fahrenheit scale corresponds to 20° C.?

Answer.—20° C. is 20 C. degs. above the temperature of melting ice, *i.e.* $20 \times \frac{9}{5}$ Fahr. degs. above 32° F $= (36+32)^\circ$ F. $= 68^\circ$ F.

When it is necessary to refer to temperatures lower than the freezing point of water, a minus sign is placed before the temperature, thus, three degrees below the freezing point of water on the Centigrade scale is written -3° C.

Marking the lower 'fixed point.'—For this purpose an arrangement like that shown in Fig. 105 is suitable. The funnel is filled with shavings or fragments of ice, which before using has been washed carefully; or snow, if more convenient, may be used. The glass dish catches the water which is formed from the melting of the ice or snow. A hole is made in the ice-shavings by thrusting in a pencil or glass rod about the size of the thermometer, and into this hole the thermometer is put and is so supported that the whole of the mercury is surrounded

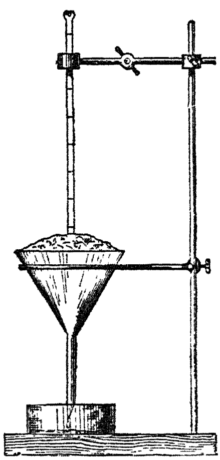


FIG 105.—Thermometer in ice for the observation of freezing point

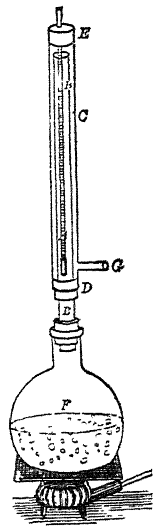


FIG 106.—Apparatus for determining the boiling point on a thermometer

by the ice or snow. The arrangement is left for about ten or fifteen minutes, until it is quite certain that the tube and mercury are at the same temperature as the melting ice. When this is so, the tube is raised until the mercury is just above the ice, and the level of the mercury is marked by a scratch on a thin layer of varnish or wax with which the stem has been covered previously.

Marking the higher 'fixed point'—Satisfactory determinations can be made by means of the apparatus shown in Fig. 106 A can or flask F is fitted with a cork, through which a glass or brass tube B passes. Surrounding this tube is a wider tube C, fitted upon the inner tube by means of a piece of thick india-rubber

tubing D. At the top of the outer tube is a cork E having a hole in which a thermometer can be fitted When the water in the flask is boiled, steam passes up the inner tube B, and down the wide tube C, and escapes at the outlet G into the open air. To use the apparatus, the top of the stem of the thermometer is pushed gently into the cork which fits in the outer tube, and adjusted so that the 100° point is just below the cork. The cork is then fitted into its place, the water boiled, and when steam has been coming off for about a quarter of an hour, the thermometer is raised so that the mercury surface is just above the cork; and, while the steam continues to pass through the apparatus, the position of the surface is indicated by means of a file mark on the outside of the stem.

EXPT. 113—**Graduation of a thermometer.** Mark on the stem of the thermometer which you are making, the lower and the upper fixed points by the methods described in the previous paragraphs Read and note the height of the mercury barometer at the time of the experiment. Fix the thermometer to a suitable strip of thin well-planed wood, the face of which has been previously covered with plain paper. Mark off on the paper the positions of the fixed points. If a *Centigrade* scale be desired, divide the interval into 100* equal divisions, and mark the fixed points 0° and 100° respectively.

EXPT. 114—**Thermometer corrections.** Determine the errors, if any, in the positions of the fixed points of a Centigrade and of a Fahrenheit thermometer. Enter your observations thus.

Height of mercury barometer, ...
Calculated temperature of steam, ...

	Centigrade thermometer		Fahrenheit thermometer.	
	Error.	Correction	Error.	Correction
Lower fixed point -				
Upper „ „				

* This is only approximately correct when the reading of the barometer is between 758 and 762 mm. When the pressure is outside these limits, the temperature of the steam must be calculated. For pressures between 750 and 770 mm, it may be assumed that the temperature of dry steam is increased or diminished by 0°·037 Centigrade for each millimetre change in the barometric height Thus, when the atmospheric pressure is 752 mm, the temperature of the steam is

$$100^\circ - \{0^\circ{\cdot}037(760-752)\} = 100^\circ - 0^\circ{\cdot}296 = 99^\circ{\cdot}70\ \text{C}.$$

If the atmospheric pressure is 770 mm., the temperature of the steam is

$$100^\circ + \{0^\circ{\cdot}037(770-760)\} = 100^\circ{\cdot}37\ \text{C}.$$

It may be necessary, therefore, to modify the above number 100.

Carefully note that when the reading is too high, the error is *positive*, and should be denoted by the sign +; when the reading is too low, the error is denoted by the sign −. The *corrections* to the readings of a thermometer are *the quantities which must be added to or subtracted from the readings in order to give the correct temperature.* Hence the signs attached to the corrections are the reverse of those attached to the errors.

Maximum and minimum thermometers.—There are several forms of thermometer which show the highest or lowest temperatures reached since they were last set. One of these is Six's self-registering thermometer represented in Fig. 107. The bulb A is filled with alcohol, and is separated from the top bulb D by a mercury thread BC. Above C there is more alcohol; but there is sufficient space left in D to allow for expansion. Expansion or contraction of the alcohol in A causes movement of the mercury thread, and the extreme positions of the thread are indicated by two steel indexes provided with springs just strong enough to prevent them from slipping.

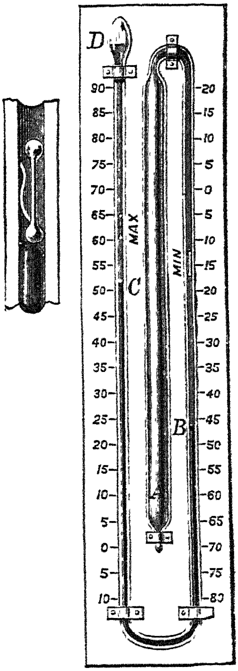


FIG 107 —Maximum and minimum thermometer.

The doctor's thermometer.—For the measurement of the temperature of the body, what is termed a **clinical thermometer** is best (Fig 108). As the temperature of the living human body in a state of health is never many degrees above or below a temperature of 98° F., a clinical thermometer is only graduated from about 95° F. to 110° F. When the bulb of such a thermometer is put into the mouth, or under the armpit, of a person in health, and left there for two or three minutes, it will be found, on taking it out, to indicate a temperature from 97°·8 F. to 98°·6 F. The thread of mercury in the stem of the thermometer remains in one position, though the air is cooling the mercury while the

thermometer is being read. This is because of the constriction at the top of the bulb, which causes the thread of mercury in the stem to be left behind while the mercury in the bulb contracts.

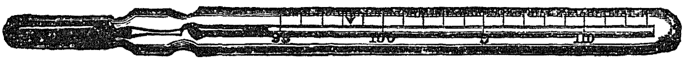


FIG 108 —A clinical thermometer

To "set" the thermometer for a fresh observation, it is only necessary to jerk it slightly, when the thread of mercury will again join up to the liquid in the bulb.

EXERCISES ON CHAPTER XII.

1. What is a thermometer, and what information concerning heat does it supply?

2. How would you test whether the two fixed points on a mercurial thermometer were marked accurately?

3. Describe how you would graduate a thermometer. Would any correction be necessary if you did it on the top of a mountain, or at the bottom of a coal mine?

4. I take two equal flasks, the mouths of which are fitted with bored corks carrying long glass tubes, and fill one with water coloured blue, and the other with methylated spirits coloured red; I then plunge them both into boiling water. Explain what will take place, giving reasons.

5. Take a glass tube open at one end and having a bulb at the other. Hold the tube so that the open end dips into water. Heat the bulb gently with a spirit lamp for a minute or two, and then take the lamp away. What will be observed? How can you account for the facts observed?

6. One-fifth of a bottle is filled with cold water; the bottle is tightly corked; the cork is pierced by a bent tube, one end of which dips into the water of the bottle and the other into water standing in an open vessel. Describe the results that may be observed if the bottle and its contents are heated up to a temperature of 99° Centigrade, and then allowed to cool.

7. A nurse cleanses a doctor's thermometer which reads up to 105° F. in boiling-hot water. The doctor now finds that the thermometer is useless. Why is this?

8. What do we mean by the 'boiling point' of a liquid? Explain why good tea cannot be made at the top of a very high mountain.

9. Convert the following into temperatures on the Centigrade scale. 71° F, 0° F., −40° F.

10. Find the temperatures on the Fahrenheit scale corresponding to. 83° C, 15° C., −5° C

11. Convert the following Centigrade temperatures into Fahrenheit.

	Melting Point	Boiling Point
(*a*) Hydrogen	−259·5°C.	−252°C.
(*b*) Mercury	−39 2	357
(*c*) Sodium	97 6	825

12. How does the distance between the two fixed points of a thermometer vary with the size of the bore, when the size of the bulb remains the same?

13. When the bulb of a thermometer is plunged into hot water, the mercury at first falls a little, and then rises. Why is this?

14. A Fahrenheit thermometer registers 110° while a faulty Centigrade thermometer registers 44°. What is the error in, and what is the correction for, the latter?

15. Two thermometers are hung up in a room. One registers a temperature of 15° and the other 59°. Explain fully the meaning of this difference.

16. In some liquid-in-glass thermometers mercury is used, in others alcohol. Discuss the advantages and disadvantages of the use of each of these two substances as the thermometric liquid.

17. Explain what you understand by the "temperature of a substance." State what conditions an instrument should fulfil which is intended to measure accurately the temperature of a liquid.

18. Why are thermometer tubes usually of very fine bore and why are they provided with bulbs?

19. How would you test the accuracy of the 'fixed points' of a thermometer? Explain how Six's thermometer shows both maximum and minimum temperatures.

CHAPTER XIII.

COEFFICIENTS OF EXPANSION.

Coefficient of expansion.—While a definite rise of their temperatures causes most bodies to expand, the amount of such expansion varies within wide limits. In the case of certain special alloys the expansion is almost negligible; while gases expand more than double their volume on being heated from 0° C. to 300° C. In the case of solids we are concerned usually with the **linear** coefficient of expansion, while for liquids and gases the coefficient of **cubical** expansion is of importance.

EXPANSION OF SOLIDS.

Coefficient of linear expansion.—The increase in length experienced by a rod of unit length, at 0° C., when its temperature is raised through 1° C is called its coefficient of linear expansion. The actual expansion being small in the case of solids, it is not necessary to know the length of the solid at 0° C.; hence a simpler definition is sufficiently accurate. It may be said that the fraction of its length which a solid body expands on being heated through 1° C. is called its *coefficient of linear expansion.*

To obtain a measurement of the linear expansion of a rod when heated the form of apparatus shown in Fig. 109 may be used. A rod of glass or metal about 18 inches long is surrounded by a glass tube having an inlet for steam at C and an outlet at D. The end A of the rod rests in a V-shaped groove and against a weight W. The other end rests on a needle which is free to roll on a glass base. To the needle is attached a cork having a split straw pointer. Any movement of the needle will be observed against the scale E

When steam is passed through the tube it heats the bar, and the expansion shows itself at B—the end A being fixed—by causing the needle to roll. To secure good contact the rod may be roughened with emery where it rests on the needle, and be pressed to it by means of an elastic band attached to the support.

After the steam has circulated for 5 or 10 minutes, the fraction of a complete circle the index has moved through is observed.

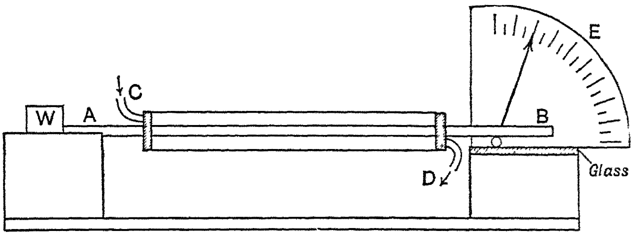


FIG. 109.—Determination of the coefficient of linear expansion of a rod.

This shows what part of a complete rotation the needle has made. To find the horizontal expansion of the rod corresponding to this rotation it is necessary to determine the diameter of the needle. To do this several similar needles are placed in a row, the total breadth is measured and divided by the number of needles.

If the diameter of the needle be d cm., the distance through which the needle rolls in one complete rotation is $(\pi \times d)$ cm. Let θ be the angle through which the index moves; then the advance of the rod, being equal to *twice* the distance through which the needle travels, is

$$2\pi d \times \frac{\theta}{360}.$$

Suppose the measured expansion to be E, the length of the rod up to the needle L, and its temperature at the commencement of the experiment 15° C. The fraction of its original length which the rod expands will be E/L

The rise of temperature having been from 15° C. to 100°C., that is 85° C., the fraction of its original length which the rod expands for *each degree rise of temperature* will be $\frac{E}{L \times 85}$.

This value is the coefficient of linear expansion.

EXPT. 115.—**Measurement of linear expansion.** Procure and examine the apparatus shown in Fig. 109, if necessary fitting the parts together. Notice that AB is a glass, or metal, rod resting in a groove at A, against a weight W, and rolling on a needle on a glass bed at B. Fix a split straw pointer to the needle to move against the graduated quadrant E. Observe that CD is a tube of wide bore fixed on the rod by means of corks and having an inlet for steam at C and an escape at D. When the apparatus is in adjustment, take the temperature of the room in the neighbourhood of the apparatus, and then pass steam through CD for 5 or 10 minutes. Note what part of a whole turn the pointer moves through Obtain the diameter of the needle by placing several similar needles in a row, measuring the total breadth and dividing by the number of needles. Obtain the actual movement of the end B of the rod by multiplying together the fraction of a rotation shown by the index and the circumference of the needle. Assuming the steam to be at 100° C obtain the rise of temperature of the rod.

Then calculate thus:

A length of......cm. of the metal or glass rod when raised......degrees expanded......cm.

Therefore 1 cm. of the rod when raised 1 degree expands......cm

The value so obtained is the coefficient of expansion of the material of the rod used.

The *coefficient of linear expansion* is usually denoted by the symbol α. From the above definition of the coefficient, it may be expressed thus:

$$\alpha = \frac{\text{change in length}}{\text{original length} \times \text{change in temperature}},$$

or, if L_t = length at $t°$ C, and L_T = length at a higher temperature T° C., then

$$\alpha = \frac{L_T - L_t}{L_t(T - t)},$$

$$\text{or} \quad L_T - L_t = \alpha L_t(T - t),$$

$$\text{or} \quad L_T = L_t\{1 + \alpha(T - t)\}.$$

EXAMPLE.—A platinum wire is 3 metres long at 0° C. What will be its length at 100° C.? (Coefficient of linear expansion of platinum, 0·000009.)

1 cm. of the wire at 0° C. becomes (1 + 0·000009) cm. at 1° C., and

" " " 1 + (0 000009 × 100) cm. at 100° C.

Hence, 300 cm. at 0° C. becomes 300(1·0009) cm. at 100° C, or

Length, at 100° C., is 300·27 cm.

Coefficient of superficial expansion.—**The coefficient of superficial expansion of a solid is the increase in area of a sheet of the substance, initially having unit area, when warmed through 1° C.**

Suppose the length of each edge of a small square sheet of a metal to be 1 cm. at 0° C. If the coefficient of linear expansion of the metal be α, then, when the sheet is warmed to 1° C., the length of each edge will be $(1+\alpha)$ cm., and the area will be $(1+\alpha)^2$ sq. cm. or $(1+2\alpha+\alpha^2)$ sq. cm But, since α is extremely small, the last term may be neglected; hence the total increase in area may be regarded as equal to 2α; or, the coefficient of *superficial* expansion is numerically equal to *twice* the *linear* coefficient.

EXAMPLE.—A sheet of brass is 40 cm long and 20 cm. wide at 0° C If it be heated to 50° C., what will be its increase in area? (Coefficient of linear expansion of brass, 0·000019.)

$$\text{Original area, at } 0^\circ \text{ C.,} = 40 \times 20 = 800 \text{ sq cm.}$$

$$\begin{aligned}\text{Length of sheet, at } 50^\circ \text{ C,} &= 40\{1 + (0{\cdot}000019 \times 50)\} \\ &= 40 \times 1{\cdot}00095 \\ &= 40{\cdot}038 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Width of sheet, at } 50^\circ \text{ C,} &= 20\{1 + (0{\cdot}000019 \times 50)\} \\ &= 20{\cdot}019 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Area of sheet, at } 50^\circ \text{ C,} &= 40{\cdot}038 \times 20{\cdot}019 \\ &= 801{\cdot}52 \text{ sq cm}\end{aligned}$$

$$\text{Hence, increase of area} = 1{\cdot}52 \text{ sq. cm.}$$

Alternate method Since the coefficient of superficial expansion is $(2 \times 0{\cdot}000019)$,

$$\begin{aligned}\text{the area of sheet, at } 50^\circ \text{ C.,} &= 800\{1 + (0{\cdot}000038 \times 50)\} \\ &= 800\{1 + 0{\cdot}019\} \\ &= 801{\cdot}52 \text{ sq. cm}\end{aligned}$$

Cubical expansion of solids.—When a solid body is heated each of its dimensions is increased in the same proportion. Suppose each edge of a cube of metal, of which the coefficient of linear expansion is α, to have a length of 1 cm. at 0° C., and that it is warmed to 1° C. The length of each edge will now be $(1+\alpha)$ cm, and the volume of the cube will be $(1+\alpha)^3$ c.c. Hence,

$$\begin{aligned}\text{Increase of volume} &= (1+\alpha)^3 - 1 \\ &= (1 + 3\alpha + 3\alpha^2 + \alpha^3) - 1 \\ &= 3\alpha + 3\alpha^2 + \alpha^3.\end{aligned}$$

But, since α is extremely small, all terms beyond the first may be neglected, hence, increase in volume $= 3\alpha =$ three times the coefficient of linear expansion.

By definition, **the coefficient of cubical expansion of a substance is the increase in volume of unit volume when it is heated through 1° C.** Hence, the coefficient of cubical expansion of a substance is equal numerically to three times the coefficient of linear expansion of the substance.

Expansion of Liquids.

Different rates of expansion.—It has been shown, in Expt. 107, that different liquids do not expand to the same amount when warmed to the same extent. The following experiment will serve to illustrate in a simple manner the great difference in the rate of expansion of different liquids, and to show that the rate of expansion is regular over a considerable range of temperature.

Expt. 116.—**Mercury, water, and spirit.** Select three 4-oz flasks, and fit each of them with a rubber stopper and a long narrow glass tube of the same bore. Fill the flasks with mercury, water, and methylated spirit (colouring the water and spirit with magenta or red ink). Adjust the quantities of liquid so that, when the cork is inserted, the liquid column in each tube reaches to the same small height above the top of the stopper Place the flasks side by side in a deep dish containing cold water. Stir the water, and note its temperature by means of a thermometer; after standing for a few minutes, measure the height of each liquid column above the top of the rubber stoppers. Pour into the dish some hot water, sufficient to raise the temperature of the water about 10° C.; stir the water continuously for a few minutes, note the temperature, and again measure the heights of the liquid columns. Add more hot water, and repeat the observations. Take other similar sets of observations until the temperature has been raised to 60° C or 70° C. Record your observations thus.

Temperature	Height of mercury column.	Height of water column	Height of spirit column

Plot the readings for each liquid on the same piece of squared paper, and state the information obtained from each of the curves.

Real and apparent expansion of liquids.—Hitherto the expansion of the glass containing a liquid has not been considered But the glass, like most substances, also expands when heated, though care must be taken to observe this expansion. as the expansion of the liquid is so much more. The momentary sinking of the liquid levels when the flasks were first placed in the warm water in Expt. 107, was caused in each case by the expansion of the flasks. The flasks became warm first and increased in size The liquid level fell. As soon, however, as the contained liquid began to get warm, with its greater rate of expansion, it outstripped the expansion of the glass and the liquid level rose again. The amount by which a liquid expands appears less than it actually is because of the expansion of the vessel. This expansion which it appears to possess is called its **apparent expansion.** To obtain the real expansion of a liquid it is necessary to add to the apparent expansion the amount the glass expands, or

$$\text{Real expansion of a liquid} = \text{its apparent expansion} + \text{expansion of the glass}$$

If any two of these values are known, evidently the third can be calculated easily.

It will be seen that in the case of a thermometer and in Expts. 106, 107 and 116 apparent expansion only is observed.

EXPT. 117. **Coefficient of expansion of liquids.**—Close one end of a glass tube about 30 cm. long and 3 mm bore Partly fill the tube with water, and fasten it to a thermometer by means of threads or india-rubber bands (Fig 110). Place the combination in melting ice, so that the water is surrounded by the ice, and observe the degree on the thermometer level with the surface of the water in the tube. Repeat the operation with the combination successively in water at 50°, 60°, 70°, 80°, and 90° C., taking care that the water in the tube is immersed completely. Now take the combination out of the water, and measure the distance from the bottom of the tube to the point at which the surface of the water stood in each

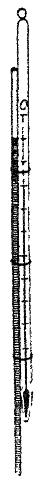


FIG 110.—Determination of the coefficient of expansion of a liquid

case, taking care that the tube does not move relatively to the thermometer. Record the observations thus:

Temperature	Length of water column	Increase of temperature	Increase of length

The apparent coefficient of expansion of a liquid may be determined with accuracy by means of a relative density bottle and a balance, as described in the following experiment

EXPT. 118.—**Alternative Method.** Clean and dry a relative-density bottle, and determine its weight (w_1) Fill the bottle with the given liquid (*e g.* turpentine), insert the stopper, and immerse the bottle to the neck in cold water. Allow it to remain in the water bath for about five minutes, frequently stir the water, and note its temperature ($t°$) just before removing the bottle. Hold the bottle in several folds of a *cold* duster, preventing direct contact of the warm hand, carefully dry the outside of the bottle, and weigh it (w_2). Pour away most of the cold water, and replace it with hot water sufficient to raise the temperature to 60° or 70° C. Suspend the bottle by a thread so that it is immersed to the neck in the warm water. Read the temperature, and, by means of a small flame under the water bath, keep the temperature constant for at least five minutes, in the meantime frequently stirring the water. Note the final temperature (T°), dry the outside of the bottle in the manner described above, and wipe off any liquid from the top of the stopper; allow the bottle to cool, and weigh it (w_3).

Weight of liquid contained in the bottle, at $t°$ C., $=(w_2-w_1)$ gm.
and " " " at T° C, $=(w_3-w_1)$ "

Hence, a weight of liquid (w_3-w_1) will expand by an amount (w_2-w_3), when warmed from $t°$ C to T° C., and the *apparent* coefficient of expansion $=\dfrac{w_2-w_3}{(w_3-w_1)(T-t)}$.

When the *real* coefficient of expansion is required, the coefficient of *cubical* expansion of glass should be added to the above result. Thus, supposing that the coefficient of linear expansion of glass is 0·000008, the above result should be increased by an amount 0·000024.

The anomalous expansion of water.—The first effect of cooling water is to make it get smaller in volume, or contract. This

goes on steadily until the temperature of four degrees Centigrade (4° C.) is reached. From this point, though the cooling is continued the water no longer contracts, but begins to expand. This expansion continues until the temperature 0° C. is reached, when the water begins to change into solid ice, which is much larger than the water from which it is formed.

Conversely, if some water be taken at a temperature of, say, 1° C., and allowed to get warmer, it will steadily get smaller in volume up to a temperature of 4° C., but after this temperature is reached the volume will increase as the temperature rises.

It has been shown already that if the volume of a body gets greater while its mass remains the same, the density of the body must get less and less. Since the same mass of water gradually gets smaller and smaller in volume as it is cooled down or warmed up to 4° C., we may say that its density becomes greater and greater as the temperature approaches 4° C. This temperature (4° C.), at which a given volume of water weighs more than the same volume at any other temperature, is known as the temperature of the **maximum density of water.**

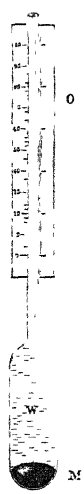


FIG. 111.—Apparatus for expansion of water.

Determination of temperature of maximum density of water.—In order to observe the changes in volume occupied by water near the temperature of maximum density, it is necessary to use, as a containing vessel, some device which has a constant capacity. This can be obtained by using a glass vessel partly filled with mercury, as shown in Fig. 111. When such a vessel is cooled, the contraction of the glass diminishes the capacity of the vessel, but the contraction of the contained mercury *increases* the capacity of the space within the tube and above the mercury. Since the coefficient of expansion of mercury is about seven times as great as the coefficient of cubical expansion of glass, a vessel of constant capacity is obtained if about one-seventh part of the total capacity is occupied by mercury. In order to fit up the apparatus, the empty tube is weighed, filled with mercury, and

again weighed. The tube is inverted, and alternately heated and cooled, until six-sevenths of this mercury has been expelled. The tube is then fixed in an upright position and filled with *boiled* distilled water.

EXPT. 119.—**Expansion of water near freezing point.** Support the apparatus shown in Fig. 111 in a wide test tube containing mercury, so as to secure uniformity of temperature. Place a thermometer in the mercury, and support the wide tube containing it and the apparatus in a beaker of cold water. Notice the position of the top of the liquid in the tube, and read the temperature shown by the thermometer. Add ice to the water, and as the temperature falls notice the level of the liquid in the tube for every degree down to 1° or 2° C

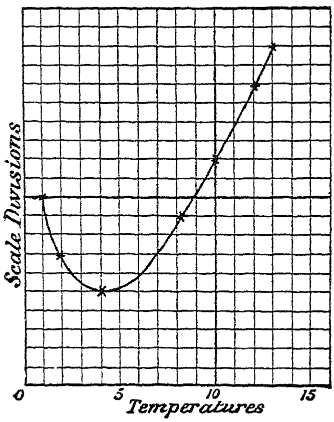


FIG 112.—Graphic representation of changes in volume of water near the temperature of maximum density.

Then let the temperature of the water in the beaker gradually rise, adding a little warm water, if necessary, and again observe the positions at the same temperatures as before. The mean of the two positions observed for each temperature should be taken as the true reading for that particular temperature. Construct a curve like that shown in Fig. 112 to represent the observations of the changes of volume of water at temperatures near the freezing point.

Hope's apparatus.—An experiment with what is known as Hope's apparatus shows very well that water is at its maximum density at 4° C. A metal cylinder provided with two side necks in the way shown in Fig. 113 is filled with water at 10° C. Into the side necks, corks with thermometers passing through them are fitted. The sides of the vessel should be wrapped round with cotton wool, and a piece of cardboard should be placed on the top of the cylinder. A freezing mixture, which can be made by mixing salt with pounded ice, is applied to the middle of the cylinder. This is done by filling a vessel, fixed round the middle of the outside of the cylinder, with the mixture in a way which the illustration makes quite clear. The freezing mixture, of

course, at once cools the water in the middle of the cylinder. On watching the thermometers it is found that the first effect of the cooling is to cause the temperature of the lower thermometer to fall. The temperature of the upper thermometer, however, remains unaltered. The only way in which this can be explained is by supposing that as the water in the middle of the cylinder is cooled it gets denser and sinks to the bottom. As the cooling proceeds it is found that the water at the bottom of the cylinder *never gets below* 4° C. But soon after the water at the bottom of the cylinder has reached 4° C., the temperature of the upper thermometer begins to fall and goes on getting lower till it actually reaches 0° C. But all this time the water at the bottom remains at 4° C.

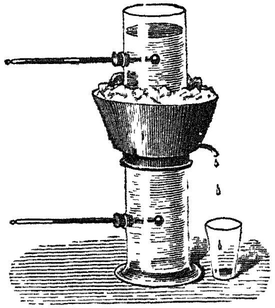


FIG 113.—Hope's apparatus for the observation of the temperature of maximum density of water

Now it is quite clear that the densest water will sink to the bottom, and as the temperature of the water there remains at 4° C. it may be concluded that water at this temperature is denser than any other.

These considerations are summed up in the statement that **water at a temperature of 4° C. expands whether it is heated or cooled.**

EXPANSION OF GASES.

Coefficient of expansion of gases.—In the case of the expansion of solids and liquids, the coefficient is so small that, although the strict definition refers to the increase in length when heated from 0° C. to 1° C., yet, in conducting an experiment, it suffices to determine the increase in length due to an increase of temperature of 1° C. at any part of the thermometer scale. But, **in the case of the expansion of gases, the coefficient is so great that it has to be defined in terms of the volume occupied at 0° C.**

Owing to the compressibility of gases (*see* Boyle's Law, p 85), the pressure to which the gas is subjected has to be taken into consideration. An increase of temperature may give rise, according to the arrangement of the containing vessel, either to an increase in *volume* of the gas or to an increase in its *pressure*. As a general rule, it is usual either (i) to keep the pressure constant, and to measure the increase in volume when

the temperature is raised, and so to determine the coefficient of expansion *at constant pressure*, or (ii) to keep *the volume constant* and measure the increase in pressure when the temperature is raised.

Expansion of gases, at constant pressure.—John Dalton, at Manchester in 1801, made the first important experiments on the expansion of gases. He claimed that the increase in volume for each equal rise in temperature is a constant fraction of the volume *at the temperature immediately preceding*. He also proved that *all gases expand equally when heated* Shortly afterwards, J. A. C. Charles (a professor of Physics in Paris) found that the expansion is a constant fraction of the volume *at some arbitrary fixed temperature* (and *not* at the temperature immediately preceding, as stated by Dalton). The law, generally known as **Charles's Law**, according to which gases expand when heated, may be expressed thus. **Equal volumes of all gases under the same constant pressure, expand by the same fraction of the volume at 0° C. for every degree rise in temperature.**

Charles's Law is practically true for the so-called permanent gases, such as hydrogen, air, nitrogen and oxygen, the increase in volume for an increase of 1° C. in temperature being in each case $\frac{1}{273}$ or 0·00366 of the volume at 0° C.; and this fraction may be termed **the coefficient of expansion of a gas at constant pressure.**

Consider a volume of 273 c.c. at 0° C., this would become 274 c.c. at 1° C, 275 c.c. at 2° C. and $(273+t)$ c.c. at t° C. Or, expressed in another way, if the volume at 0° C. be taken as unity, the volume at other temperatures may be stated as follows·

0° C.	1° C	2° C	30° C	t° C
1	$1+\frac{1}{273}$	$1+\frac{2}{273}$	$1+\frac{30}{273}$	$1+\frac{t}{273}$

Let the volume of gas at 0° C. and t° C. be V_0 and V_t respectively, and the coefficient of expansion be represented by α, then

$$V_t = V_0(1+\alpha t). \quad \ldots\ldots\ldots (1)$$

This may also be written,

$$V_0 = \frac{V_t}{1+\alpha t}. \quad \ldots\ldots\ldots (2)$$

The student must carefully bear in mind that the fraction $\frac{1}{273}$ refers to the volume of the gas *at* 0° *C.*; if the volume at some higher temperature, say 50° C., is given, it is not correct to say that it will increase by $\frac{1}{273}$ of this volume when warmed to 51° C, but it *is* correct to say that it would expand by $\frac{1}{273}$ of the volume *which it would occupy at* 0° *C.*

When the volume at some temperature other than 0° C. is given, it is necessary to calculate, by equation (2), the volume which it would occupy at 0° C., and then, by equation (1), the volume it would occupy at 51° C. An alternative method is as follows

Suppose V_t to be the volume of gas at t° C., and V_T the volume at T° C., then

$$V_T = V_0(1+\alpha T) \text{ and}$$
$$V_t = V_0(1+\alpha t).$$

Hence
$$\frac{V_T}{V_t} = \frac{1+\alpha T}{1+\alpha t} \quad \ldots\ldots (3)$$
$$= 1+\frac{T}{273}\Big/1+\frac{t}{273}$$
$$= \frac{273+T}{273+t} \quad \ldots\ldots (4)$$

or
$$V_T = V_t \times \frac{273+T}{273+t}$$

Determination of coefficient of expansion of a gas—In carrying out an experiment to determine the coefficient of expansion of a gas it is not always convenient to observe the volume occupied by the gas at 0° C., and it would be necessary to *calculate* the volume which the gas would have occupied at 0° C. An alternative method of calculating the result is to find the *apparent* coefficient of expansion of the gas between the two temperatures used in the experiment. It can be shown, as indicated below, that when the lower temperature used in the experiment is t° C., the apparent coefficient of expansion is

$$\frac{1}{273+t}.$$

Suppose α to be the apparent coefficient of expansion between the temperatures t° and T° C., and V_t, V_T, to be the observed volumes at t° C. and T° C, then

$$V_T = V_t\{1+\alpha(T-t)\}. \quad \ldots\ldots (5)$$

But (p 165) $$\frac{V_T}{V_t}=\frac{273+T}{273+t}.$$

Hence, from equation (5),

$$1+\alpha(T-t)=\frac{273+T}{273+t},$$

$$\text{or}\quad \alpha(T-t)=\frac{T-t}{237+t}, \qquad \text{or}\quad \alpha=\frac{1}{273+t}$$

EXPT. 120—**Expansion of gas at constant pressure** Dry thoroughly a 500 c c. flask, and fit it with a one-holed *rubber* stopper through which passes a short glass tube with rubber tubing and clip attached. Fix the flask in an open metal vessel (as shown in Fig. 114) sufficiently large for the flask to be immersed completely in boiling water. The flask can be held immersed by means of an inverted stout glass jar held in a clamp at its upper end In order to prevent steam from entering the flask it is well to attach a long glass tube to the open end of the rubber tubing. Keep the clip open, and gradually heat the water to boiling. Let the water boil for about 5 minutes, and close the clip. Note the temperature of the water Remove the flask, immerse it in a bath of cold water with its neck downwards. Open the clip, and keep the flask moving in the water so that the remaining air acquires the same temperature as the water. Lower the flask till the level of the water is the same inside and outside; then close the clip. Note the temperature of the cold water. Remove the flask, and measure the volume of water which has entered the flask. Find also the volume of the whole flask up to the lower surface of the stopper.

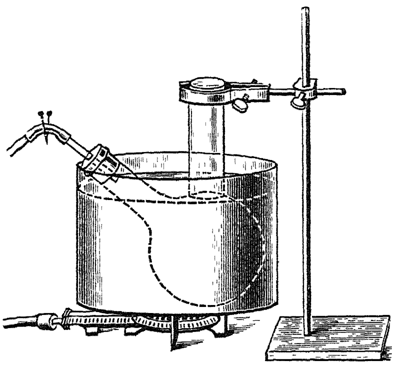


FIG. 114—Determination of the coefficient of expansion of a gas at constant pressure

A method of calculation is indicated by the following example of an experiment:

Volume of whole flask - - - - - -	340 c.c.
" water drawn into flask - - - -	77 c.c
Temperature of boiling water - - - - -	98° C.
" cold " - - - - -	15° C.

340 c.c. of air, in cooling through 83° (*i.e.* from 98° C. to 15° C.), contract 77 c.c.

340 c.c. of air, in cooling through 1°, contract $\frac{77}{83}$ c.c.

" " " 98°, " $\frac{77}{83}\times 98 = 90.9$ c.c.

Hence,

(340 − 90·9) c.c., at 0° C., expand 90·9 c.c. when warmed to 98° C.

or 1 c.c., at 0° C., expands $\frac{90.9}{249.1}$ c.c. " "

or 1 c.c., at 0° C., " $\frac{90.9}{249.1\times 98}$ c.c. " through 1° C.

or " " " 0·00372 c.c. " "

An alternative method of calculating the result is to determine the *apparent* coefficient of expansion between the temperatures 15° C. and 98° C. The result should be approximately $\frac{1}{273+15}=\frac{1}{288}$. Thus, by equation (5) (p. 165),

$$\alpha=\frac{V_T - V_t}{V_t(T-t)}$$

$$=\frac{77}{(340-77)(98-15)}=\frac{77}{263\times 83}=\frac{1}{283.5}.$$

Fig. 115 represents another method of determining the coefficient of expansion of a gas at constant pressure.

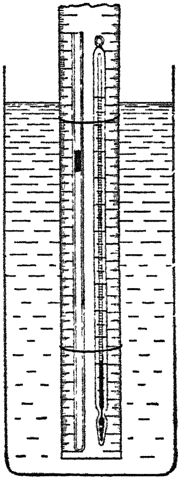


FIG. 115.—Determination of the coefficient of expansion of a gas at constant pressure.

EXPT. 121.—**Direct method.** Obtain a piece of thermometer tubing of about 1 mm. bore and 20 cm. long. Suck into it a length of about 1 cm. of mercury. Seal one end of the tube and arrange that the index of mercury comes near the middle of the tube when the end has been closed and the tube is cool. Fasten the tube to a thermometer, closed end downwards (Fig. 115). You have in it a certain volume of air, and can find the volume at different temperatures as you did with liquids. Place the combined thermometer and tube in melting ice and notice the position of the air column with reference to the thermometer scale. Repeat the operation for every 10° up to 100° C., taking care that the air column is immersed completely

in each case, and giving the tube two or three taps before making an observation, in order to make sure that the mercury is not sticking to the tube Record your observations thus

Temperature	Length of air column	Expansion for 10° C.	Average expansion for 1° C

As the tube is cylindrical and uniform in bore, the volume of the air in it is proportional to the lengths of the air column The average increase of volume for 1° C, expressed as a fraction of the volume at 0° C., is the *coefficient of expansion.* Calculate from your results, by means of equation (4) (p 165), the coefficient of expansion of air

When a gas is heated in circumstances where, as in these experiments, free expansion is possible, it is said to expand *under a constant pressure.* Both at the beginning of the experiment and after the gas has been heated, the pressure to which it is subjected is simply that of the atmosphere.

It should be remembered carefully that the same results would be obtained if any other gas, instead of air, were experimented with.

The absolute scale of temperature —Charles's Law holds good whether a gas is expanded by heating or contracted by cooling. We may therefore construct a table, such as the following, which gives the volume at different temperatures of a mass of gas which occupies 273 c c. at 0° C. ·

283 c.c.	at	10° C.
278 c c.	,,	5° C.
273 c c.	,,	0° C.
268 c.c.	,,	− 5° C.
263 c.c.	,,	− 10° C

We may thus imagine that, if such a degree of cooling were possible, a temperature would be reached at which the volume of the gas entirely disappeared. From the above Table this temperature will be at − 273° C. This is termed the **absolute zero of temperature**; and temperatures reckoned in Centigrade degrees from this zero are called **absolute temperatures** The constant coefficient of gases thus provides an absolute scale of temper-

ature. The following are corresponding temperatures on the absolute and Centigrade scales :

Absolute Scale.	Centigrade Scale
0°	−273°
200°	−73°
273°	0°
373°	100°
273 + t°	t°

Referring back to equation (4), on p. 165, where it is shown that the ratio of the volumes, V_t and V_T, occupied by a gas at temperatures t° C. and T° C. is given by the equation

$$V_T/V_t = (273 + T)/(273 + t),$$

it is evident that **the volume of a given mass of gas is proportional to its absolute temperature.**

The coefficient of increase of pressure of a gas, when its volume is kept constant.—If a gas be heated while its volume is kept constant, its pressure increases according to the same law as that which holds good for the increase of volume when the pressure is kept constant. If P_0 be the pressure exerted by any volume of a gas at 0° C., its pressure P_t at t° C. is given by the equation

$$P_t = P_0(1 + \alpha t),$$

providing that the volume remains constant. The symbol α denotes the coefficient of increase of pressure at constant volume; and it has the same value ($\frac{1}{273}$) as the coefficient of increase of volume at constant pressure.

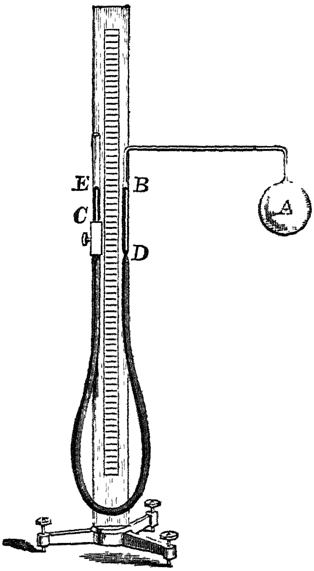


Fig. 116.—Determination of the coefficient of expansion of a gas at constant volume

Fig. 116 represents an arrangement of apparatus which may be used for determining the coefficient of increase of pressure. It consists of a glass bulb A connected by a short horizontal tube to the Boyle's tubes

EC and BD. The tube EC can be moved up and down the wooden stand. The bulb A is surrounded with water contained in a metal vessel, and the temperature of the water is observed by means of a thermometer. The tube EC is adjusted so that the mercury surface at B coincides with a mark made near to the top of the tube. If H be the height of the barometer, and if h be the difference of level of the mercury surfaces at E and B, then the total pressure on the air in A is equal to $(H \pm h)$, the sign used depending upon whether the surface at E is above or below the surface at B. The temperature of the water bath is now raised; the temperature is noted, and the tube EC is adjusted so that the mercury surface at B occupies its previous position.

If P_1 and P_2 are the total pressures at temperatures $t_1°$ C. and $t_2°$ C.,

then $$P_1 = P_0(1 + \alpha t_1) \text{ and}$$

$$P_2 = P_0(1 + \alpha t_2).$$

Hence $$\frac{P_2}{P_1} = \frac{1 + \alpha t_2}{1 + \alpha t_1}.$$

By means of this formula the value of α can be derived from the above readings

Coefficients of Expansion.

Linear (Mean, between 0° C and 100° C.).

Copper, - - -	0·0000166	Silver, - - -	0·0000192
Iron, - - -	0·0000109	Zinc, - - -	0·0000292
Lead, - - -	0·0000292	Brass, - - -	0·0000193
Platinum, - -	0·0000089	Glass, - - -	0·0000086

Cubical.

Benzene, - - -	0·00138	Alcohol (between 0° C. and 80° C.), -	0·00104
Carbon bisulphide, -	0·00147	Olive oil, - - -	0·00074
Ether (between −15° C. and 38° C.),	0·00215	Turpentine, - -	0·00105
Glycerin, - - -	0·00053	Water (between 10° and 100°), - -	0·00043
Mercury, - - -	0·00018		

EXERCISES ON CHAPTER XIII.

1. A vessel of water at the freezing point contains two small glass bulbs. One is at the bottom, the other floats, but is almost wholly below the surface. The water is heated gradually; soon the bulb that was at the bottom rises, but after a while sinks again, and remains sunk What is the meaning of this behaviour? How will the other bulb behave during the heating of the water?

2. Why may a glass stopper sometimes be loosened by warming the neck of the bottle?

3. A copper rod, the length of which at 0° C is 2 metres, is heated to 200° C What length will it be now? At what temperature will its length be 200·51 cm ?

4. Assuming the highest summer temperature to be 40° C, and the lowest winter temperature to be −20° C, what allowance should be made for expansion in one of the 1700 feet iron spans of the Forth Bridge?

5. A sheet of brass is 20 cm. long and 15 cm. broad at 0° C. What is its superficial area at 80° C.?

6. The brass pendulum of a clock beats seconds exactly at 25° C. How many seconds a day will the clock gain if the temperature falls to 0° C.?

7. The density of a liquid at 0° C. is D_0, and its coefficient of cubical expansion is k. Show that its density at t° C. is

$$D_t = D_0/(1+kt).$$

If the density of mercury at 0° C. be 13·596 grams per c.c, find its density at 125° C. if the coefficient of cubical expansion of mercury between these two temperatures be 0 00018.

8. The volume of a gram of water at 10° C. being 1·000269 c.c. and 1·002935 c c. at 25° C., find the mean coefficient of expansion between these two temperatures.

9. A graduated glass flask has a capacity of 100 c.c. at 10° C. If the mean coefficient of expansion of water between 4° C. and 25° C. is 0·00014, what weight of water will the flask hold at 25° C.?

10. An empty specific gravity bottle weighs 38 5 grams When filled with mercury at 25° C. it weighed 360·25 grams. It was then heated to 100° C. When cool it weighed 356·67 grams. Calculate the apparent coefficient of expansion between the above temperatures.

11. The bulb of a Centigrade thermometer has a volume of 1 c c. at 0° C., and the sectional area of the bore is 0·1 sq. mm. If the lower fixed point be marked 1 cm. above the top of the bulb, and the coefficient of cubical expansion of mercury in glass is 0·00015, find the distance between the points marked 20° C. and 70° C.

12. A glass rod which weighs 90 grams in air is found to weigh 49·6 grams in a certain liquid at 12° C. At 97° C its apparent weight in the same liquid is 51·9 gm. Find the coefficient of expansion of the liquid, taking the coefficient of cubical expansion of glass as 0 000024

13. A hollow lead vessel, internal capacity 20 c c at 10° C, terminating in a narrow capillary tube of 1 mm. diameter, is filled with water so that the water surface is visible and below the top of the tube. If the vessel be cooled from 10° C to 4° C, will the water fall or rise? Calculate the distance through which it moves. (Mean coefficient of expansion of water between 4° C and 10° C. is 0 000045)

14. 100 c c. of air are measured at 20° C If the temperature be raised to 50° C, what will the volume be, the pressure remaining constant?

15. 15 litres of air, measured at 27° C., are cooled to 7° C. By how much will the volume diminish?

16. On heating a certain quantity of mercuric oxide it is found to give off 380 c.c of oxygen gas, the temperature being 24° C and the barometric height 74 cm What would be the volume of the gas at normal temperature and pressure (0° C. and 76 cm)?

17. A quantity of air is contained in a straight vertical tube closed at the lower end, the air being shut off by a pellet of mercury, the weight of which may be neglected When the temperature is 13° C, the mercury is 66 cm. from the bottom of the tube; and when the temperature is raised to 52° C. this distance is increased to 75 cm Calculate the coefficient of expansion of air.

18. If the density of air at normal temperature and pressure be 0 001293 gm per c c, prove that the density at 15° C. and 76 8 cm pressure is 0 001239.

19. Calculate the weight of air in a room 20 × 10 × 2 metres when the temperature is 15° C. and the pressure 77 cm.

CHAPTER XIV.

QUANTITY OF HEAT AND ITS MEASUREMENT; SPECIFIC HEAT.

Difference between heat and temperature.—Temperature is not heat; it is only a state of a body, for the body may be cold one minute and hot the next. A hot body is one at a high temperature, a cold body one at a low temperature. When a hot body and a cold body are brought into contact there is an exchange of heat until they are both of the same degree of hotness or coldness, that is, at the same temperature. Hence, **temperature** may be defined as **that which determines the transference of heat from one part of a body to another part, or from one body to other bodies in its neighbourhood.**

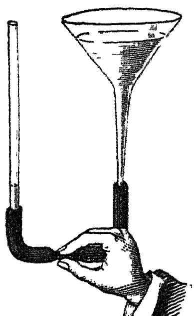


FIG 117—Tendency to equality of level of a liquid in communicating vessels.

Hydrostatic analogy.—When two vessels containing water and arranged at different levels are connected by means of a piece of india-rubber tubing, there is a flow of water from the vessel of water at the higher level towards the vessel at a lower level. This is a consequence of a property possessed by all liquids which makes them, as we say, *seek their own level.* This flow of water continues until the water in the two vessels is at the same level Evidently this is a similar state of things to that which we have in the case of a hot and cold body in contact. In one case there is a flow of water until the level is the same in the two vessels. In the other there is a passage of heat until the temperature

of the two bodies is the same. *Temperature corresponds to water-level.* It may also be said that just as different vessels may have different capacities for holding water, so different substances may have different capacities for heat. The same amount of water produces different changes of level in vessels of different sizes (Fig. 118), and, in a similar way, the same quantity of heat produces different changes of temperature (or heat-level) in substances having different capacities for heat.

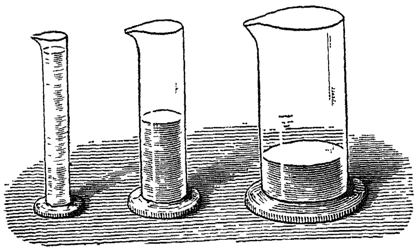


FIG. 118.—The same amount of liquid produces different rises of level when put into vessels of different capacities.

EXPT. 122.—**Mixture of equal weights of cold and warm water.** Put a certain weight of warm water in a beaker, and an equal weight of cold water in another beaker. Observe the temperature of each by means of a thermometer. Pour the cold water into the hot. It will be found on stirring them together with the thermometer (taking care not to break the thermometer), that the temperature of the mixture is about midway between the two original temperatures.

From the observations construct a table like that below, to show that the temperature, produced by mixing equal weights of the same liquid at different temperatures, is approximately equal to half the sum of the temperatures:

Temperature of water A.	Temperature of water B	$\frac{A+B}{2}$	Temperature of mixture.

Quantity of heat in water at different temperatures.—Quantity of heat may be measured by its heating effect, so that we can say that the quantity of heat in a certain quantity of water depends upon the *weight* of the water and its *temperature*. For our purpose the amount of heat in 100 grams of water at a temperature of 60° C. may be regarded as double that in 50 grams of water at 60° C., if for the sake of simplicity water at 0° C. is considered to contain

no heat When equal or unequal weights of water at different temperatures are mixed, the quantity of heat lost by the hot water is the same as the quantity gained by the cold water, neglecting the loss of heat that occurs during the experiment and the effect upon the vessels containing the water. When these influences are taken into consideration, it is found that the fall of temperature multiplied by the weight of hot water is equal to the rise of temperature multiplied by the weight of cold water

EXPT 123—**Loss and gain of heat** Weigh about 200 gm of cold water into a beaker, and observe its temperature. Put an equal weight of water into another beaker, heat it to about 45° C. Now place the beaker of warm water upon a table, with a thermometer in it, and observe its temperature When the temperature has fallen, to say 40° C, take hold of the beaker with a duster, and quickly pour the warm water into the cold Stir up the mixture with the thermometer, and observe the temperature after mixing. Record your observations as below

Weight of cold water, - - - -	.. ...gm	
Temperature ,, - - - -	. . °C.	
,, of mixture, - - -	.. ...° C	
Number of degrees through which the temperature of the cold water was raised, - - - - - -		.. . .° C
Weight of warm water, - - -	.... gm	
Temperature of warm water, - -	.... .° C	
Number of degrees through which the temperature of the warm water fell, -		... ° C

Tabulate the gain and loss of heat that occur, as shown below:

GAIN	LOSS
Weight of cold water × its rise of temperature	Weight of warm water × its fall of temperature
. . .. ×	.... × .... .
....	....

The gain will be found to be slightly less than the loss. This is not really the case, and it only appears so because the amount of heat required to raise the temperature of the glass of the beaker containing the cold water has not been taken into consideration.

EXPT. 124.—**Equality of gain and loss of heat.** Repeat the experiment, using unequal weights of hot and cold water. Notice that in each case the weight of hot water × the fall of temperature is approximately equal to the weight of cold water × the gain of temperature The difference shows the amount of heat absorbed by the glass of the cold beaker.

Unit quantity of heat.—As in all other cases of measurement, a unit or standard quantity is required with which to compare quantities of heat. The unit quantity of heat generally adopted is the **amount of heat necessary to raise the temperature of one gram of water through one degree Centigrade.** This unit is called a **calorie** or **therm.** The amount of heat required to raise the temperature of 2 grams of water through 1° C is thus 2 units or 2 calories. Similarly, if 1 gram of water at 0° C be heated until its temperature is 1° C., it will have received 1 unit of heat, or 1 calorie. When the temperature of this 1 gram of water reaches 3° C. it will have received 3 units of heat If the temperature of 10 grams of water at 0° C. be raised to 12° C, the water will have received 10 times 12 units of heat, the number of units being equal to weight (in grams) × increase of temperature (in degrees Centigrade). In fact, the number of units of heat taken up by any weight of *water* as its temperature rises, or the amount given out by any weight of *water* as it cools, may be found by multiplying the number of grams of water used by the number of degrees, as measured by a Centigrade thermometer, through which the temperature rises or falls.

Comparison of heat quantities—It has been seen that the quantity of heat in water depends upon (i) the weight of the water, and (ii) its temperature. It might be supposed, therefore, that as any weight of water at a certain temperature contains a certain quantity of heat, the same weight of another substance at the same temperature contains the same quantity of heat. This, however, is not the case. 100 grams of water at a temperature of 50° C. always contain 5000 units of heat,* but 100 grams of turpentine, mercury, lead, iron, or any other substance at the same temperature as the water, namely 50° C., do *not* contain this number of units of heat. The quantity of heat in a substance thus not only depends upon the weight and the temperature, but also upon the substance itself.

* Assuming for simplicity that water at 0° C contains no heat.

EXPT 125 —**The same quantity of heat may produce different changes of temperature.** Weigh out equal quantities of water and turpentine at the same temperature in two beakers of the same size Pour equal quantities of hot water at the same temperature into the cold water and into the turpentine. Observe the rise of temperature produced in each case. Though the equal amounts of hot water contain the same quantity of heat, the rise of temperature of the turpentine will be found to be more than the rise of temperature of the cold water, in other words, the *capacity of turpentine for heat is less than the capacity of water for heat.*

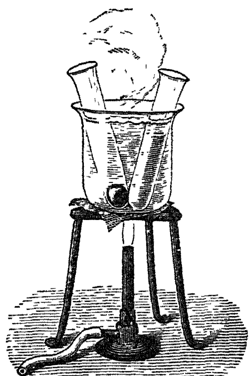


FIG 119 —Equal weights of water and mercury do not become hot at equal rates, though they both have the same opportunity

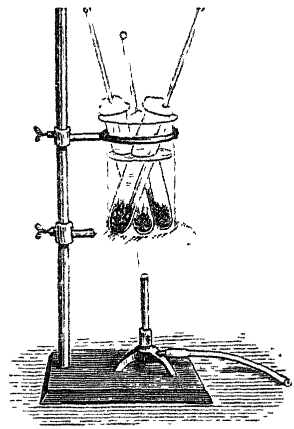


FIG 120 —Method of heating metals in test tubes for the determination of their capacities for heat Each test-tube has a loose plug of cotton wool at the top

EXPT 126 —**Comparison of rates at which water and mercury gain heat.** Weigh out equal quantities of cold water and mercury at the same temperature in two test-tubes or flasks Support the two vessels side by side at the same distance above a flame, or in a large beaker of boiling water. Let them remain for a few minutes, then observe their temperatures The rise of temperature of the mercury will be found to be greater than the rise of temperature of the water; in other words, mercury gets hot more quickly than water under the same conditions

EXPT. 127 —**Different quantities of heat in equal weights of different substances at the same temperature.** Place equal weights of lead, iron, and water in different test-tubes standing in the same beaker Put a Centigrade thermometer into each tube, so that the bulb is immersed

in the liquid or metal; and close the mouth of the tube with a loose plug of cotton wool. Heat the beaker over a laboratory burner until the water boils; and keep the water boiling until the contents of the tubes are at a constant high temperature. Provide three beakers containing equal weights of cold water at the temperature of the room. Put the hot lead into one of these, the hot iron into another, and the hot water into the third. Stir the mixtures and note the temperature in each case

Equal weights of water at the same temperature are thus shown to be heated to different degrees of temperature by equal weights of lead, of iron, and of water at the same high temperature.

Capacity of water for heat.—Of all known substances, water has the greatest capacity for heat; consequently a larger amount of heat is required to raise the temperature of a given weight of water through a certain number of degrees than is needed by an equal weight of any other substance. Similarly, in cooling through any number of degrees of temperature a definite weight of water gives out a larger amount of heat than an equal weight of any other substance, the temperature of which falls through the same number of degrees.

Comparison of capacities for heat.—When equal weights of water, iron, lead, and mercury at the same high temperature, *e.g.* that of boiling water, are each in turn stirred up with equal weights of cold water at the same temperature and in separate beakers, it is found that the hot water raises the temperature of the cold water in which it is placed through a larger number of degrees than any of the other substances. This is because the capacity for heat of water is greater than that of any of these (or any other) substances.

If the temperature be observed of the mixture formed in each of the cases supposed, namely, iron and water, lead and water, and so on, and then the number of degrees through which each has raised the temperature of the water into which it was put is calculated, a series of numbers is obtained which enables a comparison to be made of the capacities for heat of each of the substances experimented with.

The amount of heat required to raise the temperature of one gram of a substance through 1° C. (or, the amount of heat given out by one gram of a substance the temperature of which falls through 1° C)

in comparison with the amount of heat taken up (or given out) by an equal weight of water in the same circumstances, **is known as the specific heat of the substance**

Determination of specific heats.—To obtain the specific heat of a substance, usually a convenient quantity of the substance is heated to a definite temperature and then allowed to give up its heat to a known weight of water, contained in a thin brass or copper vessel known as a **calorimeter.** If losses through radiation and other causes are avoided as much as possible, the heat lost by the substance in cooling may be taken as equal to that gained by the water in having its temperature raised. The weight and rise of temperature of the water having been observed, this gain of heat can be calculated by multiplying the weight of water by its rise of temperature The heat lost by each gram of the substance, the specific heat of which is being determined, in cooling 1° C., can then be calculated, and the result is the specific heat required

In an experiment of this kind, however, a certain amount of heat is spent in warming the calorimeter, which may be regarded as equivalent to an extra quantity of water The amount of water to which the calorimeter is equivalent is called its **water-equivalent or water-value**; and it must be taken into consideration in a determination of the specific heat of a substance. Of course, the water-value of a calorimeter has only to be found once by experiment or calculation, and it can then be used in any determination of specific heat in which the particular calorimeter is employed.

The following experiments illustrate the method of determining the water-equivalent of a calorimeter and the use of this value in determining specific heats.

EXPT. 128 —**The water-equivalent of a calorimeter.** Determine the weight in grams of a copper calorimeter Observe the temperature of the air and consequently of the calorimeter

Place the calorimeter in cotton wool in a beaker Pour into the calorimeter a convenient quantity of warm water at a temperature of from 35° C to 40° C Enough to fill the calorimeter to one-third is a good amount. Notice with a thermometer, which you should use carefully as a stirrer, that, on pouring the warm water into the cold calorimeter, its temperature falls. When its temperature becomes

stationary, which it will soon do, record the temperature again. Determine the weight of the calorimeter and water. Subtract the weight of the calorimeter, and so obtain the weight of water used.

Weight of calorimeter, - - - - -	.. gm.
Temperature of calorimeter, - - - -	......° C
Weight of water, - - - - - -	. . . gm
Temperature of water, - - - - -	. .° C
Resulting temperature, - - - -	. .. ° C.

The exchange of heat which takes place may be considered as follows:

Weight of hot water × fall of temperature

.. .. × .. ...

. ... calories.

This result gives the number of heat units used in increasing the temperature of the calorimeter by an observed number of degrees. Find from the result the number of calories required to raise the temperature of the calorimeter through 1° C, that is, the water-equivalent or water-value of the calorimeter.

EXPT. 129.—**Determination of the specific heat of solids**.—Determine the weight of the copper calorimeter, the water-equivalent of which you have found already. Pour in enough water to fill it to one-third. Again weigh. Put a thermometer into the water and leave it to take the temperature of the water. When the temperature is stationary, record it. Weigh out about 50 grams of short pieces of copper wire. Heat the copper in a test tube standing in a beaker or can as shown in Fig. 120, and record the temperature shown by a thermometer standing in the copper. Quickly introduce the hot copper into the cold water, stir, note the rise in temperature of the water, and, when constant, record.

Set down your observations thus.

Weight of calorimeter and water, - -	. ... . gm.
" " alone, - - -	. ... "
Weight of water in calorimeter, - - -	. . . "
Water-value of calorimeter, - - -	. ... "
Total water, - -	... "
Temperature of mixture, - - - -	. ....° C.
Initial temperature of water, - - -	.. ... "
Rise of temperature, - - - - -	...... "
Quantity of heat gained, - - - -	...... calories

Weight of copper,	- - - - -	...... gm.
Temperature of copper before mixing,	-	... .° C.
„ „ mixture,	- - - -	...... „
Fall of temperature,	- - - - -	. .... „

.. ... grams of copper the temperature of which fell ..... degrees gave out ...... calories gained by cold water and calorimeter,

therefore

1 gram of copper the temperature of which fell ...... degrees would give out ...... calories;

and

1 gram of copper the temperature of which fell 1° C. would give out ...... calories.

.The result thus obtained is the specific heat of copper.

EXPT. 130—**Specific heats of liquids.** Weigh a calorimeter. Half fill it with turpentine, and find the weight of the turpentine. Observe the temperature of the turpentine. Observe also the temperature of some boiling water. Pour boiling water into the turpentine, keep the two liquids well stirred, and observe the temperature of the mixture. Find the weight of the water added. From these observations calculate the specific heat of turpentine.

Determine in the same way the specific heat of mercury.

(The specific heats of liquids can be determined also by the method of cooling, as described in Expt. 162, p 225.)

TABLE OF SPECIFIC HEATS.

Aluminium,	0·212	Steel,	0·118
Brass,	0·094	Zinc,	0·094
Copper,	0·093		
Glass, crown,	0·161	Glycerin,	0·576
Glass, flint,	0·117	Mercury,	0·033
Iron,	0·112	Olive oil,	0·471
Lead,	0·032	Turpentine,	0·467
Sulphur,	0·184	Petroleum,	0·511

EXERCISES ON CHAPTER XIV.

1. If a pound of water at 100° C. be mixed with a pound of water at 0° C., the temperature of the mixture is 50° C. How would the result have differed if a pound of oil at 100° C. had been substituted for the hot water? Explain the difference.

2. Explain what is meant by specific heat. How would you show that equal weights of different substances give out different amounts of heat when cooled through the same range of temperature?

3. What is the capacity for heat of a body?

Which has the greater capacity for heat, 5 c.c. of mercury or 2 c.c. of water? (Specific gravity of mercury, 13·6; specific heat, 0·033.)

4. A copper vessel, weighing 125 gm., holds 800 gm. of water at its temperature of maximum density (4° C.). How much heat must be imparted to the vessel before the water begins to boil? Assume that there is no loss of heat by radiation.

5. If 90 gm. of mercury at 100° C. be mixed with 100 gm. of water at 20° C., and if the resulting temperature be 22°·3 C., what is the specific heat of mercury?

6. In order to determine the specific heat of silver, a piece of the metal weighing 10·21 gm. was heated to 101°·9 C. and dropped into a calorimeter containing 81·34 gm. of water, the temperature of which was raised from 11°·1 C. to 11°·7 C. If the water-equivalent of the calorimeter, stirrer, and thermometer was 2·91 gm., find the specific heat of silver.

7. If you had at your command a supply of tap water at 10° C. and of boiling water, what quantities of each would you take in order to prepare a bath containing 20 gallons of water at 35° C.?

8. In order to determine the temperature of a furnace, a platinum ball weighing 80 gm. is introduced into it. When it has acquired the temperature of the furnace it is transferred quickly to a vessel of water at 15° C. The temperature rises to 20° C. If the weight of water, together with the water-equivalent of the calorimeter, be 400 gm., what was the temperature of the furnace? (Specific heat of platinum = 0·0365.)

9. For the purposes of a foot-warmer, which is preferable, a bottle containing 10 lb. of water or a 10-lb. block of iron, both initially at 100° C.? Explain your answer.

10. A silver tea-pot weighs 300 grams. One gram of silver requires as much heat to warm it as would be required by 0·056 gram of water to warm it equally. The tea-pot contains 20 grams of tea-leaves, and each gram of tea-leaves requires as much heat to warm it as would suffice to warm equally 0·5 gram of water. If 600 grams of boiling water be poured into the tea-pot, calculate the highest temperature of the tea, assuming that tea-pot and tea-leaves were originally at a temperature of 15° C.

11. I take 2 ounces of lead and 2 ounces of water, place them in the same beaker, and heat the beaker over a Bunsen flame. I then take two other beakers, each containing 2 ounces of cold water, and

add the hot lead to one and the hot water to the other beaker. After stirring, I note the temperature in each case with a thermometer. State (*a*) how the thermometer readings differ, and (*b*) the cause of this difference.

12. It is required to find the quantity of heat lost by one gram of copper when its temperature falls through one degree. Describe how this can be done.

13. What is a unit of heat? If I pour a kilogram of mercury at 100° C. into a kilogram of water at 0° C., will the result be the same as though the water had been at 100° C. and the mercury at 0° C.? Give reasons for your answer.

14. How many heat units would be required to raise 50 grams of water at 0° C. to the boiling point? If this quantity of heat were added to a litre of water at 15° C., what would be the final temperature?

15. A piece of bronze weighing 67 grams and at a temperature of 100° C. was dropped into 65 grams of water at a temperature of 16°·5 C. The temperature of the water rose to 23°·5 C. Determine the specific heat of the bronze and explain what your result means.

CHAPTER XV.

PROPERTIES OF VAPOURS EVAPORATION AND BOILING. HYGROMETRY

The kinetic theory of gases.—It has been explained previously (p. 53) how such properties as pressure, the tendency to indefinite expansion, and diffusion observed in gases, may be attributed to a state of continual and rapid motion of the molecules of which the gas consists.

The pressure which a gas exerts on the walls of a containing vessel is due to the continuous and steady bombardment of the molecules of which it is composed; and the pressure which each molecule exerts is measured by the kinetic energy which it possesses at the moment of impact. and this kinetic energy is equal to $\frac{1}{2}mv^2$, where m is the mass of the molecule and v is its velocity.

Experiments on Boyle's Law (p. 85) have shown that when the volume of a gas is halved (*i.e.* when the density is doubled) the pressure is doubled. This is just what might be expected if the pressure be due to the bombardment of the molecules against the containing walls—for, by doubling the density we double the number of molecules striking in unit time each unit area of the vessel's surface, and therefore the pressure is doubled

At the same temperature, **the average velocity of the molecules is not the same for all gases.** Thus, at normal temperature and pressure, 1 c.c. of *air* exerts a pressure of 1·033 kilogram per sq. cm, and, under the same conditions, 1 c c. of *hydrogen* exerts the same pressure. But the latter gas weighs only one-fourteenth as much as the former; hence, to exert the same pressure, the average

velocity of the hydrogen molecules must be much greater than that of the air molecules. The average velocity, at normal temperature, of hydrogen molecules is about 1800 metres per second, and of air molecules about 450 metres per second. In general, we may say that the less dense the gas, the greater is the average velocity of the molecules

Further evidence of this difference in molecular velocity is obtained in the phenomenon of **diffusion.** Suppose that a cubical metal vessel is divided into two compartments by a partition of porous material, such as unglazed earthenware, and that one compartment is filled with hydrogen and the other with air. If there are as many hydrogen molecules in each c.c. of that gas as there are of air molecules in each c.c. of air, then, since the velocity of the former is four times as great as that of the latter, the hydrogen molecules will strike the partition four times as frequently as the air molecules strike it, and the former will pass through the pores of the partition four times as rapidly as the latter. Hence, there will be an increase in the number of gas molecules, and a consequent increase of pressure, in the compartment which originally contained air. and there will be a diminution of pressure in the other compartment. This phenomenon can be verified by the following experiment.

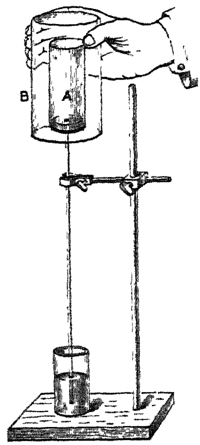


FIG 121 —Experiment on the diffusion of gases

EXPT. 131.—**Diffusion apparatus.** Close the open end of a porous cylindrical cell (such, for instance, as is used in fitting up a Daniell voltaic cell) with a rubber stopper perforated with one hole (A, Fig 121). Fix the end of a long narrow glass tube through the stopper Clamp the tube in a vertical position with its lower end dipping into a beaker of water. Fill an inverted beaker B with hydrogen, and hold it over the porous cylinder The hydrogen diffuses inwards more rapidly than the air diffuses outwards from the porous cell, and the increased pressure inside A causes bubbles of gas to escape from the lower end of the tube. Remove the beaker B; the hydrogen *inside* A now diffuses

outwards, and the diminished pressure inside causes the water to rise up the glass tube.

The Kinetic Theory of Gases also affords a simple explanation of the increase of pressure which a gas exerts when its temperature is raised. The additional energy contained in the gas when warmed is represented by an increase in the average velocity of the molecules, and this increase in velocity necessarily implies an increased pressure.

EVAPORATION AND BOILING.

Evaporation of liquids.—It is a common observation that water contained in a dish, and exposed to the air, gradually disappears. This **evaporation**, as it is termed, may be observed with many other liquids. The gradual escape of the molecules can be attributed only to the fact that the molecules of a liquid are in a state of vibratory motion; and this state differs from that which characterises a gas only in that the molecules of a liquid are packed much more closely together, and that mutual collisions are correspondingly more frequent. In these collisions some molecules may acquire occasionally a velocity considerably above the normal; if these, by chance, happen to be near the surface, they *may* escape into the air above the liquid, and there they assume the properties of gas molecules. The term **vapour** is given to the molecules in this condition.

An increase of temperature brings about an increased rate of evaporation. This would be anticipated from the theory that a rise in temperature increases the velocity of the molecules, and therefore the possibility of molecules escaping from the free surface of the liquid.

During the process of evaporation, we can imagine the space above the liquid to be occupied by a mixture of air molecules and the vapour. All these molecules are in a state of motion, and in frequent collision with each other; occasional molecules of the vapour may, as a result of collision, re-enter the liquid, while others escape to a distance and diffuse outwards. **This process of evaporation results in the atmosphere always containing more or less water vapour**; and it can be proved by simple experiments (i) that there is a limit to the quantity of water vapour which can be retained in that form by the atmosphere, and (ii) that warm

air is capable of retaining more water vapour than cold air can retain. The presence of water vapour in the air of a room can be demonstrated by the following experiment:

EXPT. 132.—**Moisture from the air.** Cool some water in a beaker, either by adding ice or crystals of sodium hyposulphite and stirring well. Notice the gradual deposition of moisture on the outside of the beaker.

It is a well-known fact that wet clothes or wet roads dry more rapidly on a warm summer's day than in the winter. Even in the summer, the air, though warm, may be very moist (or 'muggy'), and wet roads will not dry then so rapidly as when the air is fairly dry. Also, the evaporation is aided if the vapour molecules are prevented from re-entering the liquid; and this explains why a windy day is more favourable than a calm day for drying a wet surface, since the vapour molecules as soon as they escape from the liquid are conveyed by the breeze to a distance.

Evaporation takes place at all temperatures: even a lump of ice, or snow, will evaporate gradually.

All liquids do not evaporate with equal readiness, thus ether, gasolene, and alcohol evaporate more rapidly than water, providing that the temperature is the same. This fact can be demonstrated by weighing, at frequent intervals, dishes containing these liquids. Liquids which evaporate readily are termed **volatile.**

Cooling caused by evaporation.—Evaporation necessarily causes a cooling of the liquid; for the more energetic of the molecules are those which are the more likely to escape from the liquid, and this results in the *average* kinetic energy of the molecules which remain being less than before. The diminution of kinetic energy is indicated by a lowering of the temperature of the liquid. This loss of heat is more or less counterbalanced by the heat given up to the liquid from the surrounding air and from neighbouring objects, but the fall of temperature is generally sufficient to be detected readily.

EXPT. 133.—**Evaporation of ether and water.** Allow a bottle of ether and a bottle of water to stand in a room until their temperatures correspond with that of the room. Note the temperature of the room. Pour some of the ether into an empty dish and some of the water into another dish; place a thermometer in each. After a few minutes, note the reading of each thermometer.

EXPT. 134 —**Freezing by evaporation** Pour a few drops of water upon a dry piece of thin wood, and stand in the water a thin beaker containing a little ether. Blow vigorously down a tube having one end in the ether (Fig 122) or use a pair of bellows. The ether rapidly evaporates, and *in doing so takes heat from the water* between the beaker and piece of wood The beaker thus becomes attached to the wood by a layer of ice.

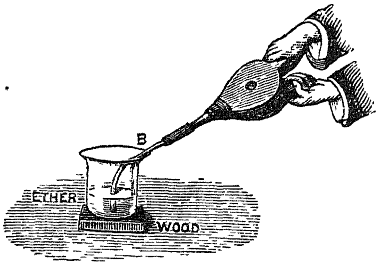


FIG 122 —Experiment to show that water may be frozen by the rapid evaporation of ether close to it.

Boiling.—At low temperatures evaporation takes place only from the exposed surface of the liquid, and the change from liquid to vapour is invisible. As the temperature is raised the rate of evaporation increases, and a temperature is reached finally when evaporation takes place anywhere within the liquid as well as from the surface. Bubbles of vapour are then formed within the liquid and rise to the surface; the evaporation is then rapid and visible, and the liquid is said to **boil**. If heat be applied to the containing vessel from below, the liquid in contact with the bottom of the vessel is always slightly hotter than the liquid above, hence the bubbles of vapour as a rule appear to form at the bottom of the vessel. For each liquid there is a definite temperature at which visible evaporation takes place, and this temperature is termed the **boiling point** of the liquid.

The temperature at which any liquid boils is influenced, to a slight extent, by the nature of the containing vessel and by the degree of cleanliness of the inner surface of the vessel It has been known for a long time that, although the temperature *of the boiling liquid* is subject to such variations, the temperature of the *vapour* immediately above the liquid is constant, providing that the pressure of the air is constant: the influence of variations of atmospheric pressure on the boiling point is explained on p. 150. For the above reason, when determining the boiling point *of a pure liquid*, it is necessary to support the thermometer with its bulb in the vapour immediately above the surface of the liquid.

EXPT. 135.—**Boiling point of water.** Fit up the apparatus shown in Fig. 123. Insert the thermometer through the cork so that the bulb is just *above* the surface of some distilled water contained in the flask. Boil the water continuously for about 5 minutes and note the reading of the thermometer. At the same time, read the height of the barometer; the reason for reading the barometer is explained on p. 192.

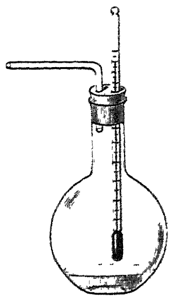


FIG. 123.—Determination of the boiling point of water.

EXPT. 136.—**Boiling point of alcohol.** Fit a boiling-tube A (Fig. 124) with a cork through which pass a thermometer and a long glass tube C. Support A in a beaker B containing water. Pour alcohol into A, and add a few fragments of glass rod so as to ensure steady boiling. The tube C serves to condense the vapour of the alcohol, and it lessens the possibility of the vapour taking fire. Heat the water in the beaker until the alcohol boils, and note the reading of the thermometer. At the same time read the height of the barometer.

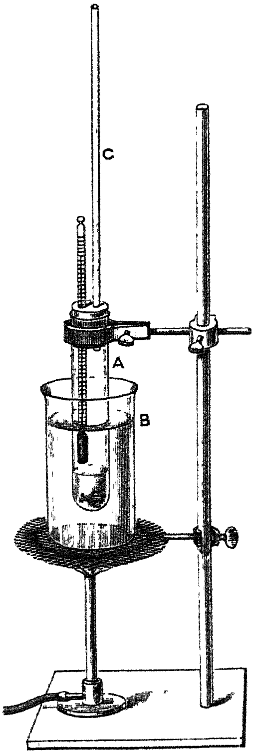


FIG. 124.—Determination of the boiling point of alcohol.

VAPOUR PRESSURE.

Vapour pressure.—In previous paragraphs we have considered the phenomenon of evaporation as taking place when the liquid is exposed to the open air. It is necessary now to consider how the evaporation is influenced when the space above the liquid is limited.

When the liquid is contained in a *closed* vessel, the escape of molecules to a distance is prevented; and after a time, a condition is set up when molecules re-enter the liquid just as frequently as they leave it. The space is then stated to be full

of the **saturated vapour** of the liquid, and the pressure which the vapour exerts on each sq cm. of the surface of the liquid is termed the **maximum vapour pressure of the liquid at the observed temperature of the experiment.** The maximum vapour pressure is expressed usually in its equivalent height of a mercury column; thus the statement that the maximum vapour pressure of water at 15° C is 12·7 mm. means that the pressure per sq. cm. of the vapour at 15° C. is equal to the pressure per sq. cm due to a column of mercury 12 7 mm. high.

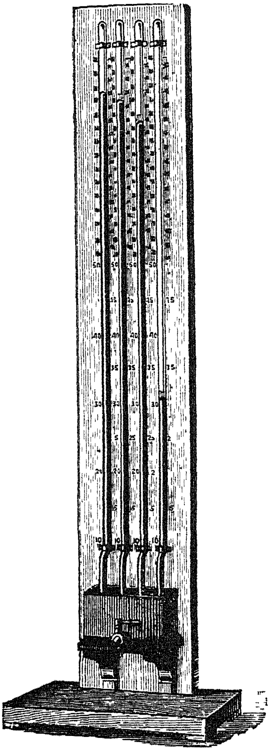


FIG. 125 —Apparatus for vapour pressure experiments.

The pressure of the vapour increases with increase of temperature—When the temperature of the containing vessel is raised, the escape of molecules from the liquid is more frequent, and more vapour molecules must accumulate above the liquid in order that equilibrium between the liquid and its vapour can be established. Since the number of molecules in the condition of vapour and contained in the same space is increased, and since their velocity of movement is increased by an increased temperature, we should anticipate that the pressure of the vapour would increase.

The density and the pressure of a saturated vapour are independent of the volume occupied by the vapour.—For, supposing that the closed vessel, containing the liquid and its saturated vapour, contract suddenly, the number of vapour molecules in each cubic centimetre of the space above the liquid will be increased, and the number of molecules striking the liquid surface will be increased correspondingly; thus, more molecules will enter the liquid than will leave it, or, in other words, some of the vapour will condense. This condensation will continue until equilibrium is again established, and then the density and pressure of the vapour will be the same as before the contraction of the containing vessel took place.

The density and pressure of the saturated vapour are independent of the presence of air—Suppose, for example, that the space above a liquid within a closed containing vessel is occupied by air. Just as much vapour will be generated from the liquid surface as would be the case if the space were a vacuum, for evaporation will continue until as many vapour molecules re-enter the liquid in a second as leave it in a second, and the number which re-enter the liquid depends solely upon the number contained in each c c. of the space above. The only effect due to the presence of air is a delay in the final establishment of equilibrium, and this is due to the frequent collisions, between the air molecules and the vapour molecules, retarding the diffusion of the vapour to the upper parts of the containing vessel

All these points may be demonstrated by means of barometer tubes, fitted up as shown in Fig 125 The first tube on the left is an ordinary barometer tube, the remaining three have had introduced into them respectively water, alcohol, and ether. The water having evaporated into the Torricellian vacuum as the space above the mercury in a barometer is called, causes but a slight fall of the mercury column The alcohol and the ether exert a greater pressure. The depression of each mercury column measures the pressure of each vapour at the temperature of the experiment. If the liquids and vapours in the tubes are warmed their pressures increase, and the mercury level falls.

EXPT 137—**Vapour pressure** Clean and dry the insides of two barometer tubes Fit them up as barometers, using clean dry mercury. Measure the heights of the mercury columns By means of a bent pipette (Fig. 126), introduce not more than *two* drops of ether into one tube, and two drops of water into the other tube. Again measure the heights of the mercury columns. Observe that there is no visible layer of either liquid at the top of the columns · the space above the columns is *not* saturated with the vapours Add more liquid in each case, sufficient to form a layer of liquid above the mercury . the spaces above the columns are now filled with the *saturated* vapours of the liquids. Measure the heights of the columns, and note the temperature of the room. *The depression of the mercury, in each case, represents the pressure of the saturated vapour of the liquid at the observed temperature.*

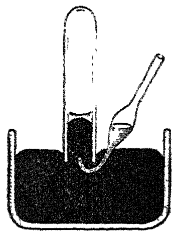


FIG. 126—Introducing a liquid into a closed tube.

Pass a Bunsen flame rapidly up and down either of the tubes, and notice how the vapour pressure increases.

Close with the thumb the open end of either of the tubes, and transfer it to a deep vessel containing mercury, so that the tube may be depressed to a considerable depth in the mercury. Notice that the height of the mercury column remains constant, thus proving that the vapour pressure is independent of the size of the space above the mercury column

EXPT 138 —**Independent vapour pressures.** Fit up a barometer tube, as before. Introduce a little air into the vacuum at the top of the tube, and measure the height of the mercury column. Introduce ether, as before. Note how the column is depressed less rapidly than before Measure the height of the column, and verify that the vapour pressure is independent of the presence of air

Vapour pressure of a liquid at its boiling point.—It has been stated (p. 188) that a liquid boils when bubbles of its vapour form within the liquid and rise to the surface. Consider the forces acting on such a bubble when near the surface (Fig. 127): the arrows directed inwards indicate the pressure due to the air and tending to compress the bubble inwards, the arrows directed outwards indicate the pressure due to the vapour within the bubble and tending to expand the walls of the bubble The fact that the bubble, during its upward movement, maintains a fairly uniform size indicates that the pressure due to the vapour is equal approximately to the atmospheric pressure. Hence, we may say that **when a liquid is heated to its boiling point its vapour pressure is equal to the pressure of the air upon it.** This is an important deduction, since it affords a simple method of determining the boiling point of a liquid; and the method is particularly useful when only a small quantity of the liquid is available. The deduction suggests also how dependent the boiling point of a liquid is upon the pressure of the surrounding air. for, supposing that the atmospheric pressure is reduced, then the vapour pressure of the liquid at a lower temperature will be equal to that of the air,—or, in other words, the liquid will boil at a lower temperature. Similarly, if the atmospheric pressure be increased,

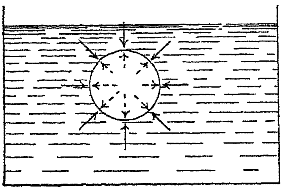


FIG 127 —Forces acting upon a bubble of vapour in a liquid

the liquid must be warmed to a higher temperature than before in order that its vapour pressure may equal that of the air,—or, in other words, the liquid will boil at a higher temperature.

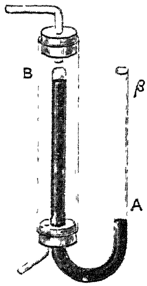


FIG. 128.—Experiment on the vapour pressure of a liquid at the boiling point

EXPT 139.—**The vapour pressure of a liquid at its boiling point is equal to the atmospheric pressure.** In Fig 128, A is a U-tube with limbs about 30 cm long, and with one limb sealed The latter limb is surrounded by a wide glass tube B closed at both ends with corks The upper cork is fitted with a glass tube for the purpose of leading steam into B; the lower cork supports the U-tube, and it has also a side tube for the escape of condensed steam Fit up the tube A as follows fill it completely to the point β with clean dry mercury, close the open end with the thumb, tilt the tube so that the air bubble at β travels round to the closed end, and finally bring the bubble back to the open end The walls of the tube should now be quite free from small air bubbles. Introduce two or three drops of well boiled distilled water into the tube so as to fill it completely, close the end with the thumb, and tilt the tube so that the water travels round to the closed end Finally, withdraw mercury from the open end, by means of a narrow pipette, until the mercury surface is just above the bend. Pass steam through the steam-jacket B, and notice how the vapour pressure of the water depresses the mercury in contact with it *until the two mercury surfaces are at the same level*

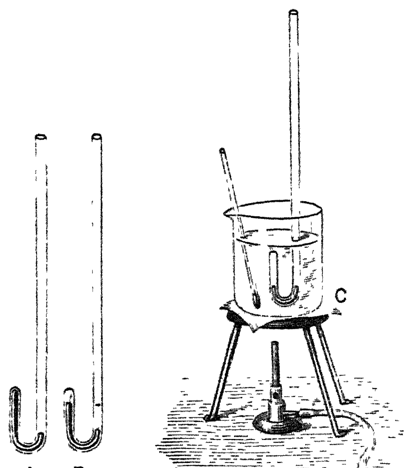


FIG. 129.—Method of determining the boiling point of alcohol

EXPT. 140.—**Vapour pressure method of determining boiling point.** Make a narrow glass U-tube, similar to Fig. 129, and with a closed

limb at least 10 cm long. Introduce mercury and alcohol, by the procedure explained in Expt 139 Support the tube in a beaker of water, and support a thermometer in the water Gradually warm the beaker, keeping the water well stirred, and note the temperature when the mercury surfaces in the two limbs of the U-tube are at the same level.

Effect of change of pressure on the boiling point.—It has been seen that at sea-level the normal pressure of the atmosphere will support a column of mercury 30 inches in length At the top of a mountain, the pressure is less because there is less air above the barometer; and at the bottom of a mine it is more for the reverse reason. At any one place, also, the pressure varies from day to day. If we wish to boil a liquid, therefore, where the pressure of the atmosphere is great, the liquid has to be heated to a higher temperature than when the pressure is less before the bubbles of vapour formed can escape at the surface of the liquid. If we heat the liquid more, its temperature gets higher before there is any conversion into vapour, and consequently its boiling point is higher when the pressure is greater. **In finding the boiling point of a liquid we must therefore know the pressure of the atmosphere at that time and place**

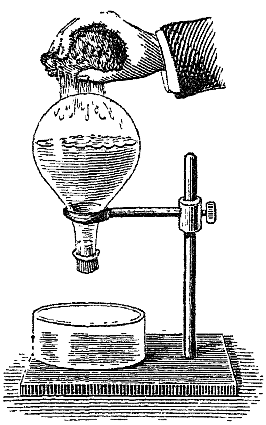


FIG. 130 —Water below 100° C boiling under diminished pressure.

A simple experiment shows that water may boil at a temperature considerably below 100° C. when the pressure upon its surface is diminished. All it is necessary to do is to take a sound cork which fits tightly the neck of a round-bottomed flask. Water is then boiled in the flask and allowed to continue boiling for some minutes so that all the air in the flask is driven out and its place taken by steam. The burner is then removed and the cork inserted into the neck of the flask as rapidly as possible. After standing to cool for a minute or two, when, owing to cooling, the temperature can no longer be 100° C. the flask is turned over and cold water poured upon its upturned under surface, or a cold wet sponge is squeezed upon it as shown in Fig. 130.

The cold water causes the steam in the flask to condense, and, as no air can get in, the pressure on the surface of the warm water is now less than it was before, and therefore the water is seen to boil quite briskly again.

EXPT. 141.—**Boiling under different pressures** Fit up the apparatus shown in Fig 131, in which C is a glass tube bent twice at right angles, tapered slightly at the lower end, and cut off in a slanting direction D is a narrow cylinder into which mercury may be poured, whereby the pressure is increased when steam is escaping through the mercury. The total pressure is obtained by adding the difference of level between α and β to the height of the barometer. By adding successive quantities of mercury obtain a series of simultaneous readings of boiling point and pressure.

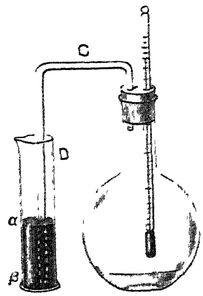


FIG. 131.—Arrangement for determining the effect of increased pressure upon the boiling point.

Obtain a similar series of readings under diminished pressure thus leave a small quantity of mercury in D, and turn down the Bunsen burner The mercury now rises up the tube C, and diminishes the pressure The pressure is obtained by subtracting from the barometer-reading the difference of mercury level in C and in D More rapid boiling is ensured by wrapping blotting-paper wetted with cold water round the neck of the flask. Verify the results by the data given in footnote, p. 150.

Effect of dissolved solids on the boiling point.—The vapour pressure of a solution is *less* than that of the pure solvent (or fluid in which a substance has been dissolved), if the temperature be the same; hence, a solution of a solid must be raised to a higher temperature than the pure solvent in order that its vapour pressure may be equal to the atmospheric pressure, or, in other words, **the boiling point of a solution is higher than that of the pure solvent.**

EXPT. 142.—**Boiling points of saline solutions.** Measure out 250 c c. of water into a large flask which has been weighed previously Weigh the flask and its contents, and support a thermometer with its bulb immersed in the water. Weigh out about six equal quantities of dry salt, each weighing about 5 grams. Heat the water to boiling. Note the temperature of the boiling water, also note the time; add 5 grams of salt, and, at the end of 2 minutes, note the temperature when boiling; add another 5 grams of salt, and again, at the end of

another 2 minutes, note the temperature. Repeat this process until all the salt has been added. Finally, cool the flask as quickly as possible and weigh it. Allowing for the total weight of salt added, find the weight of pure water still in the flask. The loss during the experiment is due to the escape of steam. Assuming that the loss takes place at a constant rate, determine the weight of water present at the moment when each reading of the thermometer is taken. Calculate the percentage of salt present in each case. Plot the observations on squared paper, taking percentages of salt as abscissae and temperatures as ordinates.

HYGROMETRY.

Moisture present in the atmosphere.—Evaporation is constantly going on from all water and wet surfaces exposed to the air, and the air would be saturated with moisture always were it not for the retarding influence (p. 191) of the air. The term **hygrometry** is applied to all those phenomena which result from the moist condition of the atmosphere

The quantity of water vapour necessary to saturate a given volume of air depends upon the temperature of the air (p. 190). Warm air can retain more water vapour than cold air. When the temperature of moist air is diminished, the air is soon cooled to a temperature at which the moisture present is sufficient to saturate the air, and when the temperature is diminished still further, part of the moisture will condense—either as **dew, hoar-frost, or cloud.**

Absolute amount of moisture present in air.—The quantity of moisture present in 1 cubic metre of the atmosphere, at a particular time, is determined by drawing a measured volume of the air through tubes containing a suitable hygroscopic substance, the tubes being weighed before and after the passage of the air.

EXPT. 142A —**Chemical hygrometer.** Heat some broken pumice-stone in a dish in a furnace, and, while hot, throw it into a vessel containing concentrated sulphuric acid. Pour off the excess of acid, and transfer the pumice to a glass stoppered store bottle. Fit up two **U**-tubes with corks and glass tubes, as shown in Fig 131A The corks should be previously soaked in melted paraffin-wax. Nearly fill the tubes with the pumice ; adjust the corks and wax them carefully to make all joints air-tight. Close the entrance and exit tubes with rubber tubing in

which a short piece of glass rod has been inserted to act as a plug Weigh the tubes accurately.

Fit up a suitable type of aspirator such as is shown in Fig 131A To prevent the possibility of moisture passing from the aspirator to the weighed tubes it is advisable to have a third drying-tube, or a bottle

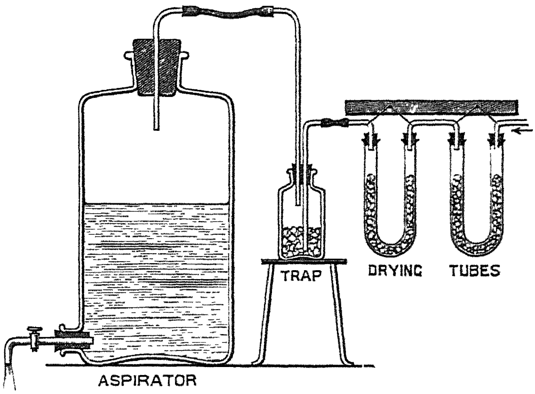


FIG 131A —A chemical hygrometer

containing more pumice which has been soaked in sulphuric acid, between the aspirator and the weighed tubes.

Fill the aspirator with water and connect it with the bottle by means of rubber tubing. Take the plugs out of the two drying-tubes and connect the tubes with the bottle. Turn on the tap of the aspirator and let the water run out gently. When a convenient quantity—four or five litres—has escaped, turn off the tap, disconnect the U-tubes, re-insert the glass plugs and again weigh the tubes. The increase of weight shows the weight of moisture in a volume of air equal to the volume of water run off. From this result, calculate the weight of moisture present in 1 cubic metre of air at the time of the experiment.

When very precise results are required, the temperature of the air in the aspirator and where it enters the U-tubes must be taken into consideration, but the foregoing experiment is sufficiently accurate to illustrate the method of determining the absolute quantity of aqueous vapour in a given volume of air.

Relative humidity.—The term **humidity** means **the degree of dampness of the air**; and the **relative humidity** is defined as **the ratio of the weight of water vapour present in unit volume of the air to the**

weight of water vapour which would be present in the same volume if the air were saturated. At any given temperature the vapour-pressure of the water present in the air is proportional to the weight of the water present in a given volume of the air, and it is more usual to express the relative humidity in terms of the pressure which the vapour exerts. Hence, we may say that

$$\text{Relative humidity} = \frac{\text{pressure actually exerted}}{\text{pressure exerted if the air were saturated}}.$$

The measurement of relative humidity is important, since it affords information as to whether any slight fall in temperature will cause condensation of moisture. The measurement is also useful in order to determine whether the air of rooms is sufficiently moist for healthy conditions.

Relative humidity is sometimes expressed as a fraction, and sometimes as a percentage. Thus, when the air is found to contain one-half as much water vapour as would be necessary to saturate it, the relative humidity may be expressed either as 0·5 or as 50 %.

The relative humidity is determined by finding out to what temperature the air must be cooled in order that the moisture present will suffice to saturate the air; and this temperature is observed by cooling the air until it begins to deposit moisture in the form of dew. This temperature is called the **dew-point.** The various devices for determining the dew-point are termed **hygrometers**; some typical devices are explained in the following paragraphs. Suppose that the temperature of the air is 21° C., and that an experiment shows that the air must be cooled to 15° C. before deposition of dew commences. The pressure exerted by the water vapour is the same in the cooled air as in the warm air, since the air and the water vapour contract in the same ratio on cooling; hence the pressure which the vapour in the uncooled air actually exerts must be equal to the maximum pressure at 15° C. Reference must now be made to the Physical Table No. 16. From such a Table we see that the maximum pressure of water vapour at 15° C. is 12·67 mm., and the maximum pressure at 21° C. is 18·47 mm. Hence, the relative humidity $= 12{\cdot}67/18{\cdot}47 = 0{\cdot}68$ (or 68 %).

The aluminium-cup hygrometer (Fig. 132).—This consists of a small well polished aluminium cup, in which water is cooled by the gradual addition of ice until dew is deposited on the outer surface.

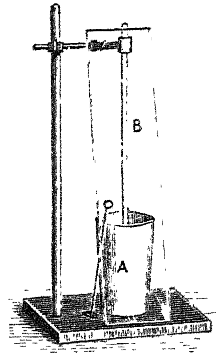


Fig. 132.—Aluminium-cup hygrometer

EXPT. 143.—**Relative humidity.** About half-fill the aluminium cup A with water. Suspend a thermometer B, graduated to 0°.2 C., in the water. Place a large sheet of glass in front of the apparatus so as to screen it from the warmth and breath of the observer. Add a *small* fragment of ice, and stir continually until the ice is melted. Add another small fragment of ice, and stir until melted. Continue this process until the deposition of dew upon the cup is observed; note the temperature of the water. Continue to stir the water and note the temperature when the deposited dew disappears. The average of these two temperatures is the *dew-point*. Observe the temperature of the room, and, from the data in Physical Table No. 16, calculate the relative humidity of the air in the room.

Regnault's hygrometer.—The principle of this hygrometer differs from that of the aluminium-cup hygrometer only in the method adopted for cooling the air. The lower end of a thin glass test-tube is fitted with a polished silver "thimble"; ether is poured into the tube, and its temperature is lowered (p. 188) by passing a stream of air bubbles through it until dew is deposited on the silver thimble. Fig. 133 represents a simple device based upon this principle. The upper end of a brightly polished cylinder A (of brass or copper) is closed with a 3-holed cork. Through the cork pass (i) a thermometer, (ii) a glass tube B terminating below the surface of the ether contained in the cylinder, and (iii) a short glass tube for the escape of ether vapour. The ether may be cooled by breathing gently down the tube B.

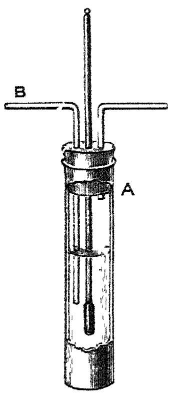


Fig. 133.—Principle of Regnault's hygrometer

Mason's hygrometer.—Mason's instrument consists of two precisely similar thermometers, suitably attached to a frame, or suspended side by side as in Fig. 134. Round

the bulb of one of the thermometers is tied a piece of muslin, to which cotton threads are attached; these hang down into water in a glass. The instrument depends for its use upon two facts which have been brought before the student's attention already. The first is that water is vaporised only at the expense of a certain amount of heat, and, secondly, the quantity of water vapour which air can take up at any temperature depends upon the amount already contained by it. Water rises up the cotton threads by the force known as capillary attraction, and consequently keeps the muslin moist. The water on the muslin evaporates, getting the heat necessary for evaporation from the bulb of the thermometer which it surrounds. The thermometer is cooled thereby, and the column of mercury sinks. This process continues until the air round the bulb is saturated and evaporation ceases. Thus the wet-bulb thermometer records a lower temperature than that with a dry bulb. The difference between the readings is greater the drier the air at the commencement of the observation, and it provides a means of estimating the amount of water vapour present by seeing how much more must be added to saturate the air.

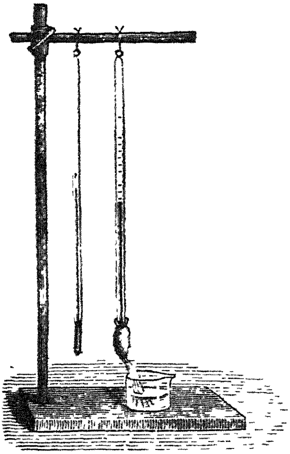


Fig. [illegible].—Wet- and dry-bulb thermometer

The **wet- and dry-bulb thermometer**, as Mason's hygrometer is called, is employed usually to indicate the relative amount of moisture in the air, but the readings may also be used to determine the dew-point by a simple calculation in connection with a set of hygrometrical tables prepared for the use of practical meteorologists. When the dew-point has been found, the relative humidity of the air or the percentage of saturation can be determined.

Cloud.—When moist air is warmed, through contact with the earth's surface, it expands and tends to rise vertically upwards. Such ascending columns of air soon become cooled, either by coming into contact with cooler strata of air or by their natural expansion under the diminished pressure of higher altitudes, and the degree of cooling may be sufficient to cause the condensation

in the form of minute drops of some of the moisture. These drops will tend to fall, and the larger the drops the more rapidly will they fall; if, in falling, they pass through strata of comparatively dry air they will be evaporated before reaching the earth; but if they pass through strata of warm air saturated with moisture, they will increase in size—owing to each cold drop condensing more moisture—and in velocity, and the drops may reach the earth's surface as **rain drops.**

If the initial condensation of the water vapour takes place at a temperature below 0° C., the particles assume the solid form, and a fall of **snow** results. **Hail** is believed to be formed by the freezing of rain-drops which have been carried up several times in succession into colder regions of the atmosphere by strong air currents. Hailstones usually have a stratified or concentric structure in which several different layers can be seen when they are cut across. This structure could scarcely be produced if hail consisted merely of rain-drops which had been frozen by passing through a layer of air below a temperature of 0° C. on their way to the earth.

Fog.—Fog closely resembles cloud, except that it is formed near the earth's surface. The condition usually necessary for its formation is that the upper strata of air are warmer than the strata near the earth's surface. The moisture contained in the upper strata diffuses downwards into the cooler strata beneath, and if the latter acquire thereby more than sufficient moisture to produce saturation the excess is condensed in the form of minute drops of water.

Dew.—Dew differs from the forms of condensed moisture seen in mists, clouds, rain, and snow, in being formed *upon* the surface of the earth. After sunset, the surface of the earth, which has been receiving heat throughout the day, begins to lose this heat by **radiation** (p. 221). Different objects and surfaces possess differing powers of radiation; those which during the day absorb heat to the greatest extent radiate it most abundantly after the sun has disappeared, and consequently become cooled before those the radiating power of which is small. Similarly, the air in contact with these bodies also becomes cooled and is

then unable to hold as much water vapour as before, and the surplus is deposited in the form of **dew**

Experiments have proved, however, that dew is only partly derived from the moisture in the air, much of it having its origin in vapour exhaled from the earth, or from the grass or other plants on which the dew appears. Moisture is being exuded by the leaves of plants continually, and in the absence of sunshine or wind it accumulates on the surface as drops of water or dew instead of being dried up as it is during the daytime. Vapour is arising also constantly from the earth, and this contributes to the formation of dew, so that though the upper surfaces of stones are not bedewed visibly on a clear night, the lower surfaces have often a heavy deposit.

Conditions favourable to the formation of dew.—For an abundant formation of dew several conditions are necessary. First, radiation must go on freely, and this happens on *bright clear evenings* when there are no clouds to obstruct the radiation. The air which is being cooled by contact with the body from which free radiation is taking place must not be disturbed before the dew-point is reached, or no dew will be thrown down, that is, the *evening must be still*. A breeze will renew constantly the air above the body which is being cooled by radiation and will prevent the dew-point being reached. Good radiating surfaces are those of leaves—whether of grass or other plants—also stones.

Hoar-frost—or as it is sometimes called *white-rime* or simply *rime*—is deposited instead of dew on these evenings when the radiation cools the overlying air to the temperature of freezing water before any deposition of moisture takes place. Hoar-frost is not frozen dew. It does not assume the liquid condition first, but is precipitated at once in the solid form. In these circumstances the dew-point is at or below the freezing-point.

EXERCISES ON CHAPTER XV.

1. A flask containing pure water is heated by a single burner, and one thermometer is placed with its bulb below the surface of the water, and another thermometer with its bulb just above the surface. When the water boils the readings of the two thermometers are taken. Will the readings be the same?

What will be the effect on the reading of each thermometer (1) of placing a second burner under the flask, and (2) of dropping some common salt into the flask?

2. Two mercury barometers are set up. Will the heights of the mercury columns be the same when the inside of *one* of the tubes is wet? If the temperature of the room be 17° C., what will be the difference in the heights?

3. Why does a muddy road dry better on a windy and warm day than on a quiet and damp day?

4. Explain why, in order to cook food at a high altitude, it is necessary to adopt a method which differs from that which would be adopted at ordinary levels

5. Define the *boiling-point* of a liquid Distinguish between *boiling* and *evaporation*. What condition determines whether a liquid will boil or evaporate?

6. If a narrow-necked flask and a wide dish, both containing ether, are placed side by side on a table, would you expect the ether in both cases to have the same temperature? If not, explain the reason for the difference.

7. Why is an iceberg frequently surrounded by fog? If a breeze be blowing, would the fog be distributed equally on all sides of the iceberg?

8. Why does fanning produce a sense of coolness to the face? Is the effect influenced by the degree of dampness of the air?

9. The temperature of a room is 14° C., and the dew-point is found to be 5° C. By means of Physical Table 16, find the pressure of the water vapour present, and determine the *relative humidity* of the air.

10. The dimensions of a closed room are $10 \times 10 \times 5$ metres If the temperature of the room be 20 C., calculate, from Physical Table No 17, the weight of water required to saturate the air.

11. A saucer containing water is left to evaporate on a window sill Explain the atmospheric conditions which will favour or retard the disappearance of the water

12. Explain what happens to the steam issuing from the funnel of a steam-engine. (*a*) on a fine warm day; (*b*) on a damp day.

13. How is the reading of a thermometer altered by wrapping a wet rag round the bulb? What will happen if the rag be wetted with (1) ether, (2) oil, instead of water? How do you explain the various results?

14. The windows of a room frequently become dimmed with moisture on dry, cold days. Is the moisture on the outside or the inside, and how do you account for its formation?

15. Describe a simple form of hygrometer, and explain as fully as you can what measurement is made with it

16. Two simple barometers are set up side by side in the same vessel of mercury, and a little water is introduced into one. Explain why the height of the mercury is different in the two cases. How will change of temperature affect the height in each case?

CHAPTER XVI.

MELTING POINT. LATENT HEAT

Temperature of melting.—When a solid is heated, the first effect is usually to make it expand But if the heating is continued long enough, when the solid reaches a certain temperature, which differs for different solids, melting begins. The solid changes into a liquid. The temperature at which the melting takes place is called the **melting point.** Thus, when a piece of lead is heated its temperature rises, it gets larger, and as the heating is continued it is converted into a silvery-looking liquid. Wax, ice, and iron are other examples of solids which melt. But ice, wax, lead, and steel differ very widely in the temperatures at which they begin to melt, as the following table shows·

Ice	melts at - - - -	0° C.
Bees-wax	„ - - - -	65° C
Lead	„ - - - -	326° C.
Steel	„ - - - -	1360° C.

So long as any of the solid remains unmelted, the temperature does not rise above the melting point. It can be shown easily by experiment that this is true in the case of ice

EXPT 144.—**Melting point of ice.** Put some small pieces or shavings of clean ice into a beaker and insert a thermometer into them. Record the temperature indicated. Pour in a little water, stir the mixture, and again record the temperature. Place the beaker on a sand-bath and warm it gently. Notice the reading of the thermometer *so long as there is any ice unmelted.*

EXPT. 145 —**Melting point of wax.** Melt a little paraffin wax in a test-tube placed in a beaker of boiling water, and immerse the bulb of a thermometer in the liquid When the thermometer is taken out, a thin film of liquid paraffin will be seen upon it. Let the

bulb cool, and notice the temperature when the wax assumes a frosted appearance, which shows that it is solidifying. When the wax on the bulb has become solid, place the thermometer in a beaker of water and gently heat the water Observe the temperature at which the wax becomes transparent again. The average of this result and the preceding one is the *melting point* of paraffin wax.

Latent heat.—The experiments which have just been described are of the very greatest importance, and should be clearly understood. It is certain that when a mixture of ice and water is heated over a laboratory burner heat is being given to the mixture continually. Yet the temperature as recorded by the thermometer gets no higher. The question arises, what becomes of this heat, as it has no effect upon the temperature of the mixture? The ice is melted gradually, and if the heating be continued long enough it is all changed into water. As soon as this has happened, every further addition of heat raises the temperature of the water. These considerations lead to the conclusion that the heat previously given to the mixture is all used up in bringing about the change of ice into water Further, it is found that not only in the case of ice, but also when any solid is turned into a liquid, there is no increase in temperature, even while heat is being added, until the whole of the solid has been changed to a liquid.

This amount of heat which is necessary to change a solid into a liquid is spoken of as **latent heat.** The word latent comes from a Latin word, meaning 'lying hidden,' and refers to the fact that the heat used up in changing a solid to the liquid condition causes no increase of temperature, but appears to be hidden away in the liquid.

EXPT. 146 —**Heat required to melt ice** Let a few lumps of ice stand in a beaker until some of them have melted. Notice that the temperature is 0° C Counterpoise two empty beakers of the same size in the pans of a balance, and put a small lump of the ice into one, and the same weight of water from the melted ice in the other You have thus equal weights of ice and water at 0° C Pour equal weights of hot water into the two beakers. When the ice is melted, observe the temperature of the water in each beaker. The temperature of the water in the beaker in which the ice was placed will be found much lower than that of the water in the other beaker, owing to the ice using up a large quantity of the heat in melting into water.

EXPT. 147.—**Ice and ice-cold water.** Take equal weights of hot water in two large beakers of the same size. Place a piece of ice in one of the beakers, and observe the temperature of the water when it has melted. Pour ice-cold water into the other beaker until the same temperature is reached. Find, by weighing, the weights of ice and ice-cold water which have been added. It will be found that a small weight of ice has as much cooling effect as a large weight of ice-cold water.

Latent heat of water.—The number of units of heat which are required to change the state of a gram of ice, converting it from the solid to the liquid condition without raising its temperature, is called the latent heat of water or the **latent heat of fusion of ice.** Thus, to melt 1 gram of ice requires 80 heat-units; that is to say, as much heat as would raise the temperature of a gram of water through 80° C., or would raise that of 80 grams of water through 1° C., is used up in changing a gram of ice into a gram of water at the same temperature. Similarly, to melt 1 lb. of ice requires as many heat-units as are necessary to raise the temperature of a pound of water from 0° C. to 80° C., or, as much heat as is wanted to raise the temperature of 80 lb. of water through one degree Centigrade.

EXPT. 148.—**Latent heat of fusion of ice.** Weigh a metal calorimeter. About half fill the calorimeter with water, previously warmed to about 35° C., and again weigh. Break some ice into small pieces, and place within the folds of some blotting-paper, in order to dry the ice, a quantity sufficient to weigh about one-fifth as much as the warm water. Stir the water with a thermometer, and when the temperature has fallen to about 30° C., note the exact temperature, and transfer the dry ice into the calorimeter. Keep the contents continually stirred until all the ice is melted, and at once note the temperature. In order to find the weight of ice added, take a final weighing of the calorimeter and its contents. Enter the observations thus:

Weight of calorimeter $= w_1$ gm.
" " warm water $= w_2$ "
" " ice $= w_3$ "
Initial temp. of water $= T_1$° C.
Final " " $= T_2$° C.
Latent heat of water $=$ L heat units.

Heat lost by warm water $= w_2 \times (T_1 - T_2)$ units.

$$\text{Heat gained by the ice} = \left.\begin{matrix}\text{Heat required to}\\ \text{melt the ice}\end{matrix}\right\} + \left.\begin{matrix}\text{Heat required to warm}\\ \text{melted ice up to } T_2^\circ\end{matrix}\right\}$$

$$= (w_3 \times L) + (w_3 \times T_2)$$

But, Heat lost by warm water = Heat gained by the ice.

Hence, $w_2 \times (T_1 - T_2) = (w_3 \times L) + (w_3 \times T_2)$,

or $$L = \frac{w_2(T_1 - T_2) - w_3 T_2}{w_3}.$$

A more accurate result is obtained when allowance is made for the fact that the calorimeter cools through the same range of temperature as the warm water contained in it If the calorimeter be made of copper, of which the specific heat is 0 09, it will give up ($w_1 \times 0{\cdot}09$) heat units in cooling through 1° C.; and, in cooling through ($T_1 - T_2$) degrees it will give up $w_1 \times 0{\cdot}09 \times (T_1 - T_2)$ heat units Hence,

Heat lost by warm water and calorimeter $= (T_1 - T_2)\{w_2 + (w_1 \times 0{\cdot}09)\}$.

Melting point, by means of a cooling curve—If a solid be melted and allowed to cool, and the temperature observed at

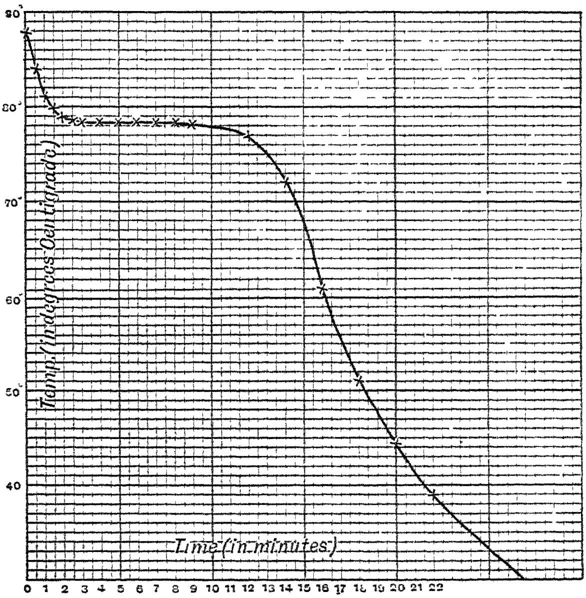


FIG. 135—Melting-point curve of naphthalene.

equal time intervals, it will be noticed that during the period of solidification the temperature will remain more or less constant.

If the readings be plotted on squared paper, this period will be indicated by part of the curve being practically horizontal, and the position on the temperature scale of this horizontal portion will indicate the temperature of solidification, which is the same as the temperature of melting (Fig. 135). The constancy of temperature during solidification is due to the fact that the latent heat given up by the liquid on solidifying more or less counteracts the loss of heat by cooling from the walls of the containing vessel.

EXPT. 149.—**Determination of melting-point.** Fit a small test-tube (5 cm × 1.5 cm) with a cork and thermometer (Fig 136) Cut a

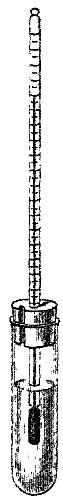


FIG. 136.—Expt. 149

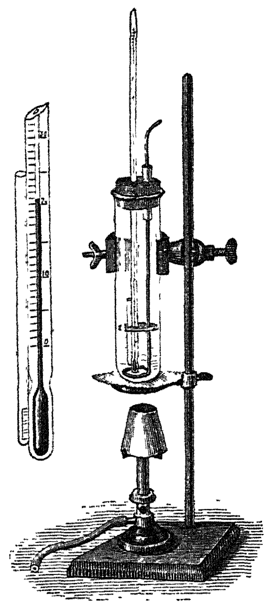


FIG 137 —Determination of the melting point of a small quantity of a substance

small groove in the side of the cork so that it shall not fit air-tight. Melt some paraffin wax in the tube. Clamp the thermometer in a vertical position and so that the tube is not touching the table. Take readings every half-minute until the wax has cooled to about 40° C. Plot the readings on squared paper, and state the temperature of solidification.

EXPT 150.—**Alternative method.** A short piece of thin glass tubing of narrow bore is dipped into some melted wax, naphthalene or butter, of which the melting-point is required, and in this way some of the substance, which soon solidifies, enters the tube. The tube is placed by the side of a thermometer, as indicated in the illustration, and fixed there. The combination is then introduced into a beaker or test-tube containing a curved stirrer which can be used to keep the water or other liquid in the vessel at a uniform temperature. When the substance in the tube is seen to melt, the temperature is recorded. The flame is then removed and the temperature noted at which the substance solidifies. Several experiments should be made and the mean temperature taken as the melting point. This method is convenient to use when only a small quantity of a substance is available for the determination of the melting point. By using a liquid like castor oil or sulphuric acid in the test tube, instead of water, the melting point of sulphur or other substance which melts below the temperatures at which these liquids boil can be determined.

Expansion during solidification.—The fact that ice floats on water is convincing evidence that ice is less dense than water, and therefore that water expands on solidifying. In fact, 100 c.c. of water expand to about 109 c.c. on solidifying.

Nearly all substances contract when they solidify, but water and a few others are exceptions. The reason that sharp castings can be made from molten cast iron is that iron contracts scarcely at all on solidifying; also, the alloy of antimony, lead, and copper used in making printers' type behaves in the same way.

The force with which water expands on freezing is extremely great: strong steel shells filled with water will burst when exposed to extreme cold. It is a common experience for water pipes in houses to be burst during cold weather, as a general rule the ice inside the pipe prevents the escape of water, and the damage becomes evident only when the ice begins to melt.

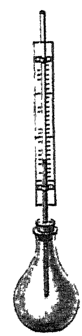


FIG. 138.—Expt 151

EXPT. 151.—**Contraction of ice when melted.** Fit up a flask, similar to Fig. 138. Introduce some broken ice into the flask, and fill it completely with tap water. Insert the cork until the surface of the water is high up the narrow

glass tube. Notice how, as the ice melts, the surface of the water falls.

EXPT. 152.—**Fracture by formation of ice.** Using a blowpipe, seal up one end of a piece of wide glass tubing; at a few centimetres from this end melt the glass and draw off the tube to capillary dimensions. Fill the tube with water up to the constriction, and seal it with a blowpipe. Place the tube in a freezing mixture of ice and salt, and cover the vessel with a cloth. In a few minutes the tube will burst.

Fig. 139 is an instructive representation of the changes in volume observed when cold ice is warmed gradually until it is converted finally into vapour. In warming up to 0° C. the ice expands, like any ordinary solid, it then contracts considerably on melting, the cold water contracts gradually until its temperature is 4° C., and then it expands until its temperature is 100° C.

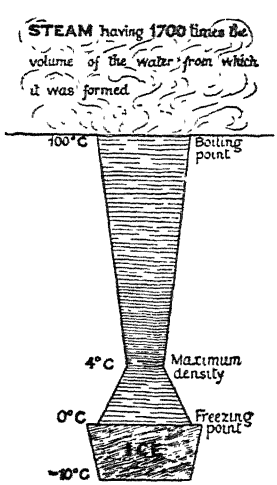


FIG. 139.—Changes in volume of ice, water, and steam

Heat disappears during vaporisation.—As previously explained, p. 187, when a liquid is changed into vapour a certain amount of heat is used up. It does not matter whether the liquid evaporates or boils; every gram of it requires a certain amount of heat before it becomes converted into vapour. In boiling, this heat is supplied by the flame or fire, and in evaporation it is taken from the objects in contact with the liquid. The faster the evaporation the more rapidly heat is absorbed in this way. When a liquid evaporates very rapidly, the cooling produced is very noticeable. For instance, if a few drops of either spirits of wine or ether are sprinkled upon the hand, the liquid soon disappears and the hand feels cold. The heat necessary for the evaporation of these liquids is taken from the hand, or from any other things with which they are in contact, consequently the hand becomes cooler and cooler as the vapour is formed. So much heat may be absorbed

in this way that, as illustrated by Expt. 134, water can be frozen by the evaporation of ether in a vessel in contact with it.

In tropical countries, where the land gets *very hot* during the day, evaporation takes place so rapidly after sunset that the water sometimes becomes so much cooled by the extraction of the heat required to bring about the change from liquid to vapour, that the water freezes

Latent heat of steam.—When once water has started to boil, its temperature gets no higher than the boiling point. So long as there is any water left, no matter how much it is heated, the temperature remains the same. All the heat is absorbed, or used up, in bringing about the change from the liquid state to that of vapour. It requires a great many more heat-units to convert one gram of water at a temperature of 100° C. into steam at the same temperature, than it does to change a gram of ice at 0° C. into a gram of water at 0° C. Whereas to bring about the latter change requires an expenditure of 80 heat-units, to convert a gram of water at 100° C into a gram of steam without changing its temperature requires no fewer than 536 heat-units Thus, the **latent heat of steam**, or, as it is sometimes called, the **latent heat of vaporisation of water**, is 536. Expressed in another way, we may say that it requires as many heat-units as would raise the temperature of 536 lbs of water through 1° C. to bring about the change of one pound of water at 100° C. into one pound of steam at the same temperature

Just as a large quantity of heat is required to convert water into steam, so a large quantity is given up when steam becomes water. It is for this reason that a scald from the steam of boiling water is worse than the scald from the boiling water itself.

A rough method of determining the latent heat of steam consists in comparing the time required for a uniform source of heat to raise ice cold water to 100° C. with the time required to convert the same weight of boiling water into steam. Thus, if the latent heat of steam were 200 heat-units, the period of time required by a steady source of heat to raise 1 gram of ice cold water to the boiling point would be one-half the time required to convert 1 gram of boiling water into steam, since the quantities of heat required in the two processes would be 100 units and 200 units respectively

EXPT. 153.—**Conversion of water into steam** Pour about 50 c.c. of ice-cold water into a metal calorimeter which has been cooled by surrounding it with ice. Quickly dry the outside of the calorimeter, and place it on gauze heated by a steady flame. Note the time (i) at the instant when heating commences, (ii) when the water commences to boil, and (iii) when all the water has evaporated. By comparing the time intervals (i-ii) and (ii-iii), calculate the latent heat of steam. The error in the result will probably amount to about 10 %.

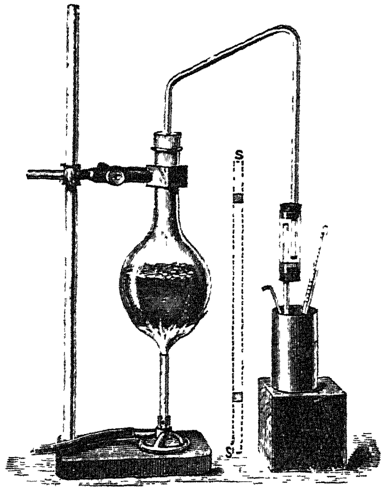


FIG 140 —Determination of the latent heat of vaporisation of water.

EXPT. 154 —**Determination of latent heat** Arrange a flask with the connections shown in Fig. 140. The short length of wider glass tube is a trap to catch condensed steam. Put some water into the flask and boil it While the water is getting hot, weigh out about 300 grams of water in a beaker or a thin metal vessel, and observe its temperature After steam has been issuing from the glass tube for a few minutes, place the vessel so that the end of the tube is well under the water, and let it stay there until the thermometer records a temperature of about 40° C. Then weigh the water again to find the weight of steam condensed. Enter the observations as follows :

Weight of calorimeter $= w_1$ gm
" " cold water $= w_2$ "
" " steam $= w_3$ "
Initial temp. of water $= T_1$° C.
Final " " $= T_2$° C.
Latent heat of steam = L heat units.

$$\text{Heat gained by cold water} = w_2 \times (T_2 - T_1) \text{ units.}$$

$$\text{Heat lost by steam} = \left.\begin{matrix}\text{Latent heat of}\\ w_3 \text{ gms. steam}\end{matrix}\right\} + \left.\begin{matrix}\text{Heat given by } w_3 \text{ gms. water}\\ \text{in cooling from } 100° \text{ to } T_2° \text{ C.}\end{matrix}\right\}$$

$$= w_3 L + w_3(100 - T_2).$$

But, Heat gained by cold water = Heat lost by the steam.

Hence, $$w_2(T_2 - T_1) = w_3L + w'_3(100 - T_2),$$

or $$L = \frac{w_2(T_2 - T_1) - w'_3(100 - T_2)}{w_3}.$$

A more accurate result is obtained if allowance is made for the fact that the calorimeter is warmed through the same range of temperature as the cold water contained in it. The student should re-calculate the result, taking this into account.

EXERCISES ON CHAPTER XVI.

1. Suppose that it requires 80 times as much heat to melt one ton of ice as would be required to warm one ton of water one degree of temperature on the Centigrade scale, how much of the ton of ice would be melted by pouring into a cavity in its surface a gallon of boiling water? A gallon of water weighs 10 lb.

2. How would you propose to prove by experiment that to boil away a gallon of water requires about five times as much heat as is needed to raise its temperature from the freezing to the boiling point?

3. Four ounces of hot lead filings and four ounces of water at the same temperature are poured upon separate slabs of ice. Will the lead or the water melt more ice? Give reasons for your answer.

4. An ounce of water at 0° C is mixed with ten ounces of water at 70° C. What is the temperature of the mixture?

An ounce of ice is dissolved in ten ounces of water at 70° C., and the temperature of the mixture is found to be something over 56° C. What can be learnt from this experiment?

5. One hundred grams of boiling water are poured upon one hundred grams of ice. What results may be observed?

6. (i) How many grams of ice must be put into 100 gm of water at 40° C to lower the temperature to 5° C?

(ii) How many grams of steam at 100° C. must be put into 100 gm of water at 5° C. in order to raise the temperature to 40° C.?

7. (i) Why does steam cause much more severe burns than are caused by water at the same temperature?

(ii) When the temperature on a winter's day rises above 0° C., why does not the snow all melt at once?

8. Into a calorimeter containing 120 gm. of water at 10° C. steam at 100° C. is passed. The total weight of steam condensed is 5 gm, and the final temperature is 34°.5 C. What value does this give for the latent heat of steam?

In this experiment the calorimeter weighed 25 gm. and the specific heat of the metal of which it is made is 0·1. Re-calculate the above result, allowing for the heat absorbed by the calorimeter

9. It is often stated that water freezes at 0° C. and boils at 100° C. Explain why this statement is not exactly true (*a*) if sea-water is used, (*b*) if the water is boiled at the top of a high mountain.

10. How would you determine the temperatures at which (*a*) butter melts, (*b*) water boils?

11. A piece of ice is placed in an evaporating basin under which a small Bunsen flame is kept steadily burning until there is nothing left in the basin. What can be learnt from this experiment?

12. Describe experiments to prove that heat is absorbed without raising the temperature when a solid is changed into a liquid and a liquid into a gas.

The temperature of some water is raised from 0° C. to 100° C. in half an hour. Assuming the supply of heat to be constant, about how long would it take for all the water at 100° C. to be converted into steam? (The latent heat of steam is 536.)

13. What is a thermometer? What temperature would you expect to be indicated by a Centigrade or a Fahrenheit thermometer when the bulb is placed in (*a*) melting ice, (*b*) boiling water, (*c*) your mouth, (*d*) ordinary drinking water?

14. Why is it that (*a*) water pipes sometimes burst during frosty weather, (*b*) the crack is not discovered until after the thaw sets in?

15. Describe two methods of determining the melting point of beeswax, and name any precautions you would take to secure accurate results.

CHAPTER XVII.

TRANSFERENCE OF HEAT

Conduction, convection and radiation.—The transmission of heat from one point to another may be effected in three ways, viz.

1. Heat may pass from one particle of a body to the next, travelling from the hotter to the colder parts, and causing no motion that can be seen of the particles of the body. This mode of transference is called **Conduction**, and is the process by which solids are heated.

2. When the heated particles actually move from one part of the body to another, causing it to become warmer throughout, the process is known as **Convection**. Liquids and gases become heated in this way.

3. When the heat passes from one point to another in straight lines with great speed, without heating the medium through which it passes, it is said to be transmitted by **Radiation**. The heat of the sun is transmitted through space in this way.

CONDUCTION.

Conduction of heat.—If you place one end of a short metal rod in a fire and hold the other, the rod soon begins to feel warm, and as time goes on it gets warmer and warmer, until at last it can be held no longer. Heat has passed from the fire along the rod, or has been **conducted** from the fire by the rod. **The process by which heat passes from one particle of a body to the next is called conduction, and the body along which it passes is known as a conductor of heat.**

Those substances which easily transmit heat in this way are called **good conductors**, while those which offer a considerable amount

of resistance to the passage of heat are called **bad conductors.** Metals are, as a rule, good conductors of heat, but some metals conduct heat better than others.

EXPT 155.—**Difference of conductivity.** In Fig 141, A, B, and C are rods of copper, brass, and iron, each 4 mm in diameter, and about 40 cm long. A short length at the end of one of the rods is bent at right angles and the three rods are bound together at D with several layers of thick copper wire.

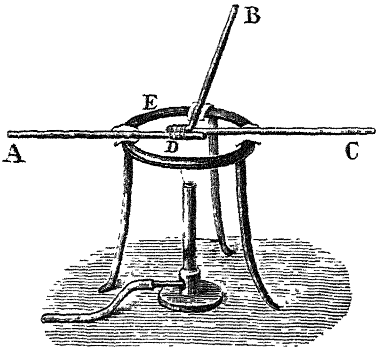


FIG 141 —Conduction of metals

Apply, by means of a brush, a coating of melted paraffin wax to the surface of the rods. Support the rods on a tripod E, placing pads of asbestos, or other bad conductor, between the rods and the top of the tripod. Heat the junction D with a Bunsen flame, and note how the wax gradually melts at a distance from the junction In a short time the melting ceases to extend; and this condition is arrived at when the heat is lost from the surface of the rods just as rapidly as it is conveyed along the rods by conduction. Notice that the melting has not extended equally far along all the rods the melting will be most extended in the case of the best conductor. Measure, and note, the distance along each rod to which the melting has extended.

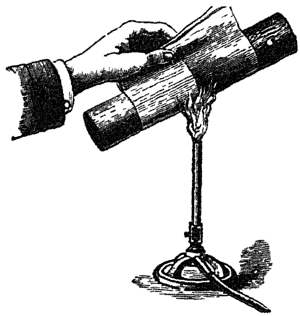


FIG. 142 —Effect of different conductivities of wood and metal.

The fact that glass is a bad conductor is demonstrated when the end of a glass rod is fused in a blow-pipe flame although the rod is held in the hand only a short distance from the heated end. Fig. 142 represents an experiment which shows that wood is a bad conductor as compared with metals. A single layer of paper is wrapped tightly round a rod

consisting partly of metal and partly of wood. If this be moved to and fro in a flame the paper touching the wood is scorched much sooner than that touching the metal This is due to the heat transmitted through the paper being conveyed away by the metal more readily than by the wood; consequently the paper touching the wood is soon heated to the temperature at which it scorches.

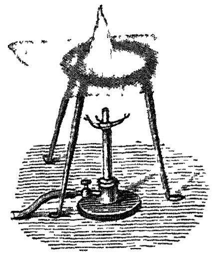


FIG 143.—Principle of the Davy safety lamp

Fig. 143 represents how the combustible mixture of coal gas and air rising from a Bunsen burner may be ignited above a piece of wire gauze held over the burner, and without the ignition proceeding downwards through the gauze. The mixture of gas and air, like all combustible substances, will not commence to burn unless it is heated up to its 'temperature of ignition.' In this experiment, the gauze conducts away the heat of the flame, and prevents the mixed gases passing through its meshes from being heated to the temperature of ignition. In a short time the gauze may become heated to dull redness, and the flame will then pass through to the top of the burner.

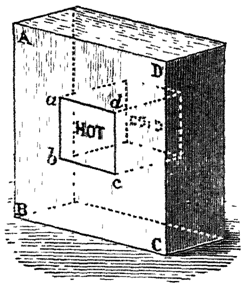


FIG. 144.—Conductivity

The principle illustrated by this experiment is utilized in the Davy safety lamp used by miners. The flame of the lamp is surrounded by wire gauze which effectually prevents it from igniting any combustible gases in its neighbourhood, so long as the gauze remains relatively cool

Coefficient of thermal conductivity.—ABCD (Fig. 144) represents a slab of metal, 1 cm. thick, the front face of which is kept 1° C. hotter than the back. This difference of temperature will cause a steady flow of heat to pass through the slab. Suppose *abcd* to be a 1 cm. cube situated in the centre of the slab; the quantity of heat conveyed through it in one second from the hot face to the cold face is termed the **coefficient of conductivity** of the metal. Hence, this coefficient may be defined as **the quantity of**

heat which passes in one second through a 1 cm. cube of the metal between opposite faces which are maintained at a difference of temperature of 1° C.

Consider the case of any slab of metal, of area s sq cm, and of uniform thickness d cm. Then, if a temperature difference of $\theta°$ C is maintained between the opposite faces, the quantity Q of heat transmitted through the slab will be proportional to (i) the coefficient of thermal conductivity k of the metal, (ii) to the fall of temperature per unit thickness, *i.e.* to $\frac{\theta}{d}$, (iii) to the area s, and (iv) to the time t. Hence,

$$Q = k \times \frac{\theta s t}{d}.$$

The following table gives the coefficient of thermal conductivity of different substances:

THERMAL CONDUCTIVITY.

Brass,	at 0° C., 0·204	Ice,	0·004
	at 100° C., 0·254	Glass,	0·002
Copper,	at 0° C., 0·719	Asbestos,	0·0004
	at 100° C., 0·722	Flannel,	0·000035
Iron,	at 0° C., 0·166		
	at 100° C., 0·163		

EXAMPLE.—Water contained in an open tank is covered with ice 3 cm. thick. If the area of the tank be 1 square metre, and if the coefficient of thermal conductivity of ice be 0·004, calculate the amount of heat transmitted in 30 minutes through the ice when the temperature of the air is $-15°$ C. Also, if the latent heat of water be 80 units, calculate what increase in the thickness of the ice will take place during this interval.

The fall of temperature per unit thickness, $\frac{\theta}{d}, = \frac{15°}{3} = 5°$ C.

Area, $s, = 10^4$ sq. cm.

Time, $t, = 30 \times 60 = 1800$ sec.

Hence, $Q = 0{\cdot}004 \times (5 \times 10^4 \times 1800) = 36 \times 10^4$ heat units.

(ii) Weight of ice formed $= \frac{36 \times 10^4}{80} = 4{\cdot}5 \times 10^3$ gm.

$$\text{Volume of ice} = \frac{\text{Weight}}{\text{Density}} = \frac{4{\cdot}5 \times 10^3}{0{\cdot}92} = 4{\cdot}9 \times 10^3 \text{ c.c.}$$

$$\text{Thickness} = \frac{\text{Volume}}{\text{Area}} = \frac{4{\cdot}9 \times 10^3}{10^4} = 0{\cdot}49 \text{ cm. approximately.}$$

Bad conductors.—Most liquids are bad conductors of heat, though quicksilver, being a metal, is an exception. If liquids were

heated only by conduction, water would boil throughout just as quickly when the source of heat was placed in contact with the top layer of liquid as it does when the heating takes place from below.

EXPT 156.—**Water as a poor conductor.** Fill a test-tube three-quarters full with cold water, and having weighted a small piece of ice by winding wire round it, or in some other way, drop it gently into the test-tube. Hold the test-tube near the bottom where the piece of ice is, and warm the top of the water in a Bunsen flame, as shown in Fig. 145. The water at the top can be heated until it boils vigorously and yet the ice is not melted.

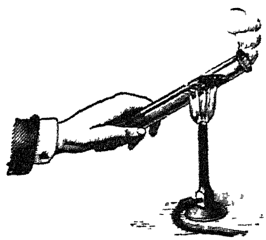


FIG. 145.—Experiment to show that water is a bad conductor of heat.

Experiments have shown that gases are very bad conductors, the conductivity of air being only about one ten-thousandth that of copper. In reckoning the conductivities of solids the proportion of heat conducted away by the air may, therefore, be neglected.

To keep ice in the warm days of summer the custom is to wrap it up in flannel and put it into a refrigerator. The flannel, because of its loose texture, encloses a quantity of air, which, being a bad conductor of heat, prevents the passage of heat from the warm outside air to the cold ice inside. Similarly, ice which has to be conveyed by rail or boat is packed in sawdust.

The refrigerator itself, too, depends upon much the same facts. The common form consists of a double-walled box with a space between the walls. This is either left 'empty,' as it is called when it is full of air, or, it is filled with some other bad conductor, such as the mineral substance *asbestos*.

If we wish to lift a hot plate we hold it with a folded cloth which does not conduct heat readily. Cylinders of engines are sometimes encased in a packing of some badly conducting material in order to prevent loss of heat.

CONVECTION.

Convection in liquids and gases.—The process by which water and other liquids are heated may be studied easily by heating

water into which some solid colouring matter has been thrown, in a round-bottomed flask over a small flame as in Fig. 146. The water nearest the flame gets heated, consequently expands, and gets lighter. It therefore rises, and causes a warm ascending current of coloured water. But something must take the place of this water which rises, and the cold water at the top, being heavier than the warm water, sinks to the bottom and occupies the space of the water which has risen. This water in its turn gets heated and rises, and more cold water from the surface sinks. Upward currents of heated water and downward currents of cool water are thus formed, until the whole of the water is heated. These currents are known as **convection currents**, and the process of heating in this manner is called **convection.**

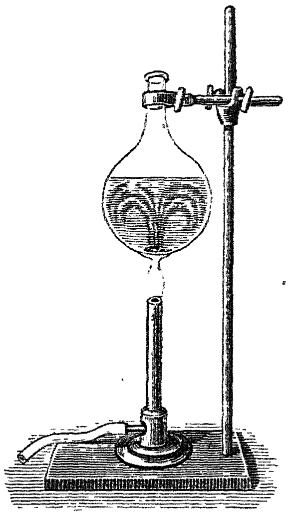


FIG 146. – Convection in a liquid.

Gases similarly are heated by the process of convection, which may be thus defined. **Convection is the process by which fluids (liquids and gases) become heated by the actual movement of their particles due to difference of density.**

EXPT. 157.—**Circulation of water.** Fit up apparatus as shown in Fig 147. A is a 6-oz. wide-mouthed, corked bottle, with the bottom knocked out (or an ordinary lamp glass may be used). A well-fitting cork with two holes is inserted, through which the bent glass tubes B, B′ pass, as shown. They are united at the bottom by a short piece of india-rubber tubing, C Pour water into A until it covers the open ends of the tubes. Now pour in about a teaspoonful of ink. Apply a small flame at B. Notice what happens.

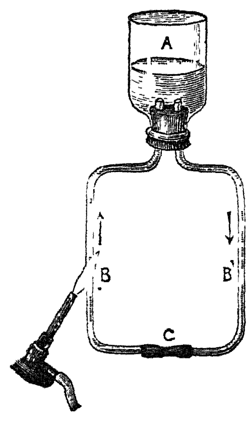


FIG. 147 —Expt 157.

EXPT 158.—**Convection currents in liquid.** Heat over a small flame a round-bottomed flask containing water (Fig. 146). Drop into the water some solid colouring matter, like cochineal, anilin dye or litmus. Notice how the hot, coloured water ascends.

EXPT. 159—**Convection currents in air.** Place a short piece of candle in a saucer, light it, put a lamp glass over it, and pour sufficient water into the saucer to cover the bottom of the lamp glass (Fig. 148). Watch how the light of the candle is affected. Next cut a strip of card less than half the height of the lamp glass, and nearly as wide as the internal diameter of the top. Insert the card into the lamp glass so as to divide the upper part into halves. Now light the candle again, and see whether it will burn with the divided chimney over it. Test the direction of the currents of air at the top of the chimney by means of smouldering brown paper.

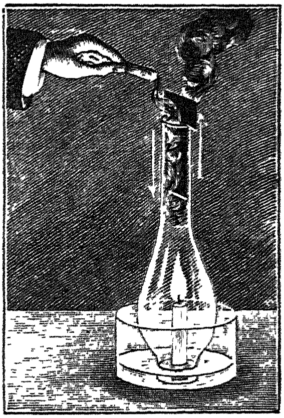


FIG. 148.—Expt. 159

Ventilation.—The ventilation of ordinary dwelling rooms is rendered possible by the way in which gases become heated by convection. The air in a room becomes warmed and rendered impure at the same time. Consequently there is a tendency for the impure air to rise, and if a suitable place near the ceiling be made for it to get out, as well as a place near the floor for the colder, purer air from outside to enter, a continuous circulation of air is set up which will keep the atmosphere of the room pure and sweet.

RADIATION.

Radiation of heat.—The fact that we feel warm in a summer sun, or that bread can be 'toasted' by holding it near the fire, is sufficient to show that heat can travel from one place to another in a third way which is neither conduction nor convection. The respects in which **radiation** differs from the other ways in which heat moves from one place to another are:

(1) it does not require a material substance for its transmission,

(2) it travels in straight lines.

Curtains have sometimes been burnt by the sun's rays being concentrated upon them by a bottle of water, though the water is not warmed much by the passage through it of the radiations from the sun. Evidently, then, the water in such a case does not pass on its heat after first becoming warmed itself—that is, it does not act as a conductor. Yet something must pass through it which can make bodies hot This something is called **radiation.** Its nature is simply a wave motion in the medium through which the rays pass. This medium is known as luminiferous ether, or shortly as the 'ether,' but it is in no way connected with the liquid ether used for scientific purposes. The ether is little more than a name, for, though something must exist to transmit waves of light and heat, nothing is certain as to its constitution.

The student will understand the phenomena of heat radiation more clearly after a short study of subsequent chapters on visible radiations (known as Light). There is indeed a close relationship between the two classes of radiation, and their chief difference may be compared to the difference between ocean waves and the small ripples on the surface of a pond, the long waves representing relatively slow rates of vibration and the short waves quicker vibrations. This difference is more apparent than real, and it is accentuated by the fact that the human eye is sensitive only to a very limited range of radiation. The close relationship between **radiant heat** and **light** is shown by the following facts:

(i) At the time of an eclipse of the sun, caused by the moon coming directly between the sun and the earth, it is observed that the heat from the sun is cut off at the same instant as the light. Hence radiant heat must travel through space with the same velocity as light, viz.: 186,000 miles per second. Compared with this, the transference of heat by conduction and convection are extremely slow processes.

(ii) Radiant heat may be *reflected* by exactly the same processes as light; and the laws of reflection are the same in both cases.

(iii) When passing from one medium into another, radiant heat is bent or *refracted* in the same manner as light. Thus, the rays of light from the sun may be refracted and focussed by means of a lens, and the fact that a piece of paper is ignited

when held at the focus proves that the radiant heat is refracted in the same manner as the rays of light

Radiant heat, like light, is a wave motion: the difference between them is simply due to the difference in the length of their waves. When a piece of metal is heated in a dark room it gives out long waves of radiant heat produced by relatively slow vibrations in the ether which can be detected by the surface of the body and by instruments of special construction, but the eye is not sensitive to these slow vibrations, and the metal is therefore invisible. As the temperature rises, the metal begins to give out shorter waves in addition to the longer ones, and the eye may perceive a radiation which it interprets as a 'dull-red' colour. At a higher temperature still shorter waves or quicker vibrations are added to the radiation, and the metal appears to be bright red.

Radiating power.—The rate at which heat is radiated outwards from a hot body depends upon (i) the difference of temperature between the hot body and the surrounding space, and (ii) the nature of the hot body's surface. Thus, a dull black surface radiates heat more rapidly than a bright metallic surface.

The quality of the surface affects in a similar manner the rate at which heat is **absorbed** by a cold body, thus, a dull black surface absorbs heat more rapidly than a bright metallic surface. In fact we may say briefly that **good radiators are good absorbers.** That this must be the case is evident if we consider two similar vessels, one having a dull black surface and the other a bright surface, containing equal quantities of warm water, and both supported near together inside a hollow vessel made of non-conducting material. Each of the vessels is radiating heat—the black one more rapidly than the bright one—and we should anticipate at first that the former would cool more rapidly than the latter. But it would be found that the temperatures of both vessels remained constant; and this is because the more rapid radiation from the black surface is counter-balanced by the fact that it absorbs far more of the heat rays falling upon it than in the case of the bright surface.

EXPT. 160.—**Radiation.** Obtain two small bright tin cans or canisters, and fit into each a cork having a hole through which a

thermometer will pass.* Cover the outside of one of the vessels with lamp-black by holding it over a candle or luminous gas flame, or over burning camphor. Put the same quantity of hot water at the same temperature in each, and then cork up the vessels, each cork having a thermometer through it so that the bulb is well immersed in the water. Observe the temperature of each vessel of water, and if the temperature of one is higher than that of the other, cool the vessel until the temperatures are equal.

Place the vessels a short distance apart, with a screen of cardboard between them. Note the reading of each thermometer, at intervals of half a minute; and continue the readings until the temperatures are at about 30° C. Plot the readings on squared paper, and deduce, from the curves obtained, which of the vessels is radiating heat more rapidly.

EXPT. 161.—**Absorption.** Similarly pour equal amounts of cold water of the same temperature into a blackened and a bright vessel, and hang them for 20-30 minutes before an even fire or closed stove; or at the same distance above an iron plate, supported on a tripod stand and heated by two laboratory burners placed so that the vessels may be in a position to receive heat equally. At the end of this time observe their temperatures. The blackened vessel will be found at a higher temperature than the bright one.

Law of cooling.—Newton's Law of Cooling states that **the amount of heat lost in a given interval of time by a particular vessel, when filled with any liquid, is proportional to the mean difference of temperature between the vessel and the surrounding air.** Hence, if when filled with warm water a given vessel takes 2 minutes to cool from $T_2°$ to $T_1°$, and if with *the same volume* of another liquid it only requires 1 minute to cool through the same range of temperature, it follows that the number of calories of heat lost in the first case must be twice as great as the number lost in the second case, for the rate of loss of heat at corresponding temperatures must be the same in both cases. In other words, the quantities of heat lost are proportional to the time-intervals required.

If w_1 be the mass of water, the heat lost is $w_1(T_2 - T_1)$ calories. If w_2 be the mass of the other liquid, of which the specific heat is s, the

* Two circular cigarette tins are suitable for this experiment. A circular hole should be filed through the centre of each lid, the hole being just large enough to admit the thermometer, and it is an advantage to solder the lid to the body of the tin.

quantity of heat lost is $w'_2 s(T_2 - T_1)$ calories. If t_1 and t_2 be the time-intervals required in the two cases, then

$$\frac{w'_1(T_2 - T_1)}{w'_2 s(T_2 - T_1)} = \frac{t_1}{t_2},$$

or

$$s = \frac{w'_1}{w'_2} \cdot \frac{t_2}{t_1}.$$

EXPT. 162.—**Specific heat by method of cooling** Weigh an empty vessel, as used in Expt 160, and nearly fill it with a measured volume of water Weigh the vessel and its contents Warm the vessel until its temperature is about 50° C Support it on a non-conducting surface and screen it from air currents Allow it to cool, and note the time required to cool (i) from 45° C to 35° C. and (ii) from 35 C. to 25° C. Empty and dry the vessel. Pour into it the same volume of the liquid of unknown specific heat, and weigh the vessel and its contents. Raise the temperature to about 50° C., and observe the time-intervals required for it to cool through the same ranges of temperature Calculate the specific heat of the second liquid from each range of cooling

Heat as a form of energy.—Heat was believed formerly to be a fluid called **Caloric**, and it was supposed that a piece of hot iron differed from a cold piece in having entered into some sort of union with this fluid, which was considered to be indestructible. But in the year 1798 Rumford boiled water by the heat developed by the friction between two metal surfaces which he rubbed together, and he found that the amount of water he could bring to the boiling temperature depended only on the amount of work he expended in rubbing. Since he could obtain an indefinite amount of heat from two definite masses of metal, it was quite clear that heat could not be matter, which cannot be created Davy made the truth even clearer by obtaining heat enough to melt ice by simply rubbing two pieces of ice together; they were both cold or without caloric; and since heat could be obtained by rubbing them together, it was quite certain that heat could not be a fluid. Joule went a step further and measured the amount of work which must be done to obtain a given quantity of heat; that is, he measured the **mechanical equivalent of heat**

His apparatus consists essentially of a large calorimeter containing water which is set in motion by paddles on a spindle turned by falling weights. The water is prevented from moving

as a whole in the calorimeter by vanes projecting radially inside this vessel. When the weights are allowed to fall, the spindle turns, the paddles move the water, the motion is stopped almost immediately by the vanes and is converted into heat, and a rise of temperature is observed. The work performed can be found from the masses of the weights and the distance through which they fall, and the heat produced can be determined in calories, knowing the water-equivalent of the calorimeter, the weight of water in the calorimeter, and the rise of temperature. Results of experiment show that **to raise the temperature of one gram of water one degree Centigrade requires about 42 million ergs of energy**, or the mechanical equivalent of heat is 42,000,000 ergs per gram-calorie.

The average of a large number of experiments made by Joule gave the value of the mechanical equivalent of heat in British units as 1,390 foot-pounds Or, **to raise the temperature of one pound of water through one degree Centigrade requires an expenditure of 1.390 foot-pounds of work.** To raise the temperature of a pound of water through 1° F. requires 772 foot-pounds.

Conversion of motion into heat —Some examples which will be familiar to the student will provide proofs that heat and work are convertible. When a brake is applied to the wheels of a train as it stops at a station, it is a common thing to see sparks fly. The resistance of friction which overcomes the motion of the train causes a sufficient amount of heat to be developed to raise the temperature of the particles of steel, which get rubbed off, to a red heat. By continually hammering a piece of iron on an anvil it can be made too hot to hold in the hand.

The following experiments show that heat appears when motion is destroyed

EXPT. 163.—i. **Heat from motion.** Hammer a piece of lead, or saw wood, and test the temperature of the lead or saw before and after the experiment.

ii. Rub a brass nail or button on a wooden seat, and notice its increase of temperature.

When a lucifer match is rubbed along a rough surface, the heat into which the work is converted is enough to ignite the match. In all such cases mechanical work is converted into heat. The converse is true also; heat is convertible into work. In the

steam-engine the heat of the furnace changes the water in the boiler into steam. The steam forces the piston along the cylinder, and this movement of the piston in a straight line is converted into the circular motion of a fly-wheel, and is used, through the intervention of suitable mechanism, in pumping water or performing some other kind of work. The steam which enters the cylinder is hotter than that which passes from it into the condenser. Thus, part of the heat of the steam has been converted into useful work and parts of it have been lost to the condenser, the air, etc.

Determination of the mechanical equivalent of heat.—Mechanical work is done when a body is raised from the ground. When the body is then allowed to fall freely, the work done in raising it is converted gradually into kinetic energy; and when the body reaches the ground all this kinetic energy is converted into heat. When the weight of the body and the distance through which it is raised are known, the work done in lifting it can be calculated; and when the specific heat of the material is known, and the rise in temperature after falling is observed, the quantity of heat developed can be calculated. Such data serve as a means of determining approximately the mechanical equivalent of heat.

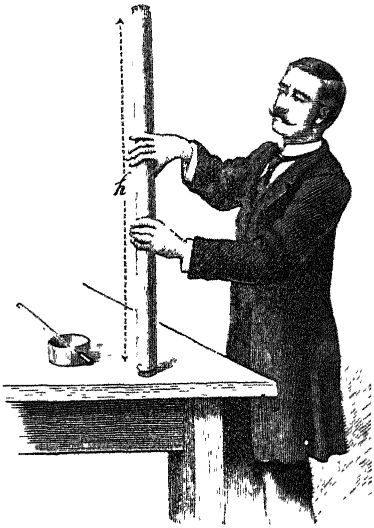


FIG. 149.—Experiment to determine the mechanical equivalent of heat.

EXPT. 164.—Obtain a cardboard tube, about 1 metre long and 5 cm diameter, fitted with a cork at each end. Weigh out about 500 gm. of small lead shot contained in a dish or beaker, and observe the temperature (t_1) of the shot, using a thermometer reading to 0°·2 C. Remove one of the corks; and, holding the tube almost horizontal, transfer the

shot to the tube. Hold the tube vertically, and measure by means of a metre scale the distance from the top of the lead shot to the top of the tube. Subtract from this the length of the tube occupied by the upper cork when firmly inserted. This gives the distance h through which the shot are raised when the tube is inverted. Insert the upper cork, and, while holding the middle of the tube firmly, rapidly invert it to a vertical position Repeat this movement at least 50 times, counting the number n of times the inverting has been repeated. Transfer the shot back to the dish, at once insert the thermometer into the shot, and note the final temperature (t_2)

If w gm. = weight of shot,
h cm. = vertical distance within the tube,
n = number of times the tube is inverted, then
mechanical work done $= n \times wh$ gm.-cm. units.

If s = specific heat of lead,
$(t_2 - t_1)^\circ$ C. = rise in temperature, then
heat developed $= ws(t_2 - t_1)$ calories

Hence, **mechanical work equivalent to one calorie** $= \dfrac{n \times wh}{ws(t_2 - t_1)}$ gm -cm.

The following is a typical result of an experiment with this apparatus

Weight (w) of lead $= 434$ gm
Specific heat (s) „ $= 0{\cdot}0315$
Vertical height (h) of fall $= 67{\cdot}7$ cm
Rise in temperature $(t_2 - t_1) = 16^\circ{\cdot}5 - 14^\circ{\cdot}5 = 2^\circ$ C.
Number (n) of inversion $= 40$
Mechanical work done $= 40 \times (434 \times 67{\cdot}7)$ gm.-cm. units.
Heat developed $= 434 \times 0{\cdot}0315 \times 2$ calories.

Hence,

work equivalent to 1 calorie $= \dfrac{40 \times 434 \times 67{\cdot}7}{434 \times 0{\cdot}0315 \times 2} = (4{\cdot}3 \times 10^4)$ gm.-cm. units.

Determination of the mechanical equivalent of heat by means of magneto-electricity. If a metal disc be rotated rapidly between the poles of a strong electro-magnet electric currents are caused to flow round it. These currents which are induced in the metal plate, cease to flow when its rotation is stopped; moreover, these induced currents flow in such a direction that they tend to stop the rotation of the disc. The first transformation of energy is the conversion of the energy of rotation into that of the electric currents flowing round the metal plate. But this is followed by the transformation of the

energy of the electric currents into that of heat. Such a plate is rotated between the poles of a very powerful electro-magnet only by the expenditure of a great amount of mechanical work, and this is eventually converted into sufficient heat in the plate to make it too hot to touch. By measuring the amount of work expended in rotating the disc, and also the quantity of heat developed in it, it is easy to calculate the mechanical equivalent of heat.

Compression and expansion of a gas.—When a gas is compressed, work is done upon it and this energy is converted into heat. Dalton showed that when air is compressed to half its volume, a rise of temperature of 50° F. (27·8° C.) is produced. Conversely, when a compressed gas is allowed to expand there is a fall of temperature. If carbon dioxide gas be allowed to escape suddenly from a cylinder in which it has been compressed, the cooling produced by the expansion is sufficient to convert the gas into a snow-like solid. This fact is utilised in apparatus for cooling air and other gases until they are converted into the liquid form.

EXPT. 165.—**Heat by compression.** Compress air into a bicycle tyre or cylinder by means of a bicycle pump. Notice that the part of the pump connected with the tyre or cylinder becomes warm.

EXPT. 166.—**Cooling by expansion.** Allow air, which having been compressed into a cylinder has again assumed the temperature of the air, to come in contact with a thermopile or other delicate means of measuring changes of temperature. Notice the cooling of the compressed air when allowed to escape.

Radiation and work.—Radiation can be converted into work, but in a less direct manner than is the case with ordinary heat; it must first be absorbed and heat some material body thus causing its molecules to oscillate in the manner described already. This form of heat has a mechanical equivalent; and it is fair to conclude that, if the whole radiation be absorbed, the mechanical equivalent of the absorbed heat is an exact measure of the energy of the radiation.

EXERCISES ON CHAPTER XVII.

1. On a cold morning a gardener grasps the iron part of his spade with one hand and the wooden part with the other. Explain why one hand feels colder than the other.

2. If a spoon made of solid silver and one made of brass and only silver plated are placed in some boiling water, the handle of the silver spoon becomes much hotter than that of the plated one Why is this?

Describe an experiment by which you would show that your explanation is correct.

3. Why is a vessel of water heated more quickly when heat is applied at the bottom than when it is heated at the top?

Draw a diagram to illustrate the movements of a liquid heated from below

4. Two test-tubes A and B are filled with water. A small piece of ice is allowed to swim in A, and a similar piece of ice is sunk by a weight to the bottom of B Heat is applied to the closed end of A and to the open end of B In which test-tube may we expect the ice first to melt? and in which may we expect the water first to boil? Give reasons for your answer.

5. State which modes of heat transference are involved in warming a room by means of an open fire-grate. Which mode is the more important?

6. Explain why a thick glass vessel usually cracks when hot water is poured into it

7. A metal plate, 1 square decimetre in area and 0·5 cm. thick, has the whole of one face covered with melting ice, while the other face is in contact with boiling water. If the coefficient of conductivity of the metal be 0·14, how many kilograms of ice will be melted in an hour?

8. Two similar thermometers are taken, and the bulb of one is blackened. They are both (i) placed in sunlight, (ii) exposed on a clear night How will the readings of the thermometers differ in each case?

9. A silver cup containing boiling water is placed on a silver tray in a room. Describe the various ways in which the water loses heat.

10. Illustrate the various ways in which heat can be transmitted from one body to another What conditions determine the rate at which the transmission is effected in each case?

11. What experiments would you perform to show that:

(*a*) Silver is a good conductor of heat;

(*b*) Water is a bad conductor of heat?

12. In the case of a shot fired at a target, state (*a*) why the velocity of the shot changes when it strikes the target; and (*b*) why the target is made hot where the shot strikes it.

13. When a brake is applied to a wheel of a moving train, red-hot sparks are seen What are these sparks, what is the source of their heat, and why do they soon disappear?

14. The point of a gimlet with which a hole has been bored is found to be hot How do you explain this? Give other instances of the same sort

15. Describe experiments to show that energy of motion can be converted into heat

16. In a shipbuilding yard a machine pierces holes in iron plates by punching out circular fragments Before beginning work the machinery and iron plates are quite cold After the operation the circular fragments are too hot to hold in the hand Why is this?

17. A brass button, when rubbed on a school form, becomes hot (*a*) What is the source of this heat?

The button is placed on a table, and in a few minutes it becomes cool. (*b*) What has become of the heat?

18. Describe the different ways in which heat may be conveyed from one place to another. Which mode of transmission is utilised in the ventilation of a sitting room?

19. It is often stated that heat and work are equivalent Explain this in the case of a steam engine, and when the brake is applied in a railway train. How may the mechanical equivalent of heat be determined?

20. What happens when a gas is compressed considerably in a water-cooled cylinder, and then is allowed to expand? Mention any practical use of this that you know.

21. Heat may be described as a form of energy. Explain precisely what the statement means Describe one method by which the amount of energy corresponding to the unit quantity of heat has been determined

22. If a hot lamp-chimney be touched with a cold knife-blade it will probably crack. If a tightly-corked bottle full of water be put out of doors on a frosty night it will burst. Explain as fully as you can the reasons for these two results

23. What is meant by convection? Illustrate your answer by sketches, taking the case of a vessel filled with water and heated from below, and explain why it is that convection is set up

24. Explain why the polished fire-irons in front of a fire are sometimes found to be only slightly warm, while the blackened fender is too hot to be touched

25. Describe experiments to show that good radiators are good absorbers of heat.

26. What relation would you expect to find between the reflecting and emissive powers of a substance? Give reasons for your answer

Describe a method of comparing the emissive powers of different substances at the same temperature (e g 100° C)

27. Describe and explain the phenomenon of the convection of heat

Two test tubes are partly filled with liquid air; the lower part of one is surrounded by a vacuous space, the other by air State and explain what you would observe.

28. Describe experiments illustrating the relation between heat and work, and explain one method of determining the mechanical equivalent of heat

29. Describe experiments by which it has been shown that when work is done against friction, the quantity of heat produced is proportional to the amount of work so done.

30. Define the mechanical equivalent of heat

Describe an experimental method of determining it, pointing out the quantities which must be measured for this purpose

PART IV.

LIGHT.

CHAPTER XVIII.

PROPAGATION OF LIGHT. SHADOWS. PHOTOMETRY

Light is a form of radiation.—In considering, in a part of the previous chapter, the ways in which heat can be transferred from one place to another, it was seen that the heat of the sun reaches the earth by radiation. These solar radiations comprise what collectively is called sunlight, they are conveyed in the form of waves through the medium ether, which is believed to pervade all space, and may be referred to conveniently as **ether-waves.** These ether-waves are of various lengths and can produce different effects. If they fall upon our bodies the longer waves may be absorbed, and the energy of the wave-motion become converted into **heat**: if they fall upon the retina of an eye, the shorter waves may produce a sensation of light, and the waves are then spoken of as **light**; falling upon a photographic plate or upon a green leaf, the shortest ether-waves may produce chemical effects, and these waves are referred to as **actinic.** But, in their passage through the ether, ether-waves do not give rise to any of these effects; they are simply waves transferring energy by wave-motion.

Opaque and transparent media.—Ether-waves are not transmitted through all media with equal facility. Substances such as iron and other metals, wood, or stone, through which the waves which produce the sensation of light cannot be transmitted, are termed **opaque**; others, such as glass, water, or air through

which such waves are transmitted readily, are termed **transparent.** A few media are known which transmit a portion only of the light falling upon their surface, and these are termed **translucent**, fog, white paper, ground glass, porcelain, and a mixture of milk and water are examples of translucent media At the same time, the distinction between opaque and transparent media is not absolutely definite, since more or less light may be transmitted through media usually regarded as opaque, providing that a layer of sufficient thinness is used. Thus, a sheet of gold leaf, when supported between two sheets of glass, is somewhat transparent; similarly, a very thin section of a stone is transparent, and a thin shaving of wood is translucent. The fact that very little light penetrates to great depths beneath the surface of water shows that this medium is transparent only when thin layers are used.

Light travels in straight lines.—It can be shown by the observation of several simple phenomena that light travels in straight lines through any homogeneous transparent medium. Any straight line in such a medium along which light is propagated may be termed a **ray** of light. Rays of light proceed in all directions from every point of a luminous body, a small assemblage of rays included within a cone of very small angle, and proceeding at its apex from a luminous point, is termed a **diverging pencil** or **beam**; if the rays converge towards a point they constitute a **converging pencil.** When the luminous point is at an infinite distance, the rays of light are practically parallel to one another, and a small assemblage of the rays is termed a **parallel pencil** or **beam.**

That light rays travel in straight lines in ordinary circumstances can be shown at once by examining the paths of the rays as they pass through a hole in the shutter of a darkened room. Though the light-waves are not themselves visible, yet the path of the light becomes apparent, because the minute particles of dust in the air are rendered luminous by the vibrations of the ether being reflected from them If there were no dust particles in the room the beam of light would be invisible. When the path of a beam is made visible by smoke or dust it is seen to be a straight line. That light travels in straight lines may, indeed, be inferred from several everyday experiences. We cannot see round a corner; if light travelled

through a uniform medium in lines that were sometimes bent, there is no reason why we should not. Or, again, everyone knows that it is only necessary to put a small obstacle in the path of the light from a luminous body to shut out completely our view of it

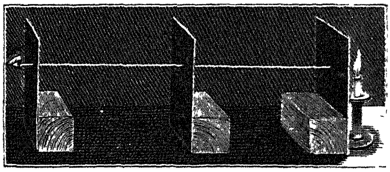


FIG 150—Experiment to show that light travels in straight line

EXPT. 167. — **Rectilinear propagation.** Place three cards together and pierce a small hole through them. Support the cards as in Fig. 150. The light of the candle can be seen only when the holes are in a straight line. What is true of light applies equally to all other kinds of radiation.

Inverted images produced by a pin-hole.—When an object is viewed through a pin-hole camera, it is seen to be upside down upon the screen. Similarly, all images produced by a small aperture are inverted. This inversion is a direct consequence of the fact that light travels in straight lines That this is really the case can be understood fully by the following simple considerations.

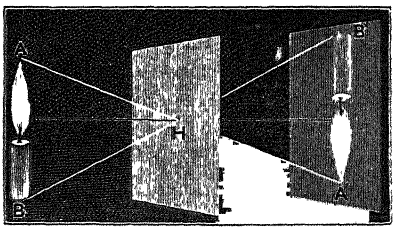


FIG. 151—Explanation of the inversion of images seen through a pin hole

Let H, in Fig 151, be the pin-hole, and AB the candle. Rays are sent out in all directions by every point of the candle, but of all the rays from one point, such as A, only that in the direction AH can pass through the hole and form an image A'. Similarly, the only ray from B which can get through the hole is BH, so an image of B is formed at B'. The same reasoning applies to any part of the candle, hence a complete inverted image is produced

If the light which passes through a small hole in the shutter of an otherwise dark room be caught by a screen of cardboard, a coloured, inverted image of the sky and landscape will be seen.

By using a pin-hole camera, a photograph of the view can be obtained (Fig. 152) A pin-hole camera can be made from a light-proof box having a minute hole at one end, or one can be purchased for a few pence. The bright circles of light seen under trees in summer are really images of the sun formed by the small spaces between the leaves.

FIG. 152.—Photograph obtained with a pin-hole camera, by Mr. F. Butterworth

Size of image produced by a pin-hole.—That the size of the image depends upon the distance of the screen from the pin-hole is proved practically by varying the distance of the screen from the pin-hole and measuring the length of the image. The greater the distance of the screen the longer the image. The reason for this alteration in the size of the image is a simple one. The rays of light from the top and bottom of the object travel through the pin-hole, and since one is travelling upwards and the other downwards, they will be farther apart the greater the distance they travel. Consequently, the image is longer the more the screen is moved from the pin-hole.

The relation between the sizes of the object and image according to their distances from the aperture is

$$\frac{\text{Length of object}}{\text{Length of image}} = \frac{\text{distance of object from aperture}}{\text{distance of image from aperture}}.$$

The larger the image the less bright it is, because the small amount of light which passes through the aperture is spread over a greater area in the case of the enlarged image.

Illumination due to overlapping of images.—When a pin-hole is made in the front face of a pin-hole camera, an image of the

bright object looked at is formed on the screen in the manner described in the preceding paragraphs. If a second hole be pierced, a second image is obtained. When the number of holes is increased steadily one at a time, the images, it is observed, start overlapping, at the same time becoming blurred. When the number of images has become considerable, no separate image can be distinguished; **diffused light**, as it is called, is produced, and the screen is illuminated in the ordinary way

Intensity of light—In proceeding from the source of illumination, light spreads out as indicated in Fig. 153, so that though each ray retains its original intensity the number of rays which illuminate a given area depends upon the distance of that area from the luminous source S. At twice the distance the rays are spread over four times the area, so their illuminating effect, as at M, is only one-fourth of what it is at *m*. The amount of light received from a luminous source is thus inversely proportional to the square of the distance from the source.

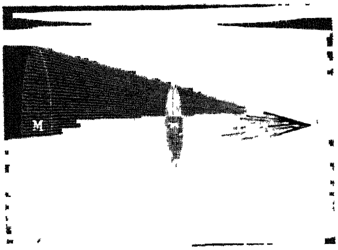


FIG 153—Intensity of light at different distances from the source.

Suppose L to be the rate per second at which luminous radiation is emitted uniformly in all directions from a point situated at the centre of a sphere of radius r cm. Since the area of the sphere is $4\pi r^2$, the quantity of radiation falling on each square centimetre of the sphere will be $L/4\pi r^2$. This quantity is termed the **intensity of illumination** of the surface: and it is evident that the illumination varies inversely as the square of the distance.

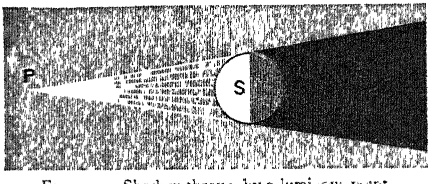


FIG. 154.—Shadow thrown by a luminous point.

Shadows.—Let P (Fig. 154) be a luminous point and S an opaque sphere intercepting the light which falls upon it. A screen held beyond S and at right angles to

the axis of the cone of rays will indicate a well-defined circular shadow of the sphere, all points of the screen within the shadow being protected totally from the rays.

If the source of light be large, as in S (Fig. 155), a more complicated shadow is obtained. Each point of S throws its own shadow cone; and by drawing the cones due to two points at the upper and lower edges of S, it is seen that only the space common to both these cones is protected completely from light; this space is indicated by the black cone terminating at the point P. When a screen AB is held between E and P, so that it is normal to the line joining the centres of S and E, there will be visible on its surface a

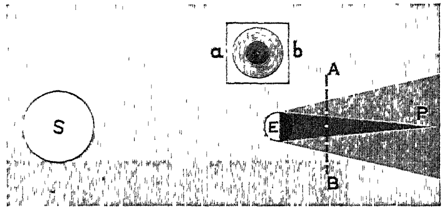


FIG. 155.—Umbra and penumbra, due to an extended source of light.

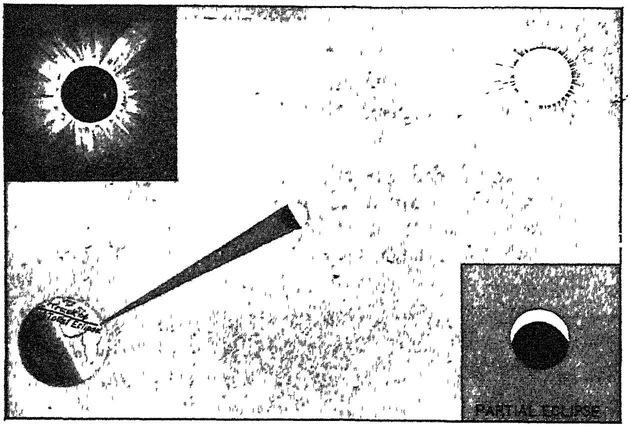


FIG 156.—Eclipse of the sun, caused by the moon coming between the sun and the earth.

black central circle called the **umbra**, surrounded by a ring of partial shadow, termed the **penumbra**. The appearance of the shadow is shown at *ab*.

A shadow of this type is thrown by the light of the sun falling upon the moon. When this shadow falls upon the earth's surface, a *total* eclipse of the sun is seen from any point within the umbra, and a *partial* eclipse is seen from any point within the penumbra (Fig. 156). Similarly, the sun throws a shadow of the earth, and the moon is partially (or totally) eclipsed when it passes into the shadow.

Dimensions of shadow-cones.—The length of a shadow-cone, or the diameter of the cone at any point, can be determined by graphical construction or by a simple calculation. For instance,

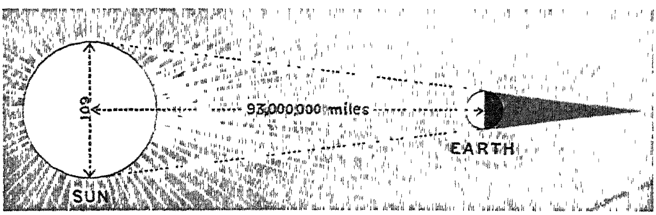


FIG. 157.—Shadow-cone of the earth

Fig. 157 represents the shadow-cone caused by the sun shining upon the earth. The diameter of a cone at any point is directly proportional to the distance from the apex. Hence we have the relation

$$\frac{\text{Earth's diameter}}{\text{Sun's diameter}}=\frac{\text{Earth's distance from apex}}{\text{Sun's distance from apex}}.$$

If the earth's diameter be taken as unity, the sun's diameter is about 109 times greater. The earth's distance from the sun is 93,000,000 miles; and the distance (x) of the earth from the apex of the shadow may be found from the simple proportion:

$$\frac{1}{109}=\frac{x}{93{,}000{,}000+x}.$$

The length of the earth's shadow is thus found to be 861,111 miles. Knowing the distance of the sun, the diameter can be calculated by the same principle based upon the fact that a half-penny, which is one inch in diameter, exactly covers up the sun's

disc when held at a distance of nine feet from the eye. For we have the proportion

$$\frac{\text{Diameter of halfpenny}}{\text{Diameter of sun}} = \frac{\text{Distance of halfpenny}}{\text{Distance of sun}},$$

or, reducing inches to decimals of a mile,

$$\frac{0\cdot000019}{x} = \frac{0\cdot000019 \times 12 \times 9}{93{,}000{,}000}.$$

From this relation the sun's diameter is found to be about 860,000 miles It is of interest to notice that the earth's shadow-cone has about the same length as the sun's diameter.

It is often easy to determine the size and shape of a shadow by a graphical construction as in the following example:

Example. A man 6 feet in height is standing 15 feet from a lamp 12 feet high: what is the length of the man's shadow upon the ground?

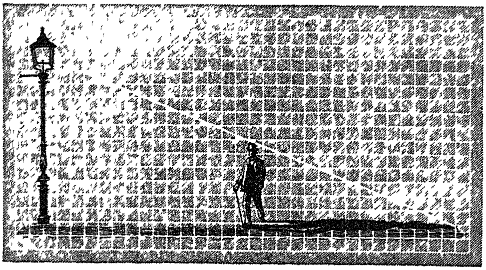


Fig. 158.—Length of shadow.

By drawing the given dimensions to scale, or on squared paper, as in Fig. 158, it will be found that the required length is 15 feet. The same result is obtained by calculation as in previous examples.

Velocity of light.—The ether-waves travel with a velocity of 186,000 miles per second. This rate of propagation has been determined in the case of light in several different ways, two of which will here be described briefly.

1. **By observations on Jupiter's satellites.**—The planet Jupiter, represented by J in Fig. 159, has several satellites or moons which revolve round Jupiter in a plane nearly coincident with that of the planet's orbit round the sun, and consequently one or other of these satellites frequently passes into the shadow of the planet thrown by the sun, and so becomes invisible to us. When the

earth E, and Jupiter J, are on the same side of the sun, let the time of disappearance of one of the moons into the planet's shadow be observed. Suppose the time at which the phenomenon should happen six months hence to be calculated by considering the period which the satellite takes to revolve around the planet. It is found that when the half-year has elapsed the

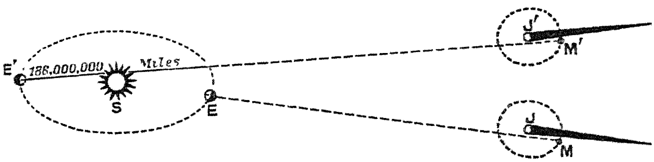


FIG. 159.—Römer's method of determining the velocity of light from observations of eclipses of Jupiter's satellites.

observed time of disappearance and the calculated time do not agree. The eclipse occurs about 1000 seconds late. This is because the earth's position relatively to Jupiter has undergone a change: it is now at E' on the other side of its orbit, and the light-message has to travel across the orbit before reaching us. This extra journey takes 1000 seconds. The distance across the earth's orbit, that is, from E to E', may be taken as 186,000,000 miles: and as light takes 1000 seconds to traverse this, the velocity is 186,000,000 divided by 1000 or about 186,000 miles per second.

2. **By terrestrial experiments.**—The velocity of light has been determined by various experimenters on the earth itself. We shall only describe the principle of the method employed by M. Fizeau in 1849 (Fig. 160). Two places about five miles apart were chosen and light sent from one station was reflected back to its starting point by a mirror which it struck normally at the more distant station. There was then interposed between the mirror and the source of light a toothed wheel, carefully constructed so that the width of the teeth and spaces between them were equal. This was placed near the source of light. It is clear that if the reflected light be received by a tooth of the wheel it will not reach an eye suitably placed on the same side of the wheel as the source of light. Moreover, if the wheel be rotated it can be given such a speed that there is always a tooth in the way to meet the light which has travelled to the mirror and back again. Similarly

the speed can be made of such a value that one of the spaces shall always fall in the path of light. When this is the condition of things it is easy to see that the wheel rotates through an angular distance equal to the width of a tooth in a time that the

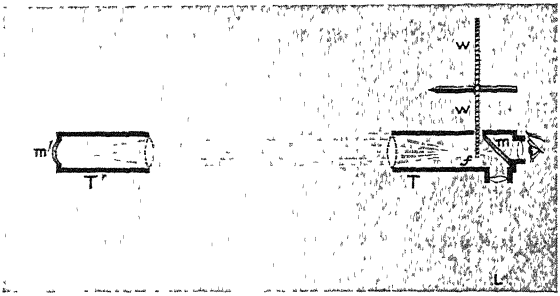


FIG 160—Fizeau's apparatus for determining the velocity of light. L, source of light, *w*, *w*, toothed wheel, *m*, plain glass mirror, *m'*, silvered mirror, *f*, place where the light strikes the toothed wheel.

light travels from the wheel to the mirror and back again to the wheel. The time occupied by the wheel in rotating the angular distance can be calculated at once from the rate of rotation, while the distance from the wheel to the mirror can be measured directly.

PHOTOMETRY.

Illumination of a surface.—When light is emitted from a point at the rate L, the illumination on a surface situated at a distance d, and at right angles to the path of the rays, is equal to $L/4\pi d^2$. Suppose two luminous points, emitting light at the rates L_1 and L_2 respectively, to be situated at a distance apart; if an opaque white screen be held between the points, at right angles to the line joining them, and if the screen be at distances d_1 and d_2 from the points, the illumination on the two sides of the screen will be represented by $L_1/4\pi d_1^2$ and $L_2/4\pi d_2^2$. If the screen be moved to and fro until the two sides are equally illuminated, then

$$\frac{L_1}{4\pi d_1^2}=\frac{L_2}{4\pi d_2^2},$$

$$\text{or}\qquad \frac{L_1}{L_2}=\frac{d_1^2}{d_2^2}.$$

This principle serves as an accurate method for comparing the luminosity of different sources of light: and this comparison is termed **Photometry.**

Candle power.—In order to give a numerical value to the illuminating power of any source of light, it is necessary to have some standard source which can be taken as a unit. It is sufficient to mention here the simplest unit available, viz. the **standard candle** This is defined as a sperm candle, weighing six to the pound, and burning 120 grains of wax per hour. The luminosity of a candle flame is influenced by the temperature and purity of the air, and therefore this unit is not absolutely constant. The ratio between the illuminating power of any source of light and that of a standard candle is termed the **candle power** of that source.

Rumford's shadow photometer.—In this photometer (Fig. 161) an opaque rod is placed in front of a vertical screen of unglazed paper (or of ground glass) and the two sources of light are placed so as to throw separate shadows of the rod upon the screen. The shadow due to one of the sources represents a strip of the screen which is illuminated by light from the other source only. The distances of the sources from the screen are varied until the shadows are equally dark, and then the above equation can be applied for comparing the luminosity. It is not necessary to have a completely dark room for experiments with this photometer.

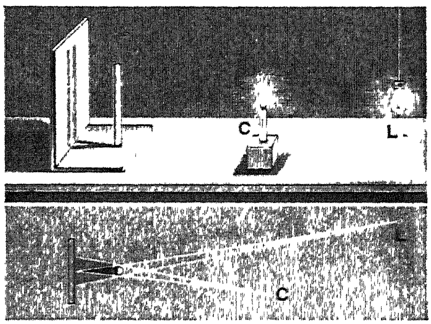


FIG. 161.—Rumford's shadow photometer

EXPT. 168.—**Law of inverse squares.** Pin a piece of white paper upon a drawing board to act as a screen. Fix the drawing board at right angles to a table in a darkened room. In front of the screen place a vertical rod about 1 to 2 cm in diameter (a retort stand will do). Beyond this, place to one side a candle fixed on a block of

wood, and to the other side four candles, fixed close together on a second block of wood. Notice that two shadows of the upright rod appear on the screen Move the candles near each other so that the two shadows of the rod touch but do not overlap Notice that one shadow, that cast by the four candles, is darker than the other. The latter shadow is illuminated by one candle, the other is illuminated by four candles Now, move back the four candles until the shadows appear equally dark, that is, in equal contrast with the bright part of the screen Then the four candles give just as much light to the screen as the single candle Measure the distance of the single candle, and the mean distance of the four.

It will be found that the latter distance is twice the former, thus showing that at double the distance of a single candle four candles are required to give the same illumination.

EXPT. 169 —**Candle-power** Using a standard candle, determine the candle-power of (i) an ordinary wax candle, (ii) a paraffin lamp, and (iii) a fish-tail gas burner. For this comparison, use the relation $L_1 : L_2 = d_1^2 : d_2^2$.

Bunsen's grease-spot photometer.—In this photometer the two sources of light are compared by placing them on either side of a screen having a grease spot in it. Its use depends on the fact that a grease spot equally illuminated on either side has the same brightness as the general surface. The light lost by transmission

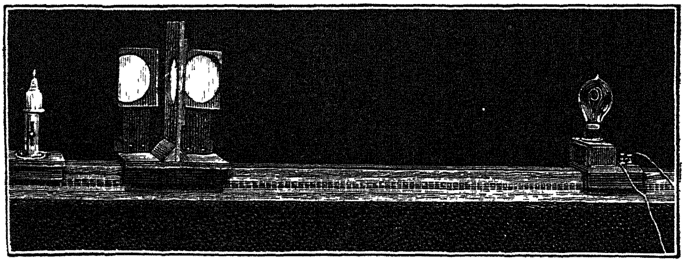


FIG. 162.—Bunsen's grease-spot photometer.

through the translucent or semi-transparent spot in one direction is compensated for by the equal transmission in the opposite direction. The eye is indifferent whether the light it receives is due to a reflection from white paper or to transmission through a translucent spot. It observes that the brightness of the grease

spot is equal to that of the rest of the surface. The intensities of the two lights are again proportional to the squares of their distances from the screen

EXPT. 170.—**Unequal illumination.** Obtain a piece of white paper. Make a grease spot in the centre. Allow a light to shine on the paper. Observe that the grease spot is darker than the surrounding surface. Observe the paper by transmitted light. Notice that the grease spot is now brighter than the general surface.

EXPT. 171.—**Equal illumination.** Use the paper as a screen and illuminate one side of it by means of a candle and the other with a lamp. Move the candle and lamp until the grease spot is barely distinguishable in point of brightness from the white surface near it. Measure the distance of the candle and lamp from the grease spot. Using the law of inverse squares, calculate the luminosity of the lamp in terms of the candle.

EXERCISES ON CHAPTER XVIII

1. Describe a pin-hole camera, and explain, illustrating your answer by a diagram, how the image of a luminous object is formed by it.

What experiment would you perform to show why it is that the image first becomes blurred and then disappears when the size of the hole is increased gradually?

2. Three candles are placed quite close together in a row at the centre of a room, and a wooden rod is held in a vertical position at a distance of about a foot from the candles. Explain, giving diagrams, why it is that as the rod is moved in a circle round the candles the shadow cast on the walls is in some positions sharp and in others very ill defined.

3. The sun shines through a crack in the shutter of a darkened room. A person inside the room says that he sees a ray of light entering the room. Put his statement in a more accurate form. What can he really see?

4. A small opaque sphere is placed between a gas burner and a white screen, and when the gas is turned down so that the flame is very small it is found that the shadow cast on the screen is quite sharp, but on turning up the gas so that the flame is large, that the edge of the shadow is blurred. Explain the reason for this change, illustrating your answer by means of diagrams.

5. A man, $5\frac{1}{2}$ ft. high, is standing at a distance of 5 ft. from a street lamp, the flame of which is 9 ft. above a horizontal roadway. Find the length of the man's shadow.

6 Draw a diagram of the sun, moon, and earth during an eclipse of the moon. How far from the earth does its umbra extend if the diameters of the sun and earth are 800,000 miles and 7900 miles respectively, and if the distance of the earth from the sun is 92,000,000 miles?

7. The sun subtends the same angle as a halfpenny at a distance of 10 feet. Give a diagram showing the size and nature of the shadow of a halfpenny cast by the sun on a surface perpendicular to the rays at a distance of 5 feet from the halfpenny

8. A shadow of a circular table 3 feet high and 3 feet in diameter, is cast on the ceiling of a room 10 feet high by a night light on the floor and vertically beneath one edge of the table Give a diagram illustrating the formation of the shadow, and find its size and shape.

9. In a comparison of the luminous intensity of an incandescent electric lamp and of a standard candle, it is observed that the shadows were of equal intensity when the distances of these sources from the screen were 42 cm and 15 cm respectively. What was the 'candle-power' of the lamp?

10. The shadows of a vertical rod on a wall, at equal distances of 2 feet each from the rod, cast by two gas flames are observed to be equally dark when the flames are at distances of 6 and 4 feet respectively from the rod Compare the illuminating powers of the flames.

11. Compare the intensities of the illumination produced on the floor of a room (i) when lit by a gas-lamp of 400 candle-power at a height of 16 ft, and (ii) when lit by an arc lamp of 1000 candle-power at a height of 40 ft

12. (i) A standard candle is 210 cm. from a 16 candle-power electric lamp. Where should a screen be placed between them in order that its two sides may be illuminated equally?

(ii) Where else, along the line joining the lights, might the screen be placed and yet be illuminated equally by each source of light?

13. What is meant by the intensity of illumination on a surface? Which would give the greater intensity of illumination on the ground immediately beneath the lamp, (i) a 100 candle-power lamp at a height of 12 feet, or (ii) a similar lamp of 1200 candle-power 45 feet high?

14. How would you arrange an experiment to determine the percentage of light that is transmitted through a neutral tinted glass plate? If a plate of such glass allowed 40 per cent. of the light incident upon it to pass through, how much light would be transmitted by a plate of the same glass of four times the thickness, assuming no light to be lost by reflection at the surfaces in either case?

15. Two lamps are placed on opposite sides of a screen, and their distances from the screen so adjusted that the two faces of it are

illuminated equally. A semi-transparent sheet is then placed between one of the lamps and the screen, and it is found that the other lamp must be moved to twice its original distance from the screen in order that the two faces may be illuminated equally again What fraction of the light falling upon it is cut off by the sheet?

16. Describe some way of comparing the candle-powers of two different sources of light.

How would you estimate by experiment the percentage of the light available which is lost by a person working in a room with a dirty window?

17. A candle is placed inside a box in a dark room. A small hole is cut in one side of the box, and a sheet of paper is held a short distance in front of the hole. Describe and explain the appearance seen on the paper.

18. A candle flame is placed on one side of an opaque screen in which there is a small hole Explain—and illustrate your explanation by a diagram—why it is that if a piece of white paper be held on the other side of the screen an image of the candle is seen on it.

Is the result affected by (i) the shape, (ii) the size of the hole?

19. Describe a method by means of which the velocity of light has been determined

CHAPTER XIX.

REFLECTION AT PLANE SURFACES

Reflection and refraction.—When rays of light fall upon the surface of any medium, some may be reflected back from the surface, and others may be transmitted through the medium. If transmitted, they may be more or less absorbed, as in the case of a translucent medium, or as in the case of coloured water or glass, which rapidly absorb light of certain colours but readily transmit those of other colours. As a rule, the transmitted rays are **bent**, or **refracted**, away from their original path.

Reflection of light rays may happen in two ways, either **regularly** or **irregularly**. In the first case, they are turned back according to simple rules, while in the second there is no uniformity about the direction of reflection. In Fig. 163 the surface AB of a medium which is denser than air is supposed to be a perfect plane, and it gives rise to regular reflection; the surface BC is uneven, and the reflection from it is irregular. The diagram shows also how some of the rays may be transmitted through the denser medium.

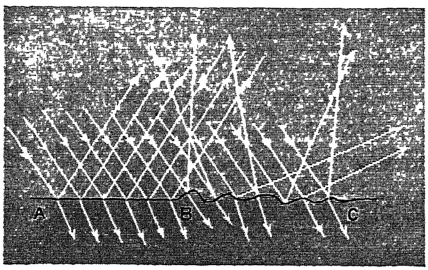


FIG. 163.—Regular and irregular reflection

The page on which this explanation is printed appears to be white because—owing to the roughness of the paper—of the

irregular reflection of the light which falls upon it. Or, if we powder a sheet of glass, the powder seems to be white for a similar reason; there are, in these and similar cases, many surfaces formed from which irregular reflection takes place.

Laws of reflection of light.—Light is reflected regularly from a plane mirror—that is, a flat reflecting surface. Such a mirror can be made from a variety of substances, but the most common is bright metal or silvered glass.

The angle between the path of the incident ray and the normal to the mirror (that is a perpendicular to the mirror at the point where the ray strikes it) at the point of incidence is termed the **angle of incidence**. The angle between the normal and the path of the reflected ray is termed the **angle of reflection.**

There is a definite connection between the angles of incidence and reflection, and it can be expressed as follows:

The paths of the incident and reflected rays are (i) in the same plane as the normal at the point of incidence, (ii) at equal angles on opposite sides of the normal.

It can be proved by experiment also that when a wave strikes a reflecting surface *normally*, *i.e.* having travelled along the normal, it is reflected back upon the same line.

EXPT. 172.—**Pin method of proving laws of reflection.** Fasten a sheet of white paper on a drawing board, and support in a vertical position on the paper a strip (about 2 inches by 1 inch) of good plane mirror AB (Fig. 164). (The mirror may be supported by fixing a small wooden cube to the back of the mirror with wax.) Fix two pins, P and Q, vertically in the board and approximately in the positions shown. View the images of these pins by looking in the direction SR, and move the eye until the image of Q overlaps that of P. Keep the eye in this position, and insert two other pins at points such as R and S, and so that these pins together with the images of P and Q all appear to be in one straight line.

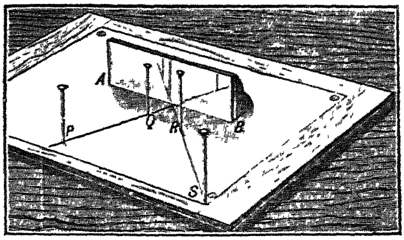


FIG. 164.—Pin method of determining the laws of reflection of light.

Draw a pencil line at the back of the mirror. Remove the mirror; draw the lines PQ and SR and produce them until they meet (Fig 165). They should meet at a point O approximately on the line of the silvered surface. Draw a line ON normal to the mirror at O. Measure the angle of incidence PON, and the angle of reflection SON, and compare them. Repeat the experiment two or three times, with different angle of incidence in each case

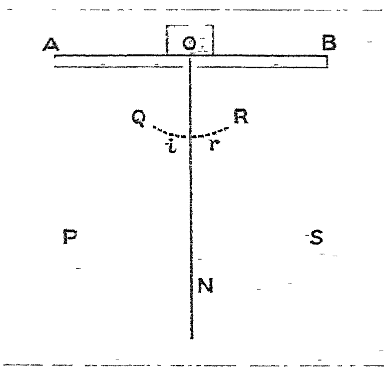


FIG. 165.—Construction to illustrate laws of reflection of light

Equality of distances of object and of image from a plane reflecting surface.—The two rules of reflection enable the formation of an image by a plane mirror to be understood easily. By a simple geometrical construction it can be shown that the image of a point is at the same distance behind a plane mirror that the point itself is in front of the reflecting surface.

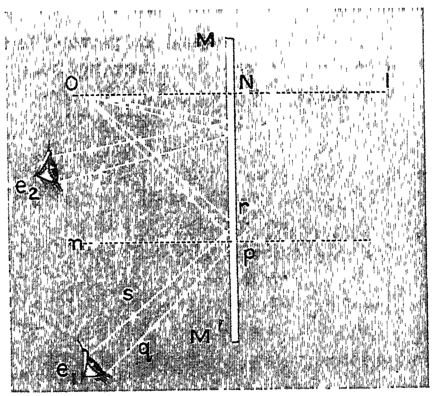


FIG. 166 —Reflection by a plane mirror.

Let MM′ (Fig. 166) be a horizontal section through a plane mirror, and O a bright point like the head of a pin. Let O*r* and O*p* be any two rays from O which are reflected by the mirror along the paths *pq* and *rs*. When an eye is situated at e_1 these rays give rise to the impression that the source of the rays is at some point I behind the mirror. Draw *pn*, a normal to the mirror at the point *p*.

Since $\angle Opn = \angle qpn$,
then $\angle Opr = \angle qpM' = \angle Ipr$.
Similarly $\angle OrM = \angle IrM$;
therefore $\angle Orp = \angle Irp$

Hence, the triangles Orp and Irp are equal; and $Or = Ir$. Join the points O and I by a line passing through the mirror at N. Then, in the triangles OrN and IrN, the sides Or and Ir are equal, the base Nr is common, and $\angle OrN = \angle IrN$. Hence, the triangles are equal in all respects (*Euc.* I. iv.). Therefore,

$$ON = IN; \text{ and } \angle ONr = \angle INr$$

Hence, **the image is situated on the normal to the mirror drawn from the source of the rays; and its distance behind the mirror is equal to that of the object in front of the mirror.**

Fig. 166 represents also how the image will appear to be situated at the same point I when the eye is in any other position, such as e_2.

The above result applies equally to the formation of the images of objects, which can be regarded as accumulations of small material particles to which the construction given already for a point may be applied.

EXPT. 173—**Image and object** Support a strip of plane mirror AB (Fig. 167) in a vertical position on a sheet of paper, and fix a pin vertically at O. View the image of the pin in the direction PQ, and insert pins at P and Q. Similarly, view the image in the direction RS, and insert pins at R and S. The image is situated somewhere along the line PQ produced, and also somewhere along the line SR produced. Hence it can only be at the point where these lines intersect. Remove the mirror, and produce the lines PQ and SR to meet at I. Draw normals to the mirror from I and O. These normals should be in the same straight line, and equal in length

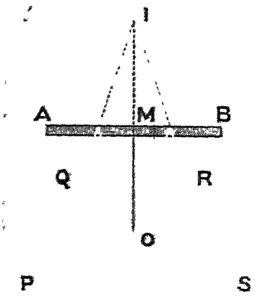


FIG. 167.—Expt. 173.

EXPT. 174—**Second method of finding an image of an object.** Use the same mirror as before. If it be 1 inch high, use pins about 2 inches long. Insert a pin at O (Fig. 167), and view its image just to one side

of the normal. Fix a pin O′ in the paper behind the mirror so that the upper part of this pin may appear to be a continuation of the image of the pin at O Move the eye slightly to the right and then to the left; if the upper part of O′ appears to move relatively to the image of O in the *same* direction as the eye, then O′ is *too far* from the mirror. Adjust the position of O′ until, from whatever direction it is viewed, it always appears to be continuous with the image of O. *The pin O′ then occupies the position of the image of* O

This second method of determining the position of an image of an object is known as the **Parallax method.** Parallax may be defined as the apparent change in the position of an object due to a change in the position of the observer. It may be demonstrated thus Place two rods, A and B, vertically, in line with the eye, and with B just behind A. If the eye is moved to the *right*, then B appears to move to the *right* of A (or, it may be said that A appears to move to the *left* of B). If the eye is moved to the *left*, then B appears to move to the left. **The further object appears to move in the same direction as the observer's eye.** If B is vertically over A, then they appear to be in the same straight line, whatever the direction of view may be.

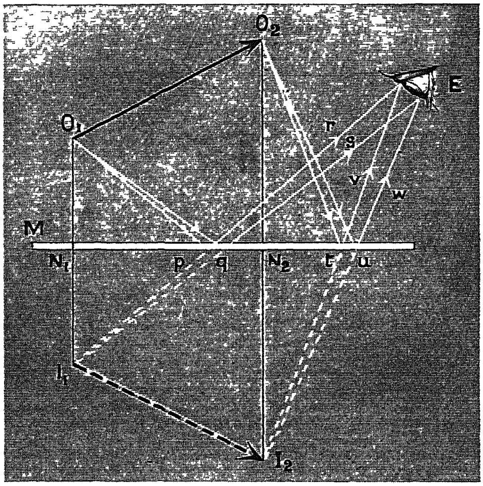


FIG. 168 —Image of an extended object, viewed by means of a plane mirror.

Image of an object in front of a plane mirror.—Let M (Fig. 168) represent a plane mirror, and O_1O_2 an object placed in front of the mirror. From O_1 and O_2 draw normals, O_1N_1

and O_2N_2, to the reflecting surface, and produce these to points I_1 and I_2, such that $N_1I_1 = N_1O_1$, and $N_2I_2 = N_2O_2$. Then I_1I_2 is the image of the object O_1O_2. The path of the pencil of rays which originates from O_1 and enters an eye situated at E is obtained by drawing the pencil (I_1pr and I_1qs) which *appears* to originate from the point I_1, thus locating the points p and q on the reflecting surface at which the rays from O_1 are reflected. Join O_1p and O_1q. The rays O_1pr and O_1qs represent the pencil of rays by which the image of O_1 is seen. In a similar manner, the rays O_2tv and O_2uw represent how the rays from O_2 are reflected so as to give the image at I_2.

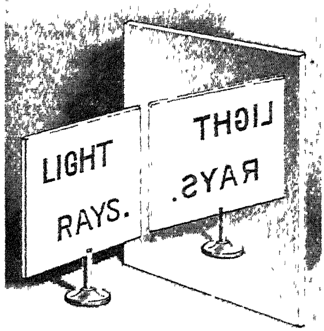


FIG. 169.—Lateral inversion due to a plane mirror.

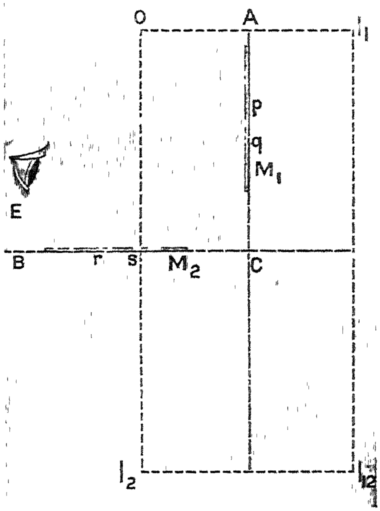


FIG 170.—Multiple images, due to two mirrors at 90°.

It is evident from this diagram, that an object, when viewed by means of a plane mirror, undergoes **lateral inversion**: the left-hand side of the object when viewed directly, appears to be the right-hand side when viewed by means of the mirror. This result is rendered more apparent when a page of printed matter is viewed in a mirror: each letter, and the sequence of the letters, is reversed sideways (Fig. 169).

Images formed by inclined mirrors.—Fig. 170 represents two plane mirrors, M_1 and M_2, placed vertically with their reflecting

surfaces coinciding respectively with two lines, AC and BC, which are at right angles to each other. When a luminous point O, situated within the angle ACB, is viewed by an eye at E, three images are visible. The images I_1 and I_2 are obtained each by *one* reflection, from the mirrors M_1 and M_2 respectively. The third image I_{12} is obtained by rays which are reflected *twice* before reaching the eye This third image may be regarded either as an image of I_1 obtained by means of the mirror M_2,

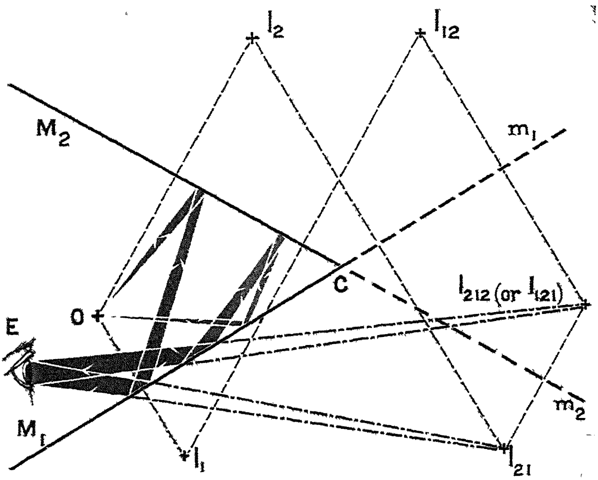


FIG 171 —Multiple images in two mirrors inclined at 60°.

or as an image of I_2 obtained by means of the mirror M_1. paths of the rays which give rise to this image, as seen eye at E, are indicated by the lines O*pr* and O*qs*.

When the angle between the mirrors is diminished, the nu of images is increased. It can be proved that when the angle is $\theta°$, the number of images is $\frac{360°}{\theta°} - 1$; thus, when the angle is 60°, the number of images is 5.

Fig. 171 indicates the position of the five images, of a luminous point O, obtained by means of two mirrors CM_1 and CM_2 inclined at 60°. It is important to remember that any image gives rise to a second image when it is situated in *front* of a reflecting surface or in front of the surface produced. For this

reason the reflecting surfaces are produced in the directions Cm_1 and Cm_2. The images I_1 and I_2 are given by single reflections from the mirrors CM_1 and CM_2 respectively. The image I_1 gives rise to a second image I_{12} by reflection from the mirror CM_2, and the image I_2 gives rise to a second image I_{21} by reflection from the mirror CM_1; in both these cases, the rays of light undergo *two* reflections before reaching the eye E, and the diagram indicates the path of the rays which give rise to the image I_{21}. Finally, the images I_{21} and I_{12} give coincident images at I_{212}, the former by means of the mirror CM_2, and the latter by means of the mirror CM_1; the path of the rays which give rise to this image are shown in the diagram, and it will be noticed that the rays undergo *three* reflections before reaching the eye. No further images are generated, since I_{212} is situated *behind* the reflecting surfaces of both mirrors. It is interesting to bear in mind that the object and all the images are situated on the circumference of a circle described round C as a centre.

EXPT 175.—**Positions of images formed by inclined mirrors.** Fasten a sheet of paper on a drawing board, and draw two lines inclined at 60°. Place two strips of mirror, similar to those used in Expt 172, so that their reflecting surfaces coincide with these lines. Fix a pin vertically at any point such as O (Fig. 171). In the first instance, count the number of images. Determine the position of each image by the method of parallax. With centre C, and radius CO, describe a circle: observe whether each image is situated on the circumference. Make a careful diagram showing the paths of the rays giving rise to at least two of the images, assuming the eye to be placed in any given position.

Rotation of a mirror.—When a mirror rotates, the angle through which the reflected ray moves is twice the angle through which the mirror moves.—This fact is utilised in the **Sextant**, an instrument used by surveyors and navigators to measure the angle subtended by two distant objects.

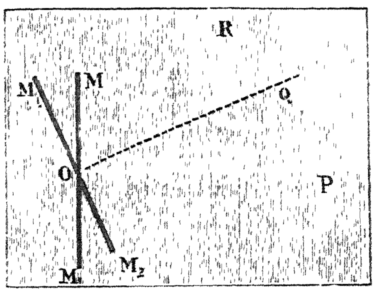


FIG. 172.—Effect of rotating a mirror.

Knowing the laws of reflection, the effect of rotating a mirror can be arrived at very easily by simple geometry. Thus, let MM

(Fig. 172) be a plane mirror, capable of rotation round an axis at O, and let PO be a ray of light incident on the mirror at O and normal to its surface. The ray is reflected back along its previous path.

If the mirror be rotated round O into the position M_1M_2, the angle through which it is rotated is equal to POQ, since this is the angle between the normals to the mirror in its initial and final positions.

When the mirror is at M_1M_2, the reflected ray is represented by OR; and, by the rotation of the mirror, the reflected ray has been rotated through the angle POR.

By the fundamental law of reflection,

$$\angle QOR = \angle POQ,$$

or

$$\angle POR = 2\angle POQ.$$

Hence, the angle through which the reflected ray moves is twice the angle through which the mirror moves.

EXERCISES ON CHAPTER XIX.

1. State the two laws in accordance with which a ray of light is reflected by a smooth surface, and describe experiments by which you would demonstrate the truth of each of these laws.

2. What is an inverted image? If the capital letter F were drawn on paper and held in front of a mirror, how would you have to draw the letter on the paper and how hold the paper, in order that the image of the letter in the mirror should present its ordinary aspect?

3. By moving a fragment of looking-glass, a boy finds that he can throw images of the sun up and down the walls and ceiling of a room. Where must he stand to be able to do this? Show by a diagram that the angle through which the image moves is twice as large as the angle through which the boy moves the glass.

4. What deviation is produced by reflection at a plane surface when the angle of incidence is 60°?

5. Make a measured drawing showing the positions of all the images of a luminous object placed between two plane mirrors inclined at 45°.

6. Two plane mirrors are inclined at an angle of 60°; give a carefully drawn diagram showing the position of the images of a luminous object placed so that the plane through it and the line of intersection of the mirrors makes an angle of 45° with one of the mirrors.

7. A mirror hangs on one of the walls of a room; show by means of carefully drawn diagrams that an observer will see by reflection more and more of the room behind him as he approaches the mirror.

8. State the laws of reflection.

Prove that a man can see the whole of his person in a mirror the length of which is half his own height.

9. Draw a diagram to show what images of an object, placed between two plane mirrors that make an angle of 50° with one another, are formed by reflection

10. How would you show by experiment that the rays from a luminous point proceed after reflection by a plane mirror as if they come from a point as far behind the mirror as the luminous point is in front?

11. A horizontal beam of parallel light is reflected by a vertical plane mirror. The mirror, remaining vertical, is turned through a small angle What is the relation between the angle through which the mirror is turned and the angle between the initial and final directions of the reflected beam? Give reasons.

12. Two mirrors are placed at right angles to one another In a plane at right angles to the mirrors an arrow is placed, lying on the bisector of the angle between the mirrors Draw a diagram to show the positions of the images of the arrow formed by the mirrors.

CHAPTER XX.

REFRACTION AT PLANE SURFACES

Refraction of light—Up to the present, rays of light have been supposed to be moving through a uniform medium When this is so, as has been seen, light travels in straight lines, and, if it meets a reflecting surface, it is turned back, according to the laws described in the last chapter. When, however, the light passes from one medium into another of a different optical density, the propagation of the wave is usually no longer rectilinear: except in those cases where the incident ray is normal to the surface of separation between the two media, the passage from one medium into the other is accompanied by a bending of its path This bending is known as **refraction**, and the ray is said to be **refracted**.

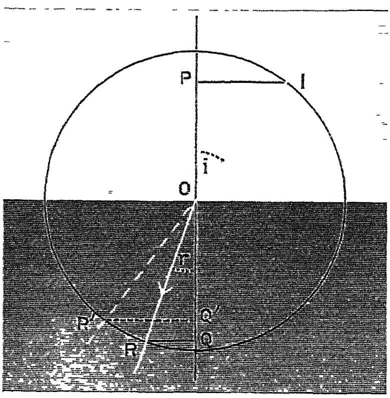


FIG 173—Law of refraction

Laws of refraction.—In Fig 173 the shaded lower part of the diagram represents a denser medium than the unshaded upper portion. The word denser is used here, and in similar connections, to mean optically denser, and must not be confused with what has been said of the density of bodies in Chapter IV. Let

IO represent a ray passing from the rarer to the denser medium, or the ray incident on the surface of the denser medium at O. The angle IO makes with the normal at O is the **angle of incidence** The ray is bent; instead of continuing its course in a straight line along OR′, it is refracted and travels in the direction of OR, which represents the refracted ray, the angle ROQ being the **angle of refraction** The angle ROR′, which represents the amount the ray has been turned out of its original direction, is termed the **angle of deviation.** Let a circle be described with the centre O and any convenient radius, and from the points where it cuts the incident and refracted rays, let perpendiculars be drawn to the normal as in Fig 173. A perpendicular is also dropped from the point R It is clear from geometry that the perpendicular R′Q′ is equal to the perpendicular IP. The ratio between the lengths of R′Q′ and RQ is constant for the same two media, *e.g* air and water, whatever the angle of incidence This ratio is called the **index of refraction** of the denser medium, or the **refractive index.** and it is usually denoted by the symbol μ Its value for air and water is about $\frac{4}{3}$, for air and glass approximately $\frac{3}{2}$, depending upon the kind of glass.

The laws of refraction may be expressed thus·

1 **The incident and refracted rays are on opposite sides of the normal at the point of incidence, and in the same plane as the normal.**

2. **If a circle be described about the point of incidence, and perpendiculars be dropped upon the normal from the intersections of this circle with the incident and refracted rays, the ratio of the lengths of these perpendiculars is constant for any two given media.**

The student who is familiar with the elements of trigonometry will find the following statement of the second law to be more convenient· **The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for any two given media** This may be written $\mu = \sin i / \sin r$, where i and r are the angles of incidence and of refraction respectively, and μ is the refractive index It can be proved experimentally that if the direction in which the light is travelling be reversed, the path of the ray remains unaltered. Consequently, Fig. 173 may be used to represent the refraction of a ray proceeding from a dense into a rarer medium; and it is evident that the refracted ray is bent away from the normal. If, in this case,

i' and r' represent the angles of incidence and refraction respectively, then

$$\frac{\sin i'}{\sin r'} = \frac{\sin r}{\sin i} = \frac{1}{\mu}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

or

$$\mu = \sin r'/\sin i'.$$

EXPT. 176 — **The refractive index of glass.** Spread a sheet of white paper upon a drawing board, and draw a straight line on the paper. Lay a thick glass slab (about $10 \times 8 \times 1\cdot5$ cm ; *rectangular edges*) on the paper with its edge AB (Fig. 174) upon this line. Close to the edges CD and AB insert pins P and Q, so that the line PQ is oblique to AB Look at P *through the block*, and move the eye until Q covers the image of P Insert a third pin, R, in such a position that it appears to be in line with P and Q. Remove the block , draw the normal MM′ , with Q as centre, describe a circle, 4-5 cm. radius. Drop perpendiculars LM and KM′, and measure their lengths. The ratio LM KM′ is the *refractive index* (μ) of glass.

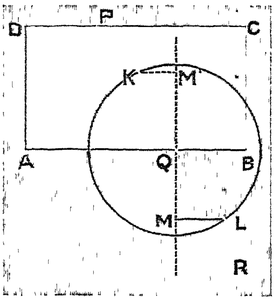


FIG 174 — Refraction by glass.

Replace the block, alter the position of the pin P, and make another determination of μ.

EXPT. 177 — **The refractive index of water.** In Fig 175 E is the vertical edge of a strip of paper fastened outside a glass trough ABCD, containing water. Look at this edge, through the trough of water, and trace the path of the rays, outside the trough, by means of two pins, Q and R. Determine the refractive index by the same construction as used in previous experiment.

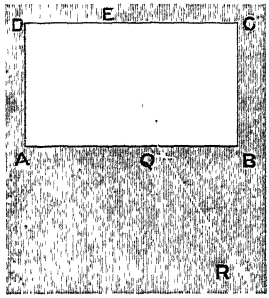


FIG 175.—Refraction by water.

In speaking of the *refractive index* of any material it is always assumed that the ray of light proceeds *from* air *into* the material, and not in the reverse direction

It will be shown subsequently that when light is proceeding from a dense into a rarer medium there is no emergent ray if the

angle of incidence exceeds a certain limit; in such a case all the light is reflected back into the denser medium.

Geometrical construction for refraction at a plane surface.—Let IO (Fig 176) be an incident ray at O, passing into an optically denser medium of which the refractive index is $\frac{4}{3}$. With O as centre describe a circle, of any radius, cutting the incident ray at I. From I draw IN perpendicular to the refracting surface. Divide ON into *four* equal parts, and measure off ON′ equal to *three* of these parts. Draw N′R perpendicular to the refracting surface, and cutting the circle at R. OR is the direction of the refracted ray.

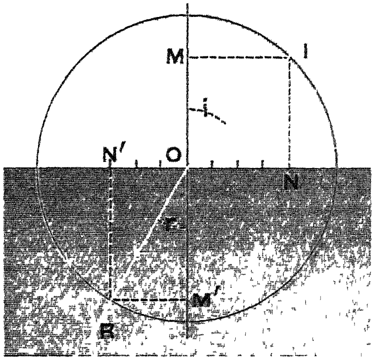


FIG 176—Refraction of incident ray passing into an optically denser medium.

The correctness of this construction is evident, since, by definition, the refractive index is equal to the ratio of the perpendiculars IM and RM′; and

$$\frac{IM}{RM'} = \frac{ON}{ON'} = \frac{4}{3}$$

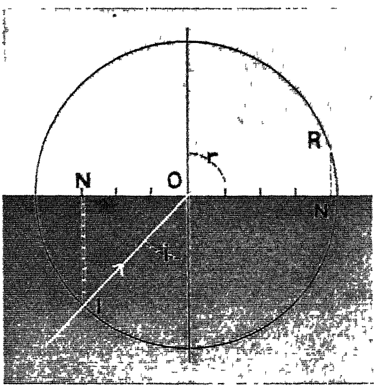


FIG 177—Refraction of incident ray passing into a less dense medium.

Fig. 177 represents the construction when the ray passes from the denser into the less dense medium. In this case ON is divided into three equal parts, and ON′ is measured off equal to four of these parts.

When the angle i is increased slightly, so that N′ coincides with the circumference of the circle, the refracted ray will coincide with the surface of separation. Any further increase in the angle of incidence will result in all the light being reflected back into the denser medium. This

limiting value for the angle of incidence is termed the **critical angle** for the denser medium

The critical angle.—When a ray of light is proceeding from a dense into a rarer medium, the maximum value which the angle of refraction can have is 90°. Since $\sin 90° = 1$, the greatest possible value of the angle of incidence such that a refracted ray may be formed is given by the equation

$$\frac{\sin i'}{\sin 90°} = \frac{1}{\mu},$$

or

$$\sin i' = \frac{1}{\mu}.$$

Hence, for any dense medium **the critical angle is the angle of which the sine is equal to the reciprocal of the refractive index of the medium.** The critical angle for water is about 49°, and for crown glass 42°.

EXPT. 178—**The critical angle for glass.** Fasten a sheet of paper on a drawing-board, and place a glass prism ABC (Fig 178) on the paper Trace the outline of the prism on the paper Fix a pin O vertically in the board and touching the face AC of the prism Place the eye to the right of B and look along the edge BA; move the eye gradually towards C and look for an image of the pin O reflected from the surface AB. When the line of sight corresponds with a direction $p_1 p_2$ the image will become dim; and if the eye be moved slightly towards C the image will disappear. Mark with pins, p_1 and p_2, the direction in which the image is barely visible. Remove the prism; join the points p_1 and p_2, and produce the lines so as to cut BC at Q. Since AB acts as a mirror, the rays after reflection will proceed as though coming from a point i which is behind AB and at a distance equal to that of O in front of AB. Join iQ. The intersection of this line with AB gives the point P from which the rays are reflected Join OP. Thus

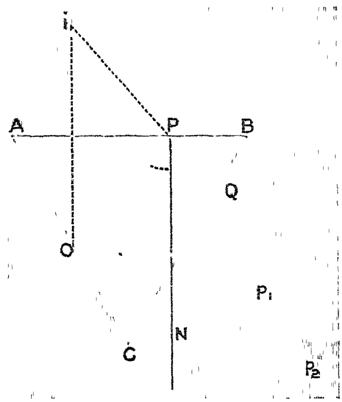


FIG 178—Determination of the critical angle for glass

OPQ, the path of the rays within the glass, is obtained. Draw a normal PN at P. Then OPN is the critical angle.

In the preceding experiment, the rays of light by means of which the pin is seen when viewed from the direction p_1p_2 are said to undergo **total internal reflection.** This phenomenon suggests how a glass prism, the angles of which are 45°, 45°, and 90°, may be used as a mirror (Fig. 179). The prism is placed so that the incident rays are normal to one of the mutually perpendicular faces The angle of incidence on the hypotenuse is therefore 45°; and, as this is greater than the critical angle, the rays are totally reflected and emerge from the prism normally to the third face. This device serves as a more perfect mirror than a glass plate which is silvered at the back, since, in the latter case, some of the light is reflected from the front surface, and some from the silvered surface, resulting in the image being made more or less indistinct

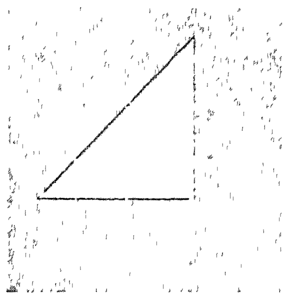


FIG 179 —Total internal reflection by a prism.

Fig 180 represents rays of light proceeding from a luminous point A beneath the surface of water Only those rays the

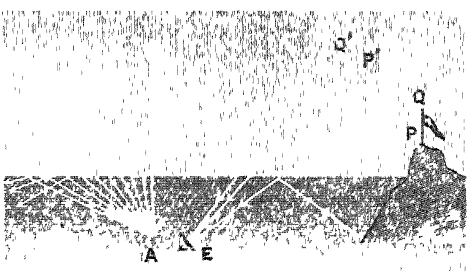


FIG. 180 —Effects due to total internal reflection in water.

angle of incidence of which is less than the critical angle emerge above the surface of the water; all others are totally reflected into the water. Similarly, to an eye situated at E below the

surface, objects above the water appear to be concentrated within a cone, of which the angle is equal to twice the critical angle; thus, the point P will appear to be situated at P′, and the point Q at Q′. Outside the boundaries of this cone objects beneath the surface of the water will be visible.

Refraction through a transparent plate.—The term plate is used to describe a slab of material with parallel faces. In Fig. 181, SP represents a ray incident at P on the side of a plate ABCD. Within the plate the ray is deviated along the path PT. On emerging from the plate the ray is deviated along the path QR, the deviation in this case being equal in amount, but opposite in direction, to the first deviation at P. Hence **in a transparent plate there is no final angular divergence, but the path of the ray is laterally displaced.** It is evident that the angle *e* of emergence must be equal to the angle *i* of incidence, since the angles *r* and *r′* are equal.

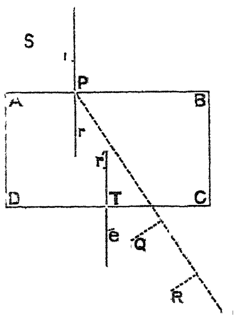


FIG. 181.—Refraction through a transparent slab.

EXPT. 179.—**Parallelism of emergent and incident rays.** Place a slab of glass ABCD (Fig. 181) upon a sheet of paper and trace its outline with a pencil. Insert a pin P near the edge AB. View the pin in an oblique direction, such as QR, and insert pins Q and R in line with the image of P. Finally, insert a pin S in line with R, Q, and P. Remove the slab; draw a line through S and P, also through R and Q cutting the slab at T. Join PT. Draw perpendiculars from Q and R on to SP produced. Measure these perpendiculars and note whether they have the same length.

The image of a point by refraction at a plane surface.—So far we have considered the refraction of a single ray only. In order to find the relative positions of an object and of its image formed by refraction at a plane surface we must consider the refraction of a small *pencil*, or assemblage, of rays.

Suppose O (Fig. 182) to be a luminous point situated in the denser of two media (X and Y), and let OCD be a narrow pencil of rays with its axis ON normal to the surface separating the two

media The paths of the rays after deviation appear to an eye looking downwards along the normal to diverge from a point I. The paths of the rays can be determined by geometry, if the refractive index between the two media is known.

If μ be the refractive index when the rays pass from X into Y, then $1/\mu$ is the refractive index when the rays pass from Y into X. By drawing a normal at C, it is evident that the angle CON is the angle of incidence of the ray OC, and that CIN is the angle of refraction. Then

$$\frac{1}{\mu}=\frac{\sin \mathrm{CON}}{\sin \mathrm{CIN}}=\frac{\mathrm{CN}}{\mathrm{CO}}\Big/\frac{\mathrm{CN}}{\mathrm{CI}}=\frac{\mathrm{CI}}{\mathrm{CO}},$$

or

$$\mu=\frac{\mathrm{CO}}{\mathrm{CI}}.$$

But when the pencil is *very narrow*, then CO and CI are approximately equal to NO and NI respectively. Hence

$$\mu=\frac{\mathrm{NO}}{\mathrm{NI}}$$

Or, **the ratio between the true distance and the apparent distance of the point below the surface of separation is equal to the refractive index.**

Hence, as the refractive index of water is 4/3, an observer looking straight down upon the surface of water sees an object in the water at three-quarters its actual distance from the surface.

This result is only true for small pencils of rays which are nearly *normal* to the surface. If the pencil entering the eye strikes the separating surface obliquely, the apparent position of the point is altered considerably, thus, in Fig. 182, the rays O*c* and O*d* are refracted so as to appear to diverge from a point I_1.

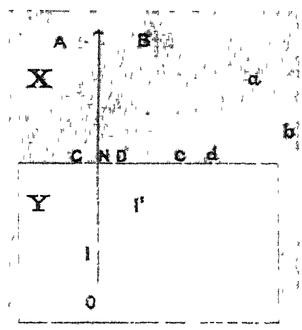


FIG. 182.—Apparent change of position caused by refraction.

EXPT. 180.—**Refractive index of water, by measurement of apparent depth.** Fill a deep glass cylinder with water, and drop to the bottom of the cylinder a small opaque object (*e.g.* a short piece of thick copper wire) Clamp a narrow

glass tube, drawn out to a jet, vertically above the surface of the water. Connect the tube to the gas supply, fix it so that the jet is horizontal, and light the gas at the jet adjusting the supply so that a *small* yellow flame is obtained View the arrangement vertically downwards and observe whether there is any parallax between the immersed object and the image of the flame reflected from the

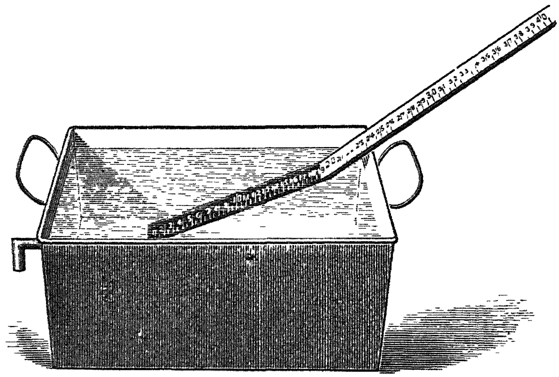


FIG 183 —Apparent bending and shortening of an immersed rod.

surface of the water. Adjust the distance of the jet above the water until there is no parallax. In this position the apparent depth of immersion of the object must coincide with the position of the flame's image, and the distance of the latter below the surface is equal necessarily to the vertical height of the jet above the surface Take the necessary measurements, and calculate the refractive index

This apparent displacement of a source of light immersed in a denser medium explains why a straight rod partly immersed slantwise in water (Fig. 183) appears to be bent and shortened just at the surface of the water, when the stick is viewed from one side

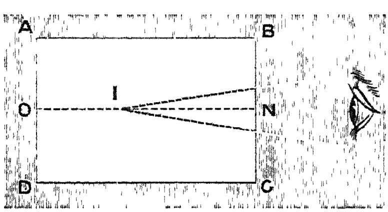


FIG. 184 —Expt 181

EXPT. 181.—**Refractive index of glass, by locating the image due to refraction at a plane surface.** Lay a slab of glass ABCD (Fig. 184) on a sheet of paper, and fix a pin vertically at O in contact with one edge of the

glass. View the pin through the glass from a position along the normal ON and beyond the opposite edge. That portion of the pin which is seen through the glass will appear to be at some point I. Determine the position of this point by holding another pin vertically with its point touching the glass and moving the point to-and fro along ON until a position is found where the movable pin appears to remain continuous with the image of O when the eye is moved slightly to right and left.* Measure the distance IN and ON, and calculate the refractive index of glass.

Refraction through a prism.—When a wedge-shaped piece of glass, or a prism, as it is called in optics, is interposed in the path of a ray of light from a small hole in the cap of a lantern, it is easy to see, by watching the image of the hole on a screen, that the image moves in a direction towards the base of the prism. This is because the ray is bent by its passage through the prism, so that on its emergence from the glass it continues in a new path inclined towards the base of the prism. The amount of bending experienced by the ray of light depends, among other conditions, upon the angle between the inclined sides of the prism meeting in its edge, or **the angle of the prism**, as it is called.

In Fig. 185 let the triangle ABC represent a section of the prism at right angles to its faces such as we should see by looking at the

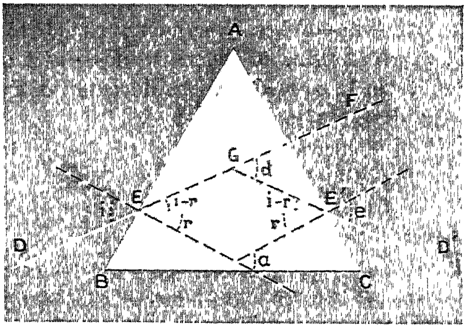


FIG. 185.—Refraction of a ray of light through a prism

end of it. Suppose DE is a ray of light striking the face AB of the prism. The light on entering the prism passes from the air

* For description of this parallax method, see p. 252

into the glass, or *from a rarer into a denser medium*, and is bent *towards* a line drawn perpendicular to the face of the prism at the point where the ray of light strikes it. It consequently travels along the line EE′ until it reaches the face AC of the prism. Here it passes from the glass into the air, *i e. from a denser into a rarer medium*, and is, in such circumstances, bent *from* the perpendicular, and travels along the line E′D′. In every such passage through a prism it is noticed that the light is always bent or refracted towards the thick part of the prism

EXPT 182 —**Pin method of tracing deviation by a prism.** Stand a prism upright, that is, upon one of its ends, upon a piece of white paper. Stick two pins into the paper in positions such as D and E (Fig 185), E being as close as possible to the face of the prism. Place two more, E′, D′, on the opposite side of the prism, so that the four appear in a straight line when looking through the prism Draw the outline of the prism ABC, and then take away the prism and the pins and connect the pin-holes as shown in the diagram. It will be found that the ray is bent towards the base of the prism both when it enters and emerges. Measure the angle FGD′, which is termed the *angle of deviation (d)*; also measure the angle of the prism at A.

Repeat the experiment, using a different angle of incidence Note whether the deviation is the same as before; and note also that the path of the ray within the prism is not necessarily parallel to the base BC of the prism

Minimum deviation.—When experiments similar to Expt. 182 are carried out with prisms of different angles and of different kinds of glass, it can be proved that the deviation of a ray passing through a prism depends upon

(i) the refracting angle of the prism,
(ii) the material of the prism,
(iii) the angle of incidence of the ray entering the prism, and
(iv) the nature of the incident light.

With a given prism there is one angle of incidence for which the angle of deviation is a minimum, and it can be proved, both theoretically and by experiment, that the deviation has a minimum value when the angle of emergence is equal to the angle of incidence (or, in other words, if the two sides of the prism are equal in length, when the path of the ray within the prism is parallel to the base of the prism).

EXPT. 183.—**Minimum deviation and calculation of the refractive index of a prism.** Place a prism ABC (Fig. 185) upon a sheet of paper, and fix two pins in positions corresponding to D and E View the two pins through the prism in the direction D′E′. Slowly rotate the prism round the point A, and in either direction notice that the line D′E′ of the image varies, and that there is *one position* of the prism in which D E′ approaches most nearly to the direction GF of the incident ray. Mark the emergent ray by means of pins D′ and E′, and trace the outline of the prism on the paper. Remove the prism and pins.

Draw the incident ray DE and the emergent ray E D′ Join EE′ The path of the ray is represented by DEE′D′. Note whether EE′ is parallel to the base BC. Produce DE to any point F, and D′E′ to G. Measure the angle of deviation (d). Draw normals at the points E and E′, and measure the angles of incidence (i) and of emergence (e). The angle A of the prism is equal, necessarily, to the angle (a) between the normals at E and E′.

Notice that, from Fig. 185, the angle a is equal to twice the angle r of refraction; hence

$$r=\frac{a}{2}.$$

Also, since the angle e of emergence is equal to the angle i of incidence,

$$\angle GEE' = \angle GE'E$$

and $$d=2(i-r)=2i-a;$$

or $$i=\tfrac{1}{2}(d+a).$$

It has been explained previously that the refractive index (μ) is equal to the ratio $\sin i/\sin r$; hence

$$\mu=\frac{\sin i}{\sin r}=\sin\frac{d+a}{2}\Big/\sin\frac{a}{2}.$$

From the measurements obtained in the experiment calculate, by means of this formula, the refractive index of the glass prism.

Graphical diagrams.—The effect of refraction upon the course of a ray can be determined easily by the construction of graphical diagrams based upon the principles described already. It is only necessary to bear in mind that when a ray passes from one medium to another of different optical density, perpendiculars should be drawn to the normal at equal distances on the rays from the point of incidence and their lengths should be in the ratio of the refractive index In the denser medium the normal

must represent always the smaller number of the ratio given as the refractive index. The following examples illustrate the construction of two diagrams of this kind:

EXAMPLE.—1. A ray of light is incident at an angle of 50° to the surface of a glass plate which is 5 cm thick. The refractive index of the glass is 1·5 Determine graphically the path of the emergent ray, and measure the lateral displacement which the ray undergoes.

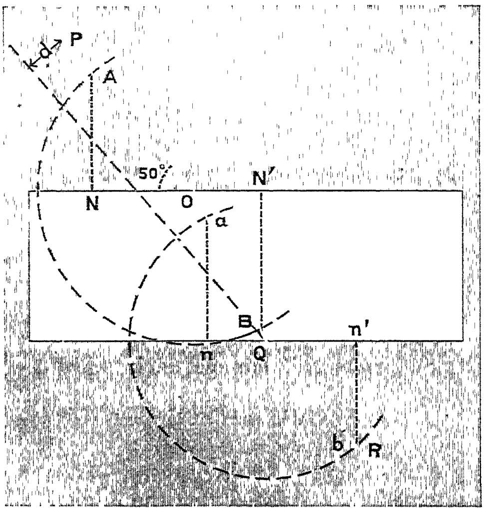


FIG. 186 —Geometrical construction for the path of a ray through a transparent slab

Let PO (Fig. 186) be the incident ray, and O the point of incidence. With centre O, and with any convenient radius, describe a circle which cuts the incident ray at A.

From A draw a normal AN to the surface of the glass. Measure ON, and mark off a distance ON′ such that ON $=(1{\cdot}5\times$ ON′). From N′ draw the normal N′B cutting the circle previously described at B. Join OB, and produce it if necessary to meet the lower surface of the plate at Q OQ is the *refracted ray.*

With centre Q, and with any convenient radius, describe a circle which cuts the ray OQ at *a*. From *a* draw the normal *an*. Measure Q*n*, and mark off a distance Q*n′* such that Q*n′* $=(1{\cdot}5\times$ Q*n*). From *n′* draw the normal *n′b* cutting the circle at *b*. Join Q*b*. Then Q*b* is the direction of the *emergent ray.*

The lateral displacement is determined by producing the ray QR backwards and measuring the perpendicular distance d between the rays QR and PO.

EXAMPLE—2. An equilateral hollow glass prism (with very thin walls) is filled with carbon bisulphide. Trace the path of a ray of light which falls on the prism making an angle of incidence of 60° on the side of the normal away from the apex of the prism. Measure the deviation. [Refractive index of carbon bisulphide = 1·7.]

In Fig. 187 let PO be the incident ray. Measure off, along the face AB, distances OD and OE such that OD = 1·7 × OE. From D and E draw normals to the face of the prism.

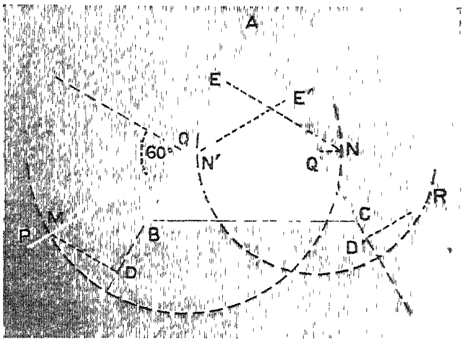


FIG. 187.—Geometrical construction for the path of a ray through a prism.

Let the normal from D cut the incident ray at the point M. With centre O, and radius OM, describe a circle. If this circle cuts the normal, drawn from E, at the point N, then the line ON is the path of the ray through the prism.

Let Q be the point of emergence. Measure off, along the face AC, distances QD′ and QE′, such that QD′ = 1·7 × QE′. From E′ draw a normal, intersecting the path OQ at N′. With centre Q, and radius QN′, describe a circle, cutting the normal from D′ at the point R. The line QR is the path of the emergent ray.

The angle of deviation is found to be 57° approximately.

EXERCISES ON CHAPTER XX.

1. A bright bead is placed at the bottom of a basin of water, and a person stands in such a position that he can just see it over the edge of the basin. While he is looking, the water is drawn off. How will this affect his view?

Draw a diagram showing the direction of a ray of light passing from the bead through the water and the air in each case.

2. A thick layer of transparent liquid floats on the surface of water. Trace the course of a ray of light from an object immersed in the water through the floating liquid to the air.

3. Describe an experiment to show the path of a ray of light which passes obliquely through a thick plate of glass. Illustrate your answer by a sketch in which you indicate clearly the path of the ray in the air before it enters the glass, in the glass, and in the air beyond the glass.

4. An upright post is fixed in the bottom of a pond which is three feet deep; the top of the post is three feet above the water. How will the post appear to an eye about the level of the top of the post and four or five feet away from it?

Draw a figure to illustrate your answer.

What will be seen as the eye moves further and further back from the post?

5. A fish swims in a glass tank; a person whose eye is above the level of the water seems to see two fish. Draw a diagram to illustrate this, and give any explanations you think necessary.

6. A vertical straight wire is viewed so that part is seen directly and part through a thick plate of glass held vertically. Describe the apparent changes in the position of the portion seen through the glass when this is slowly turned round a vertical axis

7. A glass cube is placed over a pencil mark on a sheet of paper. The mark is viewed through the cube. Explain, by means of a diagram, the apparent position of the pencil mark

8. What is meant by the refractive index of a substance? Being provided with a thick piece of glass from a box of weights, some pins, a sheet of white paper, a pencil and graduated ruler, how would you determine the index of refraction of the glass?

9. State the laws of the refraction of light. Explain why a tank of water viewed from above appears shallower than it really is.

10. Explain, by the aid of a diagram, why a pole appears to be bent when it is thrust into water in a slanting position.

11. A ray of light is incident at an angle of 30° on one face of a glass plate, 3 inches thick, the index of refraction being 1·5. Give a diagram to scale showing the refracted and emergent rays. Show that the emergent ray is parallel to the incident ray, and measure the perpendicular distance between them.

12. Explain the terms *critical angle* and *total reflection*. A glass cube of two inch edge stands upon a horizontal table; represent in plan, in a diagram drawn approximately to scale, the path of a horizontal ray of light which falls upon a vertical face of the cube and is afterwards totally reflected at a surface of contact of glass

and air. Assume the refractive index of the glass with respect to air to be $\frac{3}{2}$.

13. Two faces of a glass block are parallel and 3 inches apart; and a ray of light strikes one of them at an angle of incidence of 60°. The refractive index of the glass is 1·5. Show in a diagram the directions of the incident ray, refracted ray, and emergent ray. Measure the amount of displacement caused by the glass and record it.

14. The critical angle for a certain medium is 45°. What is its refractive index?

15. Trace the path of a ray of light which falls upon one of the perpendicular faces of an isosceles right-angled glass prism ($\mu = 3/2$) at an angle of 70° with that surface. Trace the ray until it emerges from that surface.

16. A layer of water, 2 inches deep, lies upon a slab of glass which is 1·5 inches thick and of which the lower surface is silvered. Trace the path of a ray which strikes the surface of the water at an angle of 45° ($_{air}\mu_{water} = 4/3$; $_{water}\mu_{glass} = 9/8$)

17. Find, by geometry, the critical angles for ice, glass and carbon bisulphide, having given that the refractive indices of these substances are 1·3, 1·5, and 1·7 respectively

18. Show, by geometry, why a ray of light striking one of the perpendicular faces of a right-angled isosceles glass prism along a normal cannot emerge from the hypotenuse face. Trace its path. ($\mu = 3/2$.)

19. ABCD is the square section of a glass cube ($\mu = 3/2$). P is a luminous point in AC produced, and 1 inch distant from C. PQ is a ray of light striking the side BC at R, so that ∠APR is 30°. Trace the path of the ray until it emerges from the glass. [Make AB = $3\frac{1}{2}$ inches.]

20. An equilateral hollow glass prism is filled with carbon bisulphide ($\mu = 1·7$). Trace the path of a ray which is incident upon the prism at 30° with the surface.

21. A ray of light is incident at an angle of 30° on one face of an equilateral prism. If the path of the light through the prism is parallel to the base, find the direction of the emergent ray, and the total deviation of the ray after passing through the prism.

22. A dot is made on a piece of paper, and a prism is laid on the paper over the dot. An eye in certain positions now seems to see two dots. Draw a diagram to explain this.

23. A hollow glass prism, and full of air, is immersed in a glass tank full of water. Make a diagram showing rays of light passing through both the water and the prism.

24. The minimum deviation produced by a hollow prism filled with a certain liquid is 30°. If the refracting angle of the prism is 60°, what is the index of refraction of the liquid?

25. Explain clearly how you would determine by experiment the index of refraction of a glass plate. Why is the apparent depth of a pool less than its actual depth?

26. State the laws of the refraction of light.

A ray of light, falling on one face of a plate of glass 10 cm in thickness, makes an angle of 60° with the normal. Determine the point on the other face at which the ray will emerge from the glass. [The index of refraction of the glass is 1·5]

27. State the laws of refraction of light

A ray of light is incident on the surface of water at an angle of 45°. Find by a diagram the direction of the refracted ray. [The index of refraction of water is 1·53]

28. A ray of light in a prism, which has an angle of 60°, makes an angle of 30° with the face of the prism Find the direction of the emergent ray. [The index of refraction of the glass is 1·4.]

29. State the laws of refraction of light

A ray of light is incident at 60° to the normal upon a polished glass surface. The refracted ray makes an angle of 90° with the reflected ray. Find, graphically or by calculation, the refractive index of the glass.

30. A plane mirror lies horizontally. The image of a point lying above the mirror is viewed by an eye looking at an angle of 45° with the mirror, the height of the eye above the mirror being twice the height of the point. Draw a diagram to indicate the path of two rays proceeding from the point to the eye.

Explain whether the eye would still see the image of the point in the same direction if the mirror lay at the bottom of a tank, and if water were poured into the tank (*a*) so as to just cover the point, and (*b*) so as to reach up to the eye.

31 Draw a diagram of a prism with an angle of 45°, and trace completely the course of a ray of light, which, lying in a plane at right angles to the axis of the prism, falls on the prism making an angle of incidence of 30° on the side of the normal away from the apex of the prism.

From your diagram measure the deviation. [Refractive index of the glass = 1·5.]

32. An equilateral hollow glass prism is filled with glycerine. Determine graphically the minimum deviation of a ray passing through the prism. [Refractive index of glycerine = 1 47.]

33. The refractive index of crown glass is 1·53, and its critical angle is 41° approximately. Determine geometrically the minimum angle of incidence of a ray of light which falls upon one face of an equilateral glass prism and undergoes total internal reflection from the other face.

CHAPTER XXI.

REFLECTION AT SPHERICAL SURFACES

Spherical mirrors.—A spherical mirror is a mirror having the form of part of the surface of a sphere. It may be either **concave** or **convex**, the former if the reflection takes place from the hollow side, the latter if from the bulging side.

Spherical mirrors may be considered as made up of an infinite number of very small plane facets having negligible curvature, and each such element may be regarded as a small plane mirror. It is known, from geometry, that every radius of a circle is at right angles to the tangent at the point where it cuts the circle Hence, the normals to all these plane facets will pass, like the radii of a circle, through one point which is the centre of the sphere from which the mirror has been derived. This point is called the **centre of curvature** of the mirror.

Figs 188 and 189 indicate how a concave mirror and a convex mirror respectively, each consisting of a number of small plane facets, will affect the path of the rays of a diverging pencil of rays originating from a luminous point O. For the sake of clearness each facet is represented with only one ray reflected from its middle point The path of each ray after reflection is obtained by drawing the normal to the reflecting surface, and by making the angle of reflection r equal to the angle of incidence i In the case of the concave mirror (Fig. 188) all the rays, after reflection, pass through a point I, which is called the **image** of the luminous point O; and the position of this image could be determined by holding a piece of white paper, so as to intercept the reflected rays, and altering its position until the brightly illuminated portion of its surface is restricted to a point. In the case of the convex

mirror (Fig. 189), the rays after reflection *appear* to proceed from a point I on the further side of the mirror. To an eye situated on the left-hand side of the mirror the image of the luminous point O will appear to be situated at I.

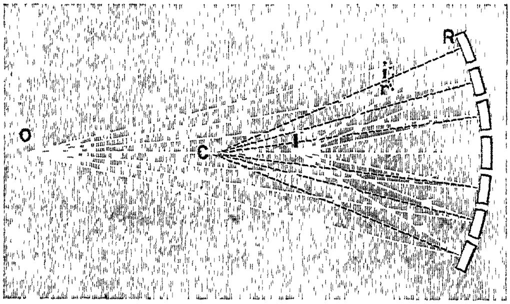


FIG 188.—The principle of a concave mirror.

It is evident that **when a luminous point is situated at the centre of curvature of a concave mirror all the rays of light from it are reflected back along the lines of incidence, and the image is formed in the same position as the luminous point.**

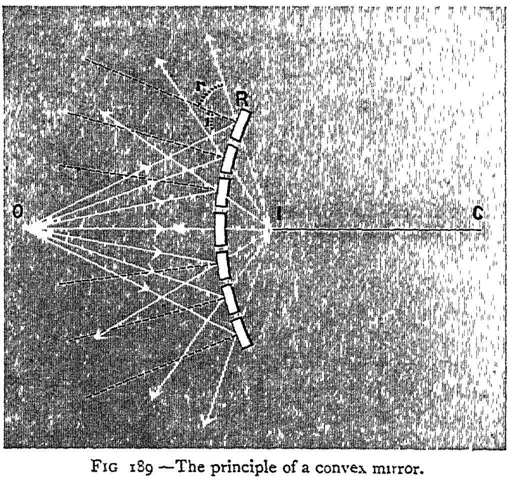


FIG 189.—The principle of a convex mirror.

In any spherical mirror, the distance from the centre of curvature to the reflecting surface is the **radius of curvature.** Thus,

in Fig 190, C is the centre of curvature and CM, CR, CM′, are all radii of curvature. MM′ is called the diameter or aperture of the mirror, and P, the centre of the reflecting surface, is often called the **pole** or **apex** of the mirror. A line going through the pole and centre of curvature is the **principal or optic axis** of the mirror, any other radius produced being a **secondary axis.**

Principal focus, and focal length.—(i) Let MPM′ (Fig. 190) be a concave mirror, of which C is the centre of curvature Let IR be a single ray of a very narrow pencil of rays parallel to the principal axis of the mirror. and let R be the point of incidence on the reflecting surface. Join CR. The line CR is normal to the surface of the mirror at R. Make the angle CRi equal to the angle IRC. Then, by the law of reflection, Ri is the path of the reflected ray Let Ri cut the principal axis at F. Then,

since $\angle CRI = \angle CRF$,

and $\angle CRI = \angle RCF$,

$\angle CRF = \angle RCF$.

Hence, FR = FC.

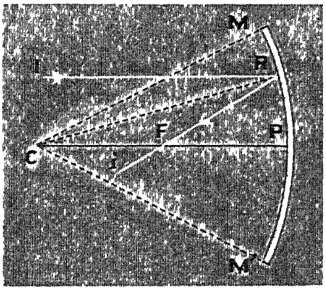


FIG 190 —Principal focus of a concave mirror

But, if the pencil of rays be very narrow, the point R coincides approximately with the point P; and, therefore, FR = FP approximately.

Hence, FP = FC.

The same reasoning will apply to all other rays in the pencil; and, therefore, **all the rays after reflection will pass through a point F which is midway between the centre of curvature and the pole of the mirror.**

(ii) Fig. 191 represents the corresponding case for a convex mirror, where IR is a single ray of a pencil parallel to the principal axis. If Cn is the normal to the reflecting surface at R, then $\angle IRn = \angle nRi$. But, by geometry, $\angle IRn = \angle FCR$, and $\angle NRi = \angle FRC$.

Hence, $\angle FCR = \angle FRC$,

and FC = FR.

But, since the pencil of rays is supposed to be very narrow, FR = FP.

$\therefore$ FC = FP.

Hence, **all the rays after reflection from a convex mirror appear to pass through a point F which is midway between the centre of curvature and the pole of the mirror.**

In each case the point F is termed the **principal focus** of the mirror, and the distance of this point from the mirror is termed

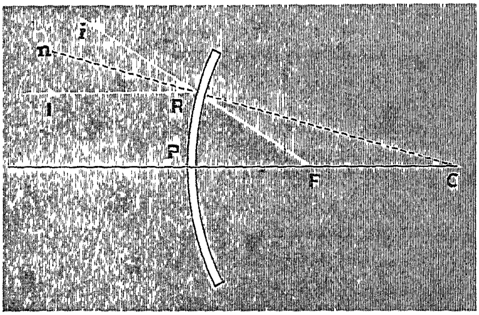


FIG 191.—Principal focus of a convex mirror.

the **focal length** of the mirror. The following definitions are of fundamental importance :

Principal focus—**When a narrow beam of rays parallel to the principal axis of a concave mirror is reflected from the surface of the mirror, the rays converge to, or diverge from, a point on the principal axis This point is termed the principal focus.**

Focal length.—**The distance of the principal focus from the pole of the mirror is termed the focal length of the mirror.**

It is evident, from the preceding paragraphs, that **the focal length of a spherical mirror is equal numerically to one half the radius of curvature.**

Real and virtual foci.—In the case of a convex mirror (Fig. 191) the reflected rays only *appear* to pass through the principal focus, and, in order to distinguish this case from that in which the rays actually pass through the principal focus, it is usual to term the former a **virtual** focus, and the latter a **real** focus.

The same terms are used in all such cases, whether the incident pencil of rays is parallel, or converging, or diverging.

Position and nature of the image of an object, due to reflection from a spherical mirror.—In determining by a graphical method the position of the image of an object, use is made of the fact that the paths of certain rays after reflection from the mirror are known,

providing that the positions of the centre of curvature and of the principal focus are given. Thus

(i) **A ray which passes through the centre of curvature of a mirror returns by the same path after reflection.**

(ii) **Rays parallel to the principal axis of a mirror pass through the principal focus after reflection.**

And, since the path traversed remains unaltered when the direction of the ray is reversed,

(iii) **Rays which pass through the principal focus assume a direction parallel to the principal axis after reflection.**

In practice, it is sufficient to make use of two only of these general principles, since the point of intersection of the paths of two rays after reflection suffice to determine the position of the image. The diagrams obtained by such methods afford information as to (i) the *position* of the image, (ii) its *nature*, whether real or virtual, and (iii) its *size*.

In calculating the position of the image, by means of the **general equation for mirrors**, which will be deduced in a subsequent paragraph, it is necessary to apply always the following rule as to the signs *plus* and *minus*: Distances measured from the mirror *towards* the source of light are **positive**; but when measured from the mirror and *away from* the source of light they are to be considered **negative**. As a result of this rule, the focal length of a concave mirror is always positive, and that of a convex mirror is always negative (Figs. 190 and 191).

Real images due to a concave mirror.—In Fig 192 let OB be an object situated further from a concave mirror than its centre

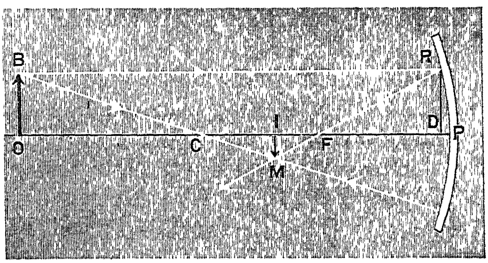


FIG 192.—Real image, by means of a concave mirror

of curvature C. A ray BR parallel to the principal axis will be reflected so as to pass through the principal focus F; and a ray

BC, passing through the centre of curvature, will be reflected back along its own path. The point M, where these reflected rays intersect, is the position of the image of the point B. It may be proved, in a similar manner, that the image of any other point on OB is situated along the line IM. Hence IM is the image of the object OB.

Since the reflected rays actually pass through the image, the image is **real**; and it is evident from Fig. 192 that the image is **inverted**.

Conjugate foci.—The nature and position of an image formed by a mirror depends upon the position of the object. Consider a point in any given position upon the principal axis. Rays diverge from it to the mirror and converge to form an image which may be real or virtual, but in either case the object and image may be regarded as interchangeable. Points at which an object and image may thus change places are known as **conjugate foci.** When the image is real, as for instance, when the object is beyond the centre of curvature of a concave mirror, the places actually can be changed, so that the image is formed where the object was previously.

Equation for mirrors.—In determining, by calculation, the relationship between the positions of the object and image, and the focal length of the mirror, it is necessary to remember that the mirror is supposed to be very narrow, and that the pole P of the mirror coincides with the point D, which is the base of the normal drawn from R to the principal axis.

If u = the distance, PO, of the object from the mirror,
v = " PI, " image "
r = the radius of curvature, CP,
and f = the focal length, FP, of the mirror;
then OB/IM = CO/CI, and RD/IM = FD/FI.
But, since OB = RD, OB/IM = RD/IM;
hence CO/CI = FD/FI = FP/FI.
But CO $= (u-2f)$; CI $= (2f-v)$; FP $= f$; and FI $= (v-f)$;

hence
$$\frac{u-2f}{2f-v}=\frac{f}{v-f},$$
or
$$uv-uf-2fv+2f^2=2f^2-fv,$$
or
$$uv=uf+vf.$$

From which the **general equation for mirrors** is derived, namely,

$$\frac{1}{\text{focal length}}=\frac{1}{\text{distance of image}}+\frac{1}{\text{distance of object}},$$

or
$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

Virtual images, due to concave mirrors. In Fig. 193 let OB be an object situated nearer to a concave mirror than its principal focus. Consider the two rays BR and BR′ proceeding from the point B. Since the path of the former passes through the

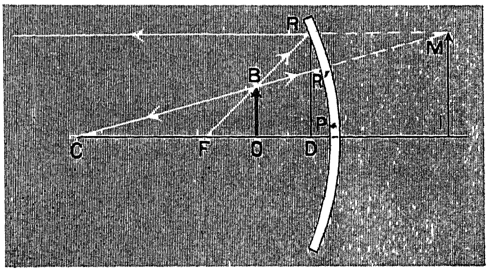


FIG. 193 —Virtual image, by means of a concave mirror.

principal focus F, its direction after reflection will be parallel to the principal axis; and, since the path of the ray BR′ passes through the centre of curvature C, it will be reflected back along its previous path. The point M where these reflected rays intersect is behind the mirror. Hence IM is the image of the object. Since the reflected rays only *appear* to originate from M, the image is **virtual**; it is evident, also, from the diagram that the image is **upright** and **enlarged.**

It is evident, from geometry, that

$$OB/IM = OC/IC, \quad \text{and} \quad OB/RD = OF/DF.$$

But
$$IM = RD;$$
$$\therefore\ OC/IC = OF/DF = OF/PF \text{ (approximately).}$$

If the symbols u, v, and f have the same meaning as on p. 280, then $OC=(2f-u)$; and, since PI is measured in the negative direction, $IC=2f+(-v)=(2f-v)$; also, $OF=(f-u)$ and $PF=f$.

Hence, $$\frac{2f-u}{2f-v}=\frac{f-u}{f},$$

or $$uv=uf+vf,$$

or $$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

This equation is identical with the general equation deduced in the previous paragraph

Virtual images, due to convex mirrors. In Fig 194 the object OB is situated in front of a convex mirror; and, as in the previous

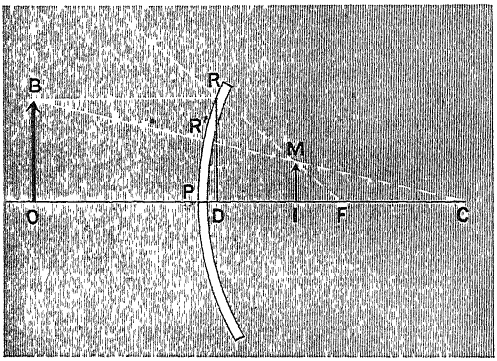


FIG 194 —Virtual image, by means of a convex mirror

cases, the position of the image IM is determined by the point of intersection of the paths, after reflection, of the two rays BR and BR′. It is evident that the image is **virtual, erect,** and **diminished.**

Since $\left.\begin{array}{l}OB/IM=OC/IC,\\ RD/IM=FD/FI;\end{array}\right\}$ $\therefore$ OC/IC = FD/FI = FP/FI (approx.).
and

Hence, $$\frac{u+(-2f)}{-2f-(-v)}=\frac{-f}{-f-(-v)},$$

or $$\frac{u-2f}{-2f+v}=\frac{-f}{-f+v};$$

$$\therefore\ uv=fv+uf,$$

or $$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

Summary.—The following deductions are obtained from the preceding paragraphs:

(i) **Concave mirrors.**

(*a*) When the object is beyond C, the image is real, inverted, and *diminished.*

(*b*) When the object is between C and F, the image is real, inverted, and *enlarged.*

(*c*) When the object is nearer than F, the image is *virtual, upright*, and enlarged.

(ii) **Convex mirrors.**

In all positions of the object, the image is *virtual, upright*, and *diminished.*

(iii) When the proper algebraic signs are given to the numerical values of the quantities u, v, and f, the general equation $1/f = 1/v + 1/u$ may be used in all problems relating to spherical mirrors.

The conditions under which an image has any of the above characters may be summarised thus:

Real	(i)	Concave mirror: object beyond C.
	(ii)	,, ,, ,, between C and F.
Virtual	(i)	,, ,, ,, nearer than F.
	(ii)	Convex mirror: object in any position.
Erect	(i)	Concave mirror: object nearer than F.
	(ii)	Convex mirror: object in any position.
Inverted	(i)	Concave mirror; object beyond C.
	(ii)	,, ,, ,, between C and F.
Enlarged	(i)	,, ,, ,, ,, ,,
	(ii)	,, ,, ,, nearer than F.
Diminished	(i)	Concave mirror: object beyond C.
	(ii)	Convex mirror: object in any position.
Equal	(i)	Concave mirror: object at C.

EXPT. 184.—**Focal length of a concave mirror.** (i) Allow a beam of parallel rays to fall upon the mirror, and adjust the position of a piece of white cardboard so that a well-defined image is formed on its surface. The distance of the cardboard from the pole of the mirror is the focal length of the mirror. A beam of sunlight is convenient for this experiment; or a distant chimney, or window frame, may be used.

(ii) Bore a small round hole in a piece of thin cardboard, and fix two thin threads, or wires, diametrically across the hole and at right angles to each other. Fix this vertically in front of a bright flame, and allow the rays passing through the hole to fall upon the mirror Adjust the mirror so that its principal axis makes a small angle with the axis of the incident beam Hold a piece of white cardboard vertically and normal to the reflected beam, and adjust its position so that a well-defined image of the cross-wires is seen on its surface Measure the distance u of the cross-wires from the mirror, and the distance v of the image from the mirror; calculate the focal length by means of the formula

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

In this, and the following, method the surfaces of a bi-concave lens —that is, a lens having both faces concave, as in Fig. 198—may be used as mirrors, providing that the room is darkened partially

(iii) Using the same source of light as in the previous method, adjust the position of the mirror so that a well-defined image of the cross-wires is seen upon the cardboard to which the cross-wires are attached and near to the round hole. Evidently the rays of light are reflected back along their own path, and the cross-wires must be situated at the centre of curvature of the mirror. Measure this distance; one-half of this distance is the focal length.

EXPT. 185.—**Focal length of a convex mirror.** Let O (Fig. 195) represent the position of the illuminated cross-wires, and let L be a bi-convex lens—that is, a lens having both faces convex, as in Fig. 197. The rays of light passing through L will converge to a point I, where a well-defined image will be formed upon a cardboard screen. Adjust the position of the screen until the best possible definition is obtained. Support the mirror M in a vertical position between L and I, and adjust its position until a well-defined image of the cross-wires is formed upon the cardboard at O. Since the rays are now reflected back along their own path they must fall normally upon the surface of M, and the distance IM between the screen at I and the mirror M must be the radius of curvature of the mirror. One half of this distance is the focal length.

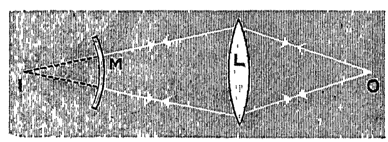


FIG 195.—Experimental method of determining focal length of a convex mirror

Magnification.—The ratio of the linear dimensions of the image and of the object is termed the **magnification.** Thus, in Figs. 192-194, the magnification obtained by means of the mirror is equal in each case to the ratio IM/OB.

It is convenient to express the magnification in terms of the quantities u and v This relationship may be deduced in the following manner: In Fig. 192,

$$\mathrm{IM/OB} = \mathrm{CI/CO},$$

or, expressed in words, the magnification is equal to the ratio of the distances of the image and of the object from the centre of curvature.

Since $\qquad \mathrm{CI} = (r - v)$, and $\mathrm{CO} = (u - r)$,

then $$\mathrm{IM/OB} = (r - v)/(u - r).$$

But $$1/v + 1/u = 1/f = 2/r,$$

or $$1/v - 1/r = 1/r - 1/u,$$

or $$(r - v)/vr = (u - r)/ur,$$

or $$(r - v)/(u - r) = vr/ur = v/u.$$

Hence, $$\mathrm{IM/OB} = v/u,$$

or, **the magnification is equal to the ratio of the distances of the image and of the object from the mirror.** The same result may be deduced by means of either Fig. 193 or Fig. 194.

NUMERICAL EXAMPLE.—A luminous object 3 inches high is situated 12 inches in front of a concave mirror, the focal length of which is 9 inches Find the position and size of the image

Using the formula $1/v + 1/u = 1/f$; since $u = +12$ in, and $f = +9$ in.,

then $$\frac{1}{v} = \frac{1}{9} - \frac{1}{12} = \frac{1}{36},$$

or $$v = +36 \text{ in}$$

Hence, the image is 36 inches *in front of the mirror.*

Also, $$\frac{\text{size of image}}{\text{size of object}} = \frac{36}{12},$$

or $$\text{size of image} = 3 \times \frac{36}{12} = 9 \text{ inches.}$$

Solution of problems on mirrors by means of squared paper.—The solution, by graphical methods, of simple problems on spherical mirrors may be rendered more simple in many cases if squared paper be used for the purpose It may suffice if suitable examples of the method are given.

EXAMPLE.—1. A luminous object, 10 cm. high, is placed in front of a concave mirror, the radius of curvature of which is 80 cm. Find the nature, size, and position of the image, when the object is (*a*) 20 cm., (*b*) 60 cm, (*c*) 110 cm. distant from the mirror.

In Fig. 196 one division of the squared paper represents 2 cm OB is the object when situated 20 cm. from the mirror; and the position

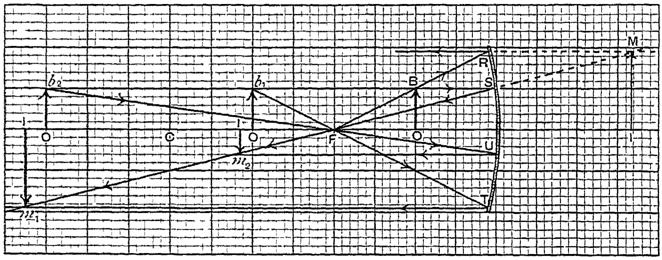


FIG. 196.—Graphical construction for image and object with a concave mirror.

of the image IM is determined by the paths of the two rays BR and BS. Similarly, the position of the image Im_1 of the object Ob_1 is determined by the rays b_1T and b_1S; and the image Im_2 of the object Ob_2 is determined by the rays b_2U and b_2S. The results obtained by measurement should be compared with those obtained by calculation from the general equation, and tabulated in the following manner:

Distance of Object.	Nature of Image.	Distance of Image		Size of Image	
		By Geometry	By Calculation	By Geometry	By Calculation
i. 20 cm.	Virtual, and upright	33·5 cm.	40 cm.	19 cm.	20 cm.
ii. 60 cm.	Real, and inverted	114·6 cm.	120 cm.	19 cm.	20 cm.
iii. 110 cm.	Real, and inverted	62·5 cm.	62·8 cm.	6 cm.	5·71 cm.

The student should endeavour to explain why the results obtained by geometry and by calculation agree more closely when the object is removed further from the mirror.

EXERCISES ON CHAPTER XXI.

1. Explain what is meant by an image. What is the difference between a real and a virtual image? In the case of a concave mirror, find the positions of the object when the image is virtual.

2. A small luminous object is placed in front of a concave spherical mirror of 12 inches focal length at distances of 3 feet, 2 feet, and 1 foot Draw figures showing the positions and relative sizes of the images, and explain your construction

3. A small object is placed six feet in front of a convex mirror of 3 feet radius. Give a diagram showing the nature and position of the image, and find its size relative to that of the object.

4. An object 5 cm. long is placed at a distance of 40 cm. from a convex mirror of 24 cm. focal length. Find the size and position of the image.

5. The middle of a small object is placed on the axis of a concave spherical mirror (i) half-way between the centre and the principal focus, (ii) half-way between the principal focus and the mirror. Draw diagrams to determine the position and circumstance of the image in each case.

6. Find the size of an image of the sun formed by a concave mirror 6 feet in radius, assuming that the distance of the earth from the sun is 100 times the diameter of the sun.

7. Prove that the focal length of a concave mirror is half its radius of curvature. How would you determine experimentally the focal length of such a mirror?

8. An object is placed 20 inches in front of a convex mirror of 10 inches radius. Find the position of the image, and draw a diagram to scale

9. Explain the terms *radius of curvature*, *focal length* How could you find by experiment the radius of curvature and the focal length of a concave mirror?

10. A mirror is fastened to the ceiling of a room and forms images of objects in the room. How can you learn from these images whether the mirror is convex or concave, and what information can you gather as to the degree of its convexity or concavity?

11. What is meant by the focal length of a mirror?
Determine graphically the position of the image formed of an object placed 30 cms. from a reflecting surface of which the radius of curvature is 40 cms.

12. Explain carefully how it is possible to find by a graphical method the position and size of the image of an object formed by reflection in a concave mirror.

A pin, 4·5 cms long, is placed 10 cms. from a concave mirror, of which the radius of curvature is 15 cms. Find the position and size of the image.

13. Similar objects are placed close up to a plane, a concave, and a convex, reflecting surface, how would you distinguish between the surfaces from the images formed in them? How will the images change as the objects are moved away from the surfaces?

14. A concave mirror has a radius of 30 cms. Where must the object be placed so that the magnification may be 3, when the image is (*a*) real, (*b*) virtual?

15. Find the relation between the radius of curvature of a mirror and the distances of an object and its image from it. How would you test the relation by experiment?

16. Distinguish between a real and a virtual image. How would you find experimentally the position of a virtual image?

17. In an experiment with a concave mirror the following measurements of u and v were obtained:

u	v	u	v
27 cm.	77 cm.	60 cm.	30 cm.
30 "	60 "	80 "	26·7 "
40 "	40 "	100 "	25 "
50 "	33 "	120 "	24 "

Plot the readings on squared paper, and find from the curve the positions of the image when (i) $u=35$ cm., (ii) $u=70$ cm., (iii) $u=90$ cm.

CHAPTER XXII.

REFRACTION AT SPHERICAL SURFACES.

IN the following paragraphs the consideration of the refraction of light rays at spherical surfaces must be limited to the deviation of rays and the formation of images by dense transparent media, which are bounded by two surfaces, of which both may be spherical, or one may be plane and the other spherical. A medium of this form is termed a **lens**.

Refraction through a lens.—Most lenses are of glass with curved surfaces, which are portions of spheres. In some lenses, however, one surface is quite plane. All lenses can be divided into two classes—**convex** or **converging**, and **concave** or **diverging**. Converging lenses are thicker in the middle than at the edges, and have the power of forming real images of objects. Concave lenses are thinner in the middle than at the edges, and are unable to form real images of objects.

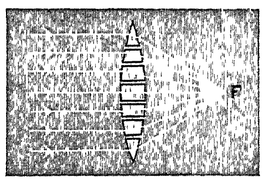


FIG. 197.—A converging lens.

To understand the action of the lenses upon the course of rays of light through them, it is simplest to regard them as being built up of parts of prisms in contact, as shown in Fig. 197, where a convex lens is built up in this way. A ray of light falling upon any one of these prisms is refracted towards its thicker part, and, as in a thin lens bounded by spherical surfaces each prism has a refracting angle proportional to its distance from the axis, the rays converge toward a point, which, if the incident rays are parallel, is known as the **principal focus** of the lens, as F in Fig. 197.

Fig. 198 represents the corresponding effect due to a concave lens. The section of such a lens may be regarded as being built up of parts of prisms of gradually increasing angle, and arranged with their bases outwards. Fig. 198 represents a pencil of parallel rays converted by a concave lens into a diverging beam of rays.

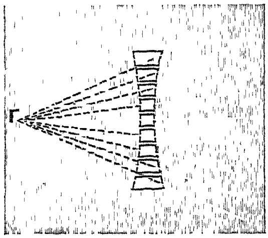


FIG. 198.—A diverging lens

Principal axis, principal focus, and focal length.—The line connecting the centres of curvature of the two surfaces of a lens is termed the **principal axis** of the lens. The **principal focus** of a converging (or diverging) lens is the point towards which the rays converge (or from which they appear to diverge) when a pencil of rays parallel to the principal axis passes through the lens. Thus, in Figs. 197 and 198, the point F represents the principal focus. The distance from the principal focus to the centre of the lens is termed the **focal length** of the lens. With thin lenses it is sufficiently accurate to take the distance from the lens to the principal focus as the focal length.

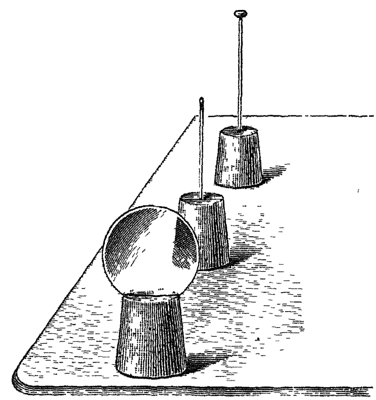


FIG. 199.—Apparatus for experiments with lenses

Images formed by lenses.—When rays of light diverge from a luminous point on the principal axis of a lens, the rays which pass through the lens will either pass through, or appear to pass through some other point, which is termed the **image** of the luminous point. The image is **real** or **virtual** according as the rays actually pass through the point or only appear to pass through it.

The preceding paragraph does not apply in the case of a converging lens when the luminous point coincides with the

principal focus of the lens; for the emergent rays are parallel, and the image is situated at an infinitely great distance.

An image of an object seen through a lens may be either (i) *real* or *virtual*, (ii) *upright* or *inverted*, and (iii) *magnified* or *diminished*.

Much information concerning the different types of image may be derived by means of the simple appliance shown in Fig. 199. It consists of three corks which serve as supports for a lens, a needle, and a long pin: the pin should be sufficiently long for its head to be well above the top edge of the lens. If a lens of 2 inch diameter be used the pin should be $2\frac{1}{2}$ inches long

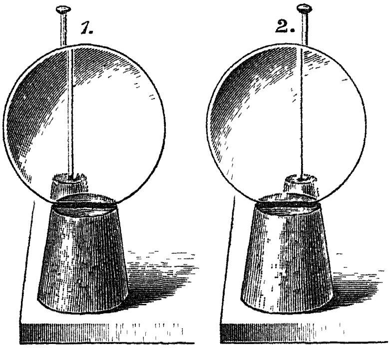


FIG. 200—Expt 186.

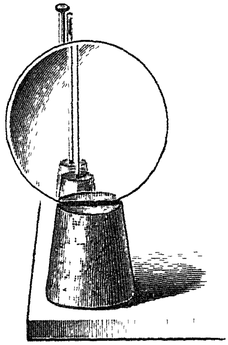


FIG 201.—Expt 187.

EXPT. 186—**A concave lens.** Place the lens vertically at distances of 5, 10, 15,.. cm. in front of the pin, adjusting the height of the pin so that its head is seen well above the edge of the lens. View the image of the pin with the aid of one eye only and looking along the principal axis of the lens. Note (i) whether the image is erect or inverted; (ii) whether it is magnified or diminished; (iii) whether it is in front of, or behind, the lens; (iv) whether it is in front of, or behind, the object.

The Parallax Method (p. 252) may be applied in order to obtain information as to the position of the image, remembering that if two objects are placed in line with the eye, and the eye is then moved to the left (or right), the more distant object appears to move to the left (or right) relatively to the near object. Thus, when viewing the image of the pin through a concave lens, if the eye is moved to the left (or right), the image appears

to move towards the left (or right) edge of the lens (Fig 200); hence the image must be behind the lens. At the same time, compare the movement of the pin's head (seen above the lens) with that of the image; the former moves relatively to the latter in the same direction as that in which the eye moves; hence, the pin must be behind the image.

It is interesting to attempt to locate accurately the position of the image. The observation is not easy, but it is worth the attempt.

EXPT. 187 —**Position of image formed by a concave lens.** Place the needle vertically between the lens and the pin, and adjust its height so that its upper end is seen just above the lens. Move the eye to-and-fro and adjust the distance of the needle from the lens until *the eye of the needle* always appears to be a continuation of *the image of the pin*: the needle then occupies the position of the pin's image. Fig. 201 represents the appearance of the experiment when the eye has been moved to the left.

Enter your observations thus:

Distance of object from lens	Is image erect or inverted?	Is it magnified or diminished?	Is it in front of, or behind, lens?	Is it in front of, or behind, object?	Distance of image from lens
10 cm.	erect	diminished	behind	in front of	7 6 cm.

EXPT. 188 —**Convex lens.** Adjust the height of the needle so that an image of its eye can be seen through the lens. Place the lens at distances of 5, 10, 15, ... cm. in front of the needle; and, for each position, find the information required in order to fill in a Table similar to that recorded in Expt 187.

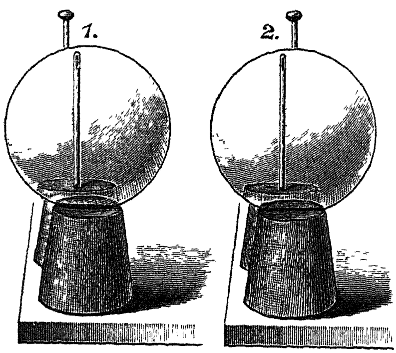


FIG. 202.—Expt. 188

Find the position of the image of the needle by the Parallax Method, using either a long pin or a short pin according as to whether the image is behind or in front of the lens. Fig. 202, (i) and (ii), represent what would be seen when the eye is moved

to the left, and when a long pin is (i) behind, (ii) in front of the image Notice that at one distance no distinct image can be seen, but if the distance of the object is increased slightly, an image is formed in front of the lens; in this case the image is located most readily by using a short, instead of a long, pin

Summary.—It is evident from the preceding experiments that images due to a concave lens are always erect, diminished, and virtual (*i.e.* on the same side of the lens as the object). In the case of a convex lens, when the object is very near to the lens the image is erect, enlarged, and virtual, at one distance only, there is no image; and at greater distances the image is inverted, real, and either enlarged or diminished (depending upon the distance of the object).

The conditions under which the image assumes any of these characters may be summarised thus:—

Real	(i)	Convex lens: object in any position except nearer the lens than F.
Virtual	(i)	Convex lens: object nearer the lens than F.
	(ii)	Concave lens: object in any position.
Erect	(i)	Convex lens: object nearer the lens than F.
	(ii)	Concave lens: object in any position.
Inverted	(i)	Convex lens: object in any position except nearer the lens than F.
Enlarged	(i)	Convex lens: object nearer the lens than F.
	(ii)	Convex lens: object between F and 2F.
Diminished	(i)	Convex lens: object at a greater distance than 2F.
	(ii)	Concave lens: object in any position.
Equal	(i)	Convex lens: object at distance 2F.

Position and nature of the images formed by lenses—Just as in the case of images formed by reflection from a spherical mirror, it is possible to determine approximately the position of the image formed by a lens by making use of the fact that the paths traced out by certain rays are known, providing that the positions of the lens and its principal focus are known. Thus,

(i) **Rays which pass through the centre of a lens do so without change of direction.**

(ii) **Rays parallel to the principal axis of the lens are refracted so as to meet at the principal focus.**

(iii) **Conversely, rays from the principal focus of a lens assume a direction parallel to the principal axis after passing through the lens.**

In practice it is sufficient to make use of two only of these general principles, since the point of intersection of the paths after refraction of any pair of these rays suffices to determine the position of the image. The diagrams so obtained afford information as to (i) the position, (ii) the nature, and (iii) the size of the image.

In calculating the position of the image, by means of the general equation for lenses (p 295), it is necessary to apply the following rule as to the signs *plus* and *minus*.

Rule of Signs —Distances measured from the lens towards the source of light are positive; but when measured from the lens and away from the source of light they are to be considered negative.

By reference to Figs. 198 and 197, it is evident that the focal length of a diverging (or concave) lens is **positive**, and that of a converging (or convex) lens is **negative.**

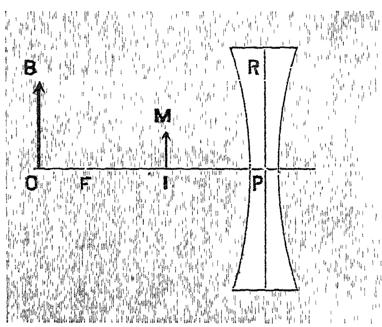


FIG 203.—Virtual image, by means of a concave lens.

Image due to a concave lens —In Fig. 203, let F be the principal focus of a diverging lens, of which PR is the median line. In determining by geometrical construction the paths of rays transmitted through the lens it is sufficiently approximate to assume that the lens is very thin, and that the divergence of the rays takes place abruptly at the median line.

Let OB be a luminous object. Consider the rays proceeding from the point B: one ray BR, which is parallel to the principal axis, is refracted in such a direction that it appears to originate from F. The ray BP passing through the centre of the lens undergoes approximately no deviation. The image can be

situated only at the point M, where the paths of these rays intersect. Similarly, the image of any other point of OB can be proved to be vertically under M. Hence, IM is the image of OB. It is evident that the image is **erect, diminished,** and **virtual** The same result would be obtained if the object were situated nearer to the lens than the principal focus.

Relationship between the focal length and the positions of the object and image—In Fig. 203, let $PO = u$, $PI = v$, and $PF = f$, then

$$OB/IM = PO/PI = u/v,$$

also

$$PR/IM = PF/IF = f/(f - v).$$

But

$$OB = PR.$$

Hence,

$$u/v = f/(f - v),$$

or

$$uf - uv = fv,$$

or

$$uf - fv = uv,$$

which gives the **general equation for lenses,** viz.,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \quad \text{or} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

This equation is applicable to all types of lenses, providing that the rule given already (p. 294), as to positive and negative values, is observed. The points at which object and image may change places without necessitating any change of construction except the reversal of the directions of the rays are known as **conjugate foci.**

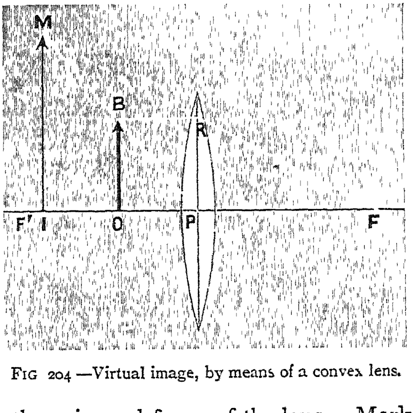


FIG 204 —Virtual image, by means of a convex lens.

Image due to a convex lens.—The character of the image due to a convex lens depends upon the nearness of the object to the lens In Figs. 204 and 205, let F be the principal focus of the lens. Mark off, to the left of the lens, a distance PF′ equal to the focal length PF.

CASE 1.—**Where the object is nearer to the lens than** F.

Consider the rays proceeding from a point B of a luminous object OB (Fig 204). A ray BR, which is parallel to the principal axis, is refracted so as to pass through the principal focus F; and a ray BP, passing through the centre of the lens, is practically undeviated The image of B must be at the point M where these paths intersect. Hence, IM represents the image of OB; and it is **erect, magnified** and **virtual.**

Fig. 204 explains the principle of the **pocket-lens** or **reading-glass** when used for slight magnification.

The general equation for lenses may be deduced from Fig. 204 in the following manner:

Since $$OB/IM = PO/PI = u/v,$$
and $$PR/IM = FP/FI = -f/(-f+v),$$
then, since $$OB = PR, \quad u/v = -f/(-f+v).$$
Hence, $$-uf + uv = -fv$$
or $$uf - fv = uv,$$
or $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

CASE 2.—**Where the object is more distant than** F.

In Fig 205 let OB be the object. By the same construction as before, the image of the point B is situated at M; and IM is the

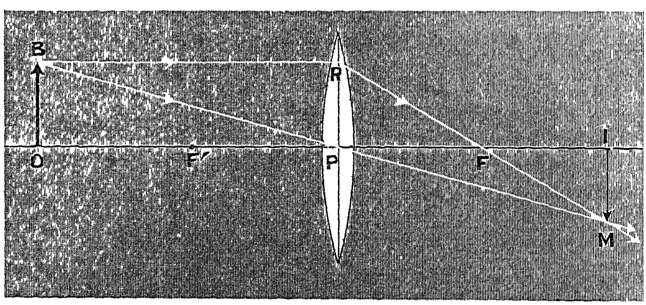


FIG. 205.—Real image, by means of a convex lens.

image of OB. Evidently the image is **real** and **inverted**; but the relative size of the image depends entirely upon the relative distances of the object and the image from the lens.

It will serve as a useful exercise for the student to deduce the general equation for lenses from Fig. 205.

Experimental determination of focal lengths.—The choice of methods of determining focal lengths of lenses depends entirely upon the apparatus available. Although a simple optical bench is desirable, yet fairly accurate results can be obtained without the aid of this appliance. The student is recommended to attempt the following methods.

CONVEX LENSES.

EXPT. 189.—**Parallel rays.** Support the lens in a vertical position and adjust its distance from a cardboard screen so as to form on the screen a real inverted image of some distant object (*e.g.* the window bars, or a distant chimney). The rays from the distant object are practically parallel, and the image is formed at the principal focus of the lens. Measure the distance of the screen from the lens.

EXPT. 190.—**Reflection method.** Bore a small circular hole through a white cardboard screen, and fix thin cross-wires across the hole with sealing-wax. Place a bright light behind and near the cross-wires. Support the lens so that the centre of the hole is on the principal axis of the lens. Place a plane mirror M (Fig. 206) close behind the lens, and adjust the distance of the lens L from the screen until an image of the cross-wires is formed on the screen P and close beside the circular hole. Evidently each ray of light is reflected back along its own path; and this can only be the case if the rays fall normally on the surface of M. Hence, the rays after passing for the first time through the lens are parallel to the principal axis, and they must therefore have originated from the principal focus. Therefore the distance f is the focal length of the lens.

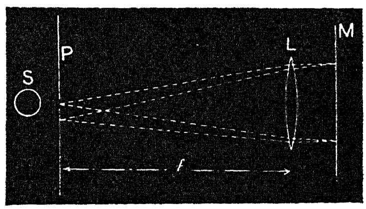


FIG. 206.—Determination of the focal length of a convex lens.

EXPT. 191.—**Image and object.** In this method, use is made of the general equation $1/f = 1/v - 1/u$. The student may use the simple appliances (lens and two pins or needles) as described on p. 291, determining the position of the image by the Parallax Method. Or, he may use the illuminated cross-wires (p. 284) and a second cardboard

screen. In either case, several perfectly independent observations should be made, the distance u being different in each case In calculating the value of f it must be remembered that, if the image is real, the distance v is negative.

Concave Lenses.

Expt. 192 —**Combination method.** Select a convex lens of known focal length, and of such converging power that, when fastened to the concave lens the combination is still converging. If necessary, determine the focal length (f_1) of the convex lens, preferably by the method of Expt 190. Fasten the two lenses together and determine the focal length (F) of the combination. The focal length (f_2) of the concave lens can then be calculated by means of the equation $1/F = 1/f_1 + 1/f_2$. When substituting the values of f_1 and F it must be remembered that the focal length of a convex lens is always negative.

Expt. 193 —**Parallax method.** Determine the focal length of a concave lens by locating the position of the virtual image of a pin or needle by the Parallax Method (Expt. 188), and applying the equation $1/f = 1/v - 1/u$

Magnification —The ratio of the linear dimensions of the image and the object is termed the **magnification** (m) Thus, in Figs. 203-205, the magnification due to the lens in each case is equal to the ratio IM/OB.

It is evident from either of the diagrams that

$$m = \frac{IM}{OB} = \frac{v}{u}.$$

It is more convenient in some problems to express the magnifying power of a lens in terms of u and f. This can be deduced from the general equation, thus:

Since $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u},$$

then $$\frac{u}{f} = \frac{u}{v} - 1,$$

or $$\frac{u}{v} = \frac{u+f}{f}.$$

Hence, $$m = \frac{v}{u} = \frac{f}{u+f}.$$

Power of a lens.—The **power** of a lens is defined as the reciprocal of the focal length. A lens of focal length 100 cm. is recognised as having unit power; and this unit is termed the **dioptre.** It is also generally recognised that the power of a converging lens is *positive*, and that of a diverging lens is *negative.* Hence, the power of a converging lens of 50 cm. focal length is equal to $+1/0{\cdot}5=2$ dioptres; and that of a diverging lens of 25 cm. focal length is equal to $-1/0{\cdot}25=-4$ dioptres.

EXAMPLES.—1. The focal length of a converging lens is 50 cm. Find the nature, position, and size of the image of an object 5 cm. long which is placed vertically (i) 25 cm., (ii) 75 cm., (iii) 125 cm. from the lens

(i) Since $f=-50$ and $u=+25$, then

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$$

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f},$$

or $$v=\frac{uf}{u+f}=\frac{25\times(-50)}{25-50}=\frac{-(50\times 25)}{-25}=+50 \text{ cm.};$$

$\therefore$ the image is virtual, and 50 cm. distant from the lens.

Also, $$m=\frac{v}{u}=\frac{+50}{+25}=2.$$

Hence, the length of image $=2\times 5=10$ cm.

(ii) Since $u=+75$ cm., then

$$v=\frac{uf}{u+f}=\frac{75\times(-50)}{75-50}=\frac{-(50\times 75)}{25}=-150 \text{ cm.};$$

$\therefore$ the image is real, and 150 cm. distant from the lens.

Also, $$m=\frac{v}{u}=\frac{150}{75}=2.$$

Hence, the length of the image is $(2\times 5)=10$ cm.

(iii) Since $u=+125$ cm., then

$$v=\frac{uf}{u+f}=\frac{125\times(-50)}{125-50}=\frac{-(50\times 125)}{75}=\frac{-250}{3}=-83{\cdot}3 \text{ cm.};$$

$\therefore$ the image is real, and 83·3 cm. distant from the lens.

Also, $$m=\frac{v}{u}=\frac{83{\cdot}3}{125}=0{\cdot}664.$$

Hence, the length of the image is $0{\cdot}664\times 5=3{\cdot}32$ cm.

2 A piece of cardboard, in which a narrow slit is cut, is placed in front of a bright light. A screen is placed vertically at a distance of 49 inches from the slit. Where must a convex lens, of focal length 6 inches, be placed so as to give on the screen a well-defined image of the slit?

The data given are $u+v=49$ inches and $f=-6$ inches

Since the image is real and on the opposite side of the lens to that of the slit, its distance from the lens must be regarded as *negative.* Hence, in the general equation $1/v-1/u=1/f$, we may substitute $v=-(49-u)$ and $f=-6$.

$$\therefore\quad -\frac{1}{49-u}-\frac{1}{u}=-\frac{1}{6},$$

or
$$\frac{-49}{u(49-u)}=-\frac{1}{6},$$

or
$$u^2-49u+294=0,$$

or
$$(u-42)(u-7)=0.$$

Hence, $u=+42$ inches, or $+7$ inches.

A distinct image will be obtained, therefore, if the lens is placed either 42 inches or 7 inches from the slit In the former position the image is diminished, and in the latter position enlarged.

Solution of simple problems by means of squared paper.—Squared paper may be used with advantage in solving simple problems on lenses, especially if the diagram so obtained is used as a verification of the result obtained by calculation The following examples will indicate the method·

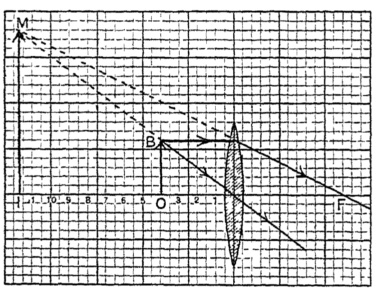


FIG 207—Graphical construction for image and object with convex lens

EXAMPLES—1 An object 3 cm high is placed at a distance of 4 cm. from a convex lens of focal length 6 cm. Find the position and size of the image.

In Fig. 207, each division of the squared paper represents 0·5 cm. The position of the image IM is determined by the construction explained on p. 295. The diagram shows that the image is (i) virtual, (ii) approximately 11·8 cm. distant from the centre of the lens, and (iii) 11·8 cm. in height.

This result will be found to agree closely with that obtained by calculation from the general equation for lenses.

2. When an object is placed at a distance of 15 cm. from a converging lens the image formed on a screen is found to be twice the size of the object. Find the focal length of the lens

Since the image is twice the size of the object, the distance of the former from the lens must be twice the distance of the latter from the lens. Hence, since the image is real, and therefore on the opposite side of the lens to that of the object, the distance of the screen from the object must be 45 cm.

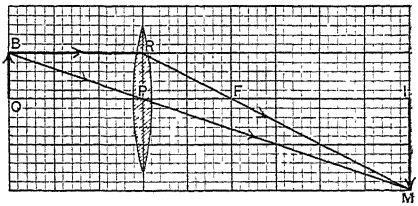


FIG. 208.—Focal length of a convex lens by graphical construction

In Fig 208, each division of the paper represents 1 cm.; mark off OP=15 cm, and OI=45 cm. Draw OB to any convenient scale, *e g* 5 cm., and make IM=2OB. Draw the ray BR parallel to the principal axis, and join RM. The point F, where this ray intersects the principal axis, must be the principal focus; and PF is the focal length. By inspection PF=9·8 cm. approximately.

EXERCISES ON CHAPTER XXII.

1. You are given a lens through which you can look, but which you are not allowed to handle. What tests would you apply in order to determine if it be concave or convex?

2. How does the image seen in a plane glass mirror differ from that seen on the ground glass of a camera, and how does each differ from the object in respect of size, and of position or inversion?

3. Define the principal foci of a thin convex lens. Draw a diagram showing the paths of the rays through such a lens from a luminous object situated on the axis of the lens at a distance of twice the focal length from its centre.

4. An object at a distance of 10 inches from a lens, when viewed through the lens appears to be at a distance of 30 inches from the lens. Find the focal length of the lens, and give a diagram showing the nature of the image.

5. A converging lens is employed to form an image of an object placed in front of it at a distance of 20 inches from the lens. If the image behind the lens is twice as large as the object, find, by a geometrical construction, the focal length of the lens.

6. A converging lens having the same focal length (5 cm.) as a concave mirror is placed with its axis coinciding with that of the

mirror, and with its centre at the centre of curvature of the mirror. Rays of light diverge from the principal focus of the lens remote from the mirror and fall on the lens. Draw a diagram to indicate their path through the lens to the mirror, and thence, after reflection, back through the lens. What difference would there be if the lens were placed with its centre at the principal focus of the mirror?

7. A pin, $\frac{1}{4}$ inch long, is placed at a distance of 3 inches from a convex lens of focal length 2 inches. Draw to scale a diagram to show the position and length of the image. Explain your construction.

8. A luminous point is situated 30 cm in front of a lens, and an image is formed 10 cm behind the lens. What kind of lens is used? and what is its focal length? Draw a diagram to scale, and verify the result of the calculation.

9. A pocket magnifying glass has a focal length of 5 cm Find, by means of a diagram drawn to scale, the position and length of an object, 0 2 cm. long, placed 3 cm. from the lens.

10. The focal length of a camera lens is -20 cm. How far from the lens should the sensitized plate be in order to photograph an object 180 cm. in front of the lens? How large a surface at this distance could be photographed on a $\frac{1}{2}$-plate ($6\frac{1}{2}$ in. $\times$ $4\frac{3}{4}$ in.)?

11. A candle stands at a distance of 3 ft. from a wall. In what position must a convex lens of 8 in. focal length be placed between them so as to produce upon the wall a distinct image of the candle?

12. An object, 2 in. long, is placed 8 in. from a concave lens of 4-inch focal length. Find, by means of a diagram, the position and length of the image.

13. In order to find the focal length of a concave lens, it was blackened, with the exception of a circle 4 cm. in diameter at its centre. A beam of sunlight was allowed to pass through this, when it was found that an illuminated circle of 20 cm. diameter was formed on a screen held 64 cm. behind the lens and parallel to it. What was the focal length of the lens?

14. An object is placed 100 cm. from a concave lens. What must be the focal length of the lens so that a virtual image of the object can be formed at a distance of 25 cm. from the lens?

15. The focal length of a concave lens is 6 in., and a small object is placed 18 in. from the lens. Draw a diagram to scale, showing the path of the rays by which the image is formed, and determine its position.

16. A screen is fixed at a convenient distance from a lighted candle. It is found that a convex lens may be placed in two positions between the candle and screen so as to throw a distinct image of the flame on the screen. Explain this.

Being allowed to measure any distances you like, how would you determine the focal length of the lens?

17. A boy has a convex lens the focal length of which is 10 cm. How far from a screen must it be to get an image of the sun on the screen? How far from the screen must it be to get an image of a candle which is a metre from the lens? What is the power of the lens?

18. How would you find the focal length of a convex lens? An object is placed 20 cm from a convex lens, and an inverted image is formed 4 times as large as the object. Find the focal length of the lens.

19. On a screen 1 foot behind a lens an image 6 inches long is formed of a man 5 feet 6 inches in height standing in front of the lens. Find the distance between the man and the lens, and the focal length of the latter

20. Describe some optical (or other) method for measuring the radii of curvature of the surfaces of an ordinary lens. Upon what does the focal length of a lens depend besides the curvatures of its surfaces?

21. What procedure would you adopt to determine most correctly and easily the power of a concave lens?

22. A converging beam of light falls upon a diverging lens, the axis of the beam being coincident with the axis of the lens Explain by means of a figure, or otherwise, the conditions for the emergent beam being convergent, parallel, or divergent.

23. A lantern slide is $3\frac{1}{4}$ inches square, and an enlarged image of it is to be formed by the aid of a lens of 6 inches focal length upon a screen 20 feet distant from the lens. What kind of lens should be used, at what distance from the slide must it be placed, and what will be the size of the image?

24. A converging lens has one of its faces plane, and the plane face is silvered The lens is held in the path of light diverging from a small hole in a screen, the curved surface being towards the hole When the lens is at a certain distance from the screen, a sharp image of the hole is thrown upon it. Explain, by means of a drawing, the path of the rays in their course from the hole to the image. What is the distance from the lens to the screen? Would the effect be the same if the experiment were repeated with the curved face of the lens silvered and the plane face turned towards the hole? Give reasons.

CHAPTER XXIII.

THE EYE AND OPTICAL INSTRUMENTS.

The eye.—Fig. 209 represents a horizontal cross-section through the centre of an eye. The outer coating consists of a hard thick membrane *s*, called the **sclerotic**. The front part *c* of the sclerotic is transparent, and is termed the **cornea**. A transparent lens-shaped body *o* of hard gelatinous consistency, termed the **crystalline lens**, is supported from the walls of the eye near to the cornea. The lens divides the eye into two chambers; the anterior chamber is filled with a watery liquid *a*, termed the **aqueous humour**, and the posterior is filled with a jelly-like substance *v*, termed the **vitreous humour**. Just in front of the lens is a contractile diaphragm *i*, the **iris**, with a circular orifice (the **pupil**) near to its centre. The retina (*r*), which is the portion of the eye's inner surface sensitive to light, is liberally supplied with nerve-fibres and blood-vessels The nerve-fibres originate from the **optic nerve** (*n*), which enters the eye on the inside of the centre of the retina. Light falling upon these nerve-fibres appears to set up nervous stimuli which are transmitted to the brain, and these are interpreted as the phenomenon termed sight

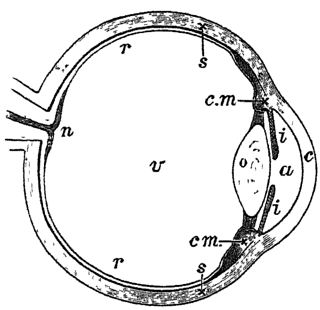


FIG 209 —Horizontal cross-section through the centre of an eye.

Fig 210 indicates how the cornea and crystalline lens give rise

to an inverted image of a distant object on the retina Although the image is inverted, yet the mental interpretation of the effect upon the retina is just as though the images were erect

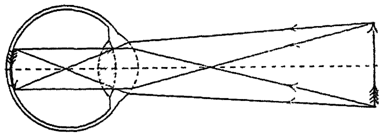


FIG 210 —Formation of an inverted image on the retina

It might be thought that a clear image of any object is only obtained when the object is at one given distance from the eye; but there is really a wide range of distance through which distinct vision is possible to a normal eye. This **power of accommodation**, as it is termed, is effected by an involuntary change in the curvature of the surfaces of the crystalline lens The change in curvature is brought about by the unconscious action of the **ciliary muscle** (*c m*, Fig. 209) which surrounds the edge of the lens and is connected to the inner walls of the eye When a near object is looked at, the ciliary muscle contracts and causes the lens to bulge more, thus increasing its diverging power. On the other hand, when looking at a distant object the ciliary muscle is relaxed, and the lens is thereby flattened The power of accommodation is limited, for objects which are very near cannot be focussed clearly. nor can the details of very distant objects be seen clearly. The distance at which objects are seen with greatest distinctness by a normal eye varies from 25 to 30 cm.

Defective eyes.—The most common ocular defects are (i) **Short-sight** (or, Myopia), (ii) **Long-sight** (or, Hypermetropia), and (iii) **Loss of accommodative power** (or, Presbyopia).

(i) **Short-sight.**—A short-sighted eye cannot see distant objects distinctly. The eye-ball is usually too long; and parallel rays falling upon the cornea are brought to a focus F (Fig. 211, A) in front of the retina. If the object from which the rays proceed is brought nearer to the eye, the position of the image recedes; and, at a certain distance, vision becomes distinct. If brought still nearer, the power of accommodation enables the lens to thicken, and vision remains distinct. Since the power of accommodation is limited, vision becomes indistinct again if the object is brought too near.

Fig. 211, A, represents how parallel rays may be brought to a focus on the retina by placing a diverging lens L in front of the eye.

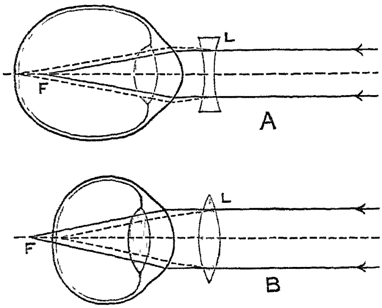


Fig. 211.—In A a double concave lens corrects the short-sighted eye, and in B the double convex lens corrects the long-sighted eye

The dotted lines indicate the path of the rays after passing through the lens. Thus, a short-sighted eye needs a diverging lens in order to see distant objects.

(ii) **Long-sight.**—A long-sighted eye cannot see near objects distinctly. When the eye is at rest, parallel rays are focussed behind the retina; but, as a rule, the accommodative power is sufficient to enable distant objects to be seen clearly, though this is not sufficient to focus near objects. The dotted lines in Fig. 211, B, indicate how, by using a converging lens L, the focus of parallel rays may be moved forwards on to the retina of a long-sighted eye.

(iii) **Loss of accommodative power.**—This defect is found usually in elderly people. It is noticed generally that, as the condition develops, the nearest point at which vision is distinct gradually recedes. For this reason, it is often the case that an elderly man finds that printed matter can be read only if held at arm's length.

An object (such as printed matter) held near the eye can only be seen clearly by an eye with this defect if seen through a convex lens, since by this means the image is more distant than the object (Fig. 204, p. 295). On the other hand, distant objects can only be seen with the aid of a concave lens, since the image is brought nearer than the object. Persons suffering from loss of accommodative power find it necessary, therefore, to use two different kinds of lenses, according to the distance of the object viewed.

Near point and far point.—The power of accommodation varies in different individuals, and in the same individual it

changes with progressive age. The normal eye of an adult can focus clearly an object which is not more than 10 or 12 inches distant from the cornea.

The nearest point to the eye at which a small object can be seen clearly is termed the **near point.** The point to which the eye is focussed when at rest is termed the **far point**; and, in the case of a normal eye, the far point is obviously at an infinite distance; but it is frequently found that the far point is within measurable distance of the eye

The determination of the near point and the far point constitutes an instructive experiment. One method is based upon the fact that when an object is viewed through a convex lens of known focal length and placed quite near to the eye, a well-defined image is seen only when the image is situated between the near and far points. When the object is brought gradually nearer to the lens a point is reached when the image just ceases to be clearly defined; by using the formula on p. 296 the position of the image, and therefore of the near point, can be calculated. Similarly, when the object is withdrawn gradually, a position is reached when the image again ceases to be defined clearly, and the position of the image, obtained by calculation, gives the distance of the far point

EXPT. 194.—**Determination of near point and far point.** Select a convex lens of known focal length (20 cm. is convenient). Draw a small square, in pencil, on a piece of cardboard, and support this vertically on a movable support behind the lens Place one eye as near as possible to the lens, and move the cardboard slowly towards the lens until the image is just becoming indistinct; measure this distance. Repeat the measurement twice, and take the average d_1 The distance of the image, *i.e.* the distance of the *near point*, can be calculated by means of the formula $1/f=1/v-1/u$. Thus, in an actual experiment, $f=-20$ cm, and $d_1=10{\cdot}1$ cm.; hence

$$-\frac{1}{20}=\frac{1}{v}-\frac{1}{10{\cdot}1},$$

or

$$\frac{1}{v}=\frac{1}{10{\cdot}1}-\frac{1}{20},$$

or

$$v=\frac{202}{99}=20{\cdot}4 \text{ cm.}$$

Similarly, note the maximum distance, d_2, at which the image is just distinct, and calculate the distance of the far point. Enter the observations thus .

Eye.	d_1	Near Point (calculated)	d_2	Far Point (calculated)
Right	6 9	10·5	13 6	42 5
Left	10·1	20 4	14·7	55·5

The visual angle.—The apparent size of an object is proportional to the size of the image of the object upon the retina, and this depends upon the distance of the object from the eye—the nearer the object the larger the image. The apparent size of the object is measured usually by the angle which the object subtends at the eye, and this angle is called the **visual angle.** Thus, in Fig 212, if E represents the eye and AB the object, the visual angle is AEB; if *ab* represents the object the visual angle is aEb.

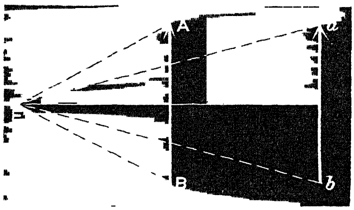


FIG 212.—Visual angles AEB and aEb

The simple magnifying glass.—The apparent size of an object, when viewed by the naked eye, is a maximum when the object is situated at the near point. The apparent size would be increased if the object were brought still nearer, but the image would be indistinct. When, however, the object is viewed through a convex lens held near to the eye, the object may be brought nearer to the eye than the 'near point,' and still the image may be well-defined, since the virtual image due to a convex lens is further from the lens than the object. Fig. 213 represents how a virtual image of an object AB is seen

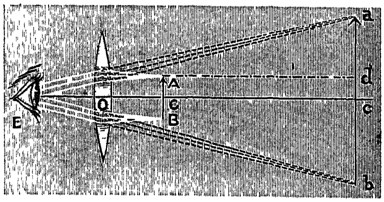


FIG 213.—The simple magnifying glass.

at *ab*; and it is evident that the visual angles of the object and of its image are the same. Hence the lens serves the purpose of enabling the visual angle to be increased while maintaining at the same time distinct definition.

In determining the **magnifying power** of a lens it is usual to adjust the position of the object so that the image is situated at the near point; and the magnifying power may be defined as *the ratio of the visual angle of the image to that of the object if it were situated at the 'near point.'*

In Fig. 213 suppose that c is the 'near point,' then

$$\text{magnifying power } (m) = \frac{ac}{dc} = \frac{ac}{AC} = \frac{Oc}{OC}.$$

If Oc be the distance d of the 'near point,' then, since the lens is converging and its focal length therefore negative,

$$-\frac{1}{f} = \frac{1}{d} - \frac{1}{u},$$

and
$$\frac{1}{u} = \frac{1}{d} + \frac{1}{f},$$

or
$$u = \frac{df}{d+f}.$$

Hence,
$$m = \frac{Oc}{OC} = \frac{d}{u} = \frac{d+f}{f}.$$

EXAMPLES.— 1. A long sighted person cannot see objects clearly at a distance of less than 50 cm. Find the power, in dioptres, of the glasses required in order to read print held 15 cm. in front of the glasses.

The focal length must be such that when an object is 15 cm. in front of the glasses, the image is at least 50 cm. distance. Hence, $u = +15$ cm., and $v = +50$ cm.

$$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{50} - \frac{1}{15} = -\frac{35}{750},$$

or
$$f = -21{\cdot}4 \text{ cm.},$$

or
$$\text{power} = +\frac{1}{0{\cdot}214} = +4{\cdot}67 \text{ dioptres.}$$

2. A person with short-sight is able to read print only when held 15 cm. from the eye. What kind of glasses, and of what focal length, are necessary in order that print held at a distance of 25 cm. from the eye may be read clearly?

In this case, $u=+25$ cm, and $v=+15$ cm.;

$$\therefore \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$=\frac{1}{15}-\frac{1}{25}=+\frac{2}{75},$$

or
$$f=+37{\cdot}5 \text{ cm.}$$

Hence, concave glasses of focal length 37 5 cm. are required.

The photographic camera.—The photographic camera consists of a rectangular box with two opposite sides vertical and rigid.

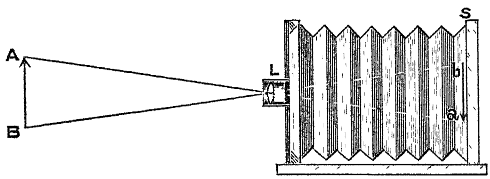


FIG. 214 —Principle of the photographic camera

The distance between these sides can be adjusted by the remaining sides being made extensible, and consisting of folded leather or other opaque material. A circular opening in one of the rigid sides serves to carry a converging lens (L, Fig. 214), and the opposite side is fitted with a translucent screen S of matt glass, which can be replaced by a photographic plate.

When the camera is placed with its lens directed towards a distant object AB, a real inverted image *ab* is formed on the other side of the lens. The screen S is moved towards the lens until its position coincides with *ab*, when a well-defined image of the object can be seen on the screen. A photographic plate consists of a glass sheet covered on one side with a gelatine film loaded with chemical compounds sensitive to actinic (p. 233) light rays. When such a plate is substituted for the screen, and exposed for a definite period to the rays passing through the lens, subsequent development of the film reveals a permanent reproduction, in black and white, of the distant object. The plate, when developed, is termed a **photographic negative.**

If the upper part of the picture be more distant than the lower part from the lens, then in order that the whole image may be defined with equal clearness in all parts, the lower part of the screen must be placed nearer to the lens than the upper part. For this reason, the frame carrying the screen is frequently hinged along its lower side to the base of the apparatus, so that it may be tilted backwards or forwards according to the relative distances of the upper and lower parts of the picture.

The optical lantern.—The optical lantern consists essentially of a bright source of light and a system of converging lenses

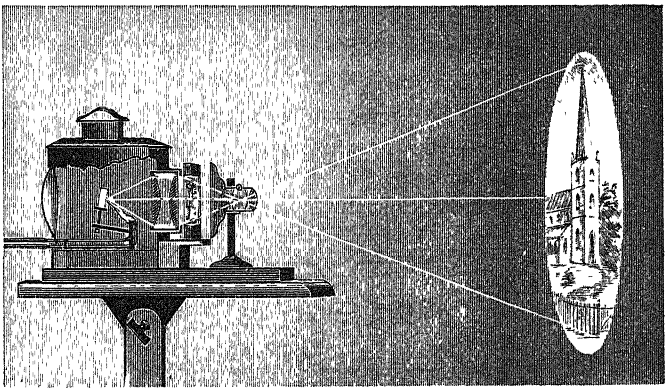


FIG 215 —The optical lantern

arranged so as to throw on a screen an enlarged inverted image of a transparent photograph or drawing, or of any object constructed in a manner suitable for projection on a screen The rays of light proceeding from the source of light (Fig. 215) pass through the transparency (or 'lantern slide') which is supported in a suitable carrier, the rays which are not absorbed by the dark parts of the slide then pass through a **focussing lens,** which is adjusted in position so that the slide is slightly beyond its principal focus, and a real inverted and enlarged image of the slide is thrown on a distant screen. The **condensing lenses** which are usually a combination of two lenses and are nearest the source of light, serve two purposes: (i) to increase the total amount of light concentrated on the slide, and (ii) to deflect the rays of light

passing through the outer portions of the slide sufficiently for their paths to pass through the focussing lens In the absence of a suitable condenser, the image on the screen would be much less bright, and it would represent the middle part only of the slide.

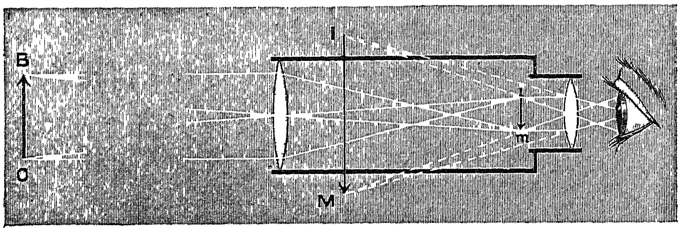


FIG. 216 —Principle of the astronomical telescope

The astronomical telescope.—In order to obtain a magnified image of a distant object it is necessary to use more than one lens Fig. 216 represents the principle of the astronomical telescope, which consists usually of a large lens, of considerable

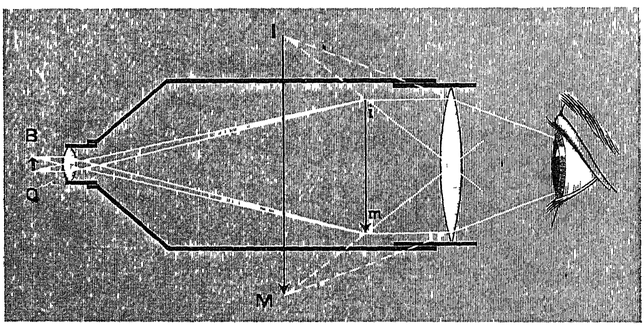


FIG 217.—Principle of the compound microscope.

focal length, called the **object-glass**, and a smaller lens of short focal length called the **eye-piece**. The diagram represents several rays coming from a point of a distant object, and, since the distance is assumed to be great, these rays are practically parallel. A real, inverted, and diminished image *im* of the object is formed in a position which practically coincides with the focus of the object-glass. The eye-piece is adjusted so that *im* is just within

its principal focus. On looking through the eye-piece a virtual and enlarged image of *im* is seen at IM. The dotted lines in the diagram indicate how the position of this image is determined.

The compound microscope.—In the compound microscope (Fig. 217) a small object-glass of short focal length is used instead of the large object-glass of long focal length employed in the telescope. If the object OB under observation be placed just beyond the principal focus of the object-glass, a magnified, real, and inverted image is obtained at *im*. When this image is viewed through the eye-piece, a virtual image IM of a much larger size is seen.

EXERCISES ON CHAPTER XXIII.

1. A man who can see distinctly at a distance of 1 foot, finds that a certain lens when held close to his eye magnifies small objects 6 times. Determine the focal length of the lens.

2. A man who can see most distinctly at a distance of 5 in. from his eye, wishes to read a notice at a distance of 15 ft. off. What sort of spectacles must he use, and what must be their focal length?

3. A long-sighted person can only see distinctly objects which are at a distance of 48 cm. or more. By how much will he increase his range of distinct vision if he uses convex spectacles of 32 cm. focal length?

4. A long-sighted person uses convex glasses of 40 cm focal length, and finds that he cannot read print through them comfortably when it is held nearer than 30 cm. What is his nearest point of distinct vision?

5. Describe the simple magnifying glass and explain its use. Draw a figure showing the position of object and image and the course of light when a magnifying glass (focal length = 3 cm.) is used by a person whose distance of most distinct vision is 15 cm.

6. A person, whose distance of most distinct vision is 10 cm., uses a simple magnifying glass to view a certain object. The focal length of the magnifying glass is 2·5 cm. Draw a figure showing the relations between the linear dimensions of object and image, and trace the course of a small pencil of light from a point of the object to the eye.

7. Describe, explaining how the image is formed, either a simple microscope or a simple astronomical telescope.

8. A patient cannot see objects clearly at a distance of less than 36 inches; find the focal length of the glasses required to enable him to read at 10 inches.

9. A short-sighted person cannot see objects clearly at a distance greater than 6 inches What spectacles would be required to enable him to see distant objects clearly? If his least distance of distinct vision without glasses is 3 inches, what would it be with the above spectacles?

10. Two converging lenses, A and B, are fixed vertically with their axes coinciding and with their centres 70 cm apart. The focal lengths of A and B are +35 cm. and +20 cm. respectively. A capital letter 10 cm high, cut from a poster, is fixed vertically 100 cm from the centre of A and on the distant side from B. The letter is viewed through both lenses by an eye placed near to B. Draw a diagram to scale, showing the position and size of the image of the letter.

11. A reading-glass of 8 cm focal length is used by a person whose 'near point' is at a distance of 24 cm. What is the magnifying power of the glass?

12. How would you arrange two convex lenses to form a telescope? What is the purpose of each lens?

CHAPTER XXIV.

COMPOSITION OF LIGHT.

Dispersion.—The multicoloured rainbow, varying from violet at the inner edge to red at the outer, is a familiar phenomenon; and the colour effects seen when the edge of a window frame is viewed through a prism, held with its refracting edge parallel to the object, are also familiar appearances. In this simple experiment we are repeating that historical experiment with which Newton proved that ordinary white light is a complex mixture of coloured lights. Fig. 218 represents the nature of this experiment. A beam of sunlight was caused to fall upon a glass prism held so as to refract the light downwards upon the opposite wall of a dark room. The light was found to be drawn out into a long coloured band, violet at the lower end and red at the upper. This coloured band Newton termed the **spectrum**. The colours violet, indigo, blue, green, yellow, orange, and red are clearly distinguishable, although each colour changes, by insensible

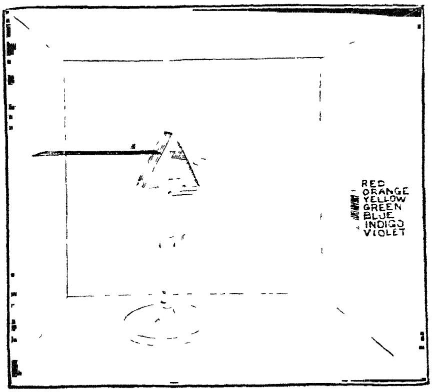


FIG 218.—Dispersion of light by a prism (Newton's experiment)

gradations, into the next. It is evident that, when passed through a prism, violet rays are bent more than yellow rays, and yellow rays more than red rays; or, expressing the same fact in other words, violet rays are more **refrangible**, that is, deflected more from their original path than yellow rays, and yellow more refrangible than red rays. The separation of the different colours, owing to this difference in refrangibility, is termed **dispersion.**

In order to understand why we can obtain a spectrum by this simple means, it is necessary to realize that the condition which gives rise to the sensation of light really consists in the rapid transmission of a kind of wave-motion through the luminiferous ether (p 233)—a medium which fills all space, even a perfect vacuum, and occupies the intermolecular spaces in all forms of matter. There is reason to believe that when light is being transmitted through the medium, there exists within it a vibratory

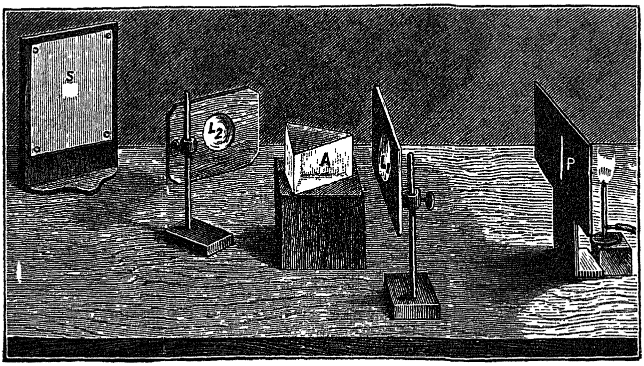


FIG. 219.—Experiment to show the dispersion of light by a prism

condition, the vibrations taking place at right angles to the direction in which the light is travelling; in this sense light waves may be compared to the ripples on a liquid surface. In just the same way that we can set up either long or short ripples in water, so the waves of light may be of different length, and it is this difference in wave length which determines the colour of the light. The shortest waves to which the eye is sensitive give rise to the sensation of violet, and the longest to that of red By special means it is possible to detect waves shorter than violet, or longer than red; but they cannot be observed by means of the unaided eye.

EXPT. 195.—**Dispersion by a prism.** In a piece of card cut a slit (P) about 2 cm. long and 1 mm. wide. Place the card, with the slit vertical, in front of a fish-tail gas flame (Fig. 219); and adjust the position of a converging lens (L_1) so that it is approximately at a distance equal to its focal length from the slit. Place a second lens (L_2) a few inches in front of L_1 and so that their axes coincide. Adjust the screen (S) so that a well-defined image of the slit is seen on its surface. Measure the distance between L_2 and S. Arrange a prism (A) on a stand, so that it is of the same height as the slit, and has its refracting edge vertical; and adjust its position so that the rays emerging from L_1 are incident at a suitable angle upon one of its faces. Catch the light emerging from the prism by a second lens (L_2). Move the position of the screen (S) until the coloured band of light is best seen. The distance of the screen from L_2 should be the same as before. Observe that the light is refracted towards the base of the prism, and that it is decomposed into constituent colours, which are differently bent by the prism. The violet light is refracted most and the red light least. Colours between these limits are bent by intermediate amounts. Name the colours you can see.

Analysis of light by a prism.—When a beam of sunlight is made to pass through two prisms similarly arranged instead of one, the coloured band or spectrum produced is longer, that is, the dispersion is greater. The amount of dispersion also depends upon the material of which a prism is made. Glass produces a much greater amount of dispersion than water; flint glass possesses twice the dispersive power of crown glass; carbon bisulphide, again, has even more dispersive power than flint glass.

Although a continuous band of colour is observed when sunlight, or limelight, or a gas- or candle-flame is seen through a prism, this **continuous spectrum** is not always produced. For when substances such as sodium, strontium, and lithium, or their compounds, are burnt in a non-luminous flame, and the light from the coloured flame observed through a prism, a spectrum is seen consisting of bright lines, which are different for different substances. A prism may thus be used, and is used, to analyse light. The light of incandescent sodium vapour, produced by burning common salt in a flame, when observed through a prism is characterised by a yellow line, and the light emitted by other substances when burning are each distinguished by rays of a

particular colour and position in the spectrum. It is thus possible to analyse a substance by examining the light it emits when rendered luminous by suitable means The instrument used in such investigations is termed a **spectroscope.**

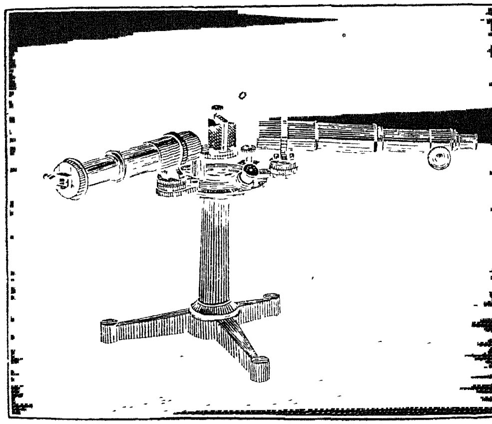


FIG 220 —A spectroscope

Spectrum analysis.—A spectroscope (Fig. 220) consists essentially of one or more prisms, with an arrangement known as a slit for limiting the breadth of the beam and a convex lens for making the rays parallel, and a small telescope for viewing the analysed light. When such an instrument is fitted upon a telescope and the telescope is directed towards the sun, a rainbow-coloured band having numerous dark lines at right angles to its length is observed These lines are the representatives of substances the luminous vapours of which exist in the sun, and by identifying them with lines produced by burning terrestrial substances, the materials of which the sun is composed have been found. The same principle applies to the stars or other celestial bodies.

Formation of white light from its constituents.—Just as it is possible to analyse white light, splitting it up into its constituent colours or wave-lengths, so, by suitable arrangements, these separated or dispersed colours can be made to recombine, forming white light over again. This building up, or synthesis, of white light can be effected in the following ways :

1. By interposing a second prism of the same material and angle as the first, with its angle reversed The dispersion of the first prism is neutralised, and the beam of light leaves the second prism in a direction parallel to the beam incident upon the first prism.

2. By the colour disc.

EXPT. 196.—**Recomposition of light by means of a second prism.**—Form a spectrum as described in Expt. 195. Place a second prism (B, Fig. 221), of the same material and having the same refracting

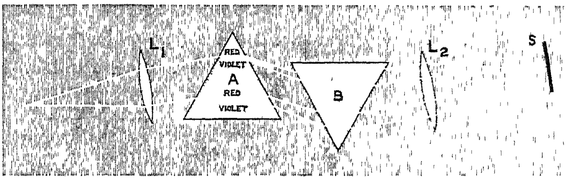


FIG 221 —Recomposition of light.

angle as prism A, in the position shown in the diagram, adjusting the second prism so that the rays fall upon one of its faces, and so that its faces are parallel to those of the first prism. Place the lens (L_2) and the screen (S) in the same relative position as before, and observe on the screen the *white* image of the source of light. Evidently the different colours of the spectrum have been recombined to form white light.

It is an important fact that, by the simple laws of refraction, the path of each of the rays through B must be parallel to its path through A; also, since the rays emerging from B must be parallel to the rays incident upon the prism A, the former rays must constitute a parallel beam, and they will be brought to a focus at the principal focus of the lens L_2, and a white image therefore is formed.

EXPT. 197.—**Recomposition by colour disc.** Upon a round piece of card paint sectors of the different colours contained in the spectrum, arranging the areas of the coloured sectors as nearly as possible in the proportion in which they occur in the spectrum.

Place the card upon a whirling table or upon a top, and rotate it rapidly, when it will be found that light from the card gives rise to the sensation of an impure white or grey.

The colour disc.—The explanation of the recombination of the separate colours of the spectrum by means of a rapidly revolving disc, as in Expt. 197, is very simple. It is due to

what is called the **persistence of images** on the retina of the eye. Each impression the retina receives lasts for a certain length of time—about one-tenth of a second. It is not an instantaneous impression only. Think of the common trick of whirling round a stick with a spark on the end which gives rise to the impression of a continuous circle of light. This is because the second impression of the spark is received by the eye before the first impression has died away. Similarly, the impression of one sector, say, a red one, has not disappeared before the next is received, and while these compounded impressions linger a third one comes along. The blurred total of all these rapidly occurring impressions produces the greyish white tinge seen when a colour disc is whirled.

Rainbows.—Rainbows are caused by sunlight falling upon drops of water whether in the form of rain or of spray. The observer must have his back to the sun; and the centre of the bow is the point in the sky directly opposite to that occupied by the sun at the time of observation. In the **primary rainbow** the red colour is on the outer edge and the violet on the inner edge, the other colours of the spectrum being between them; in the **secondary rainbow** sometimes seen above the primary one this order of colours is reversed, the red being on the inner edge and the violet on the outer. Suppose the sun to be on the horizon when an observer sees these rainbows. The centre of the bow would be on the opposite point of the horizon, the red top of the primary rainbow would be at an angle of about 42° above the horizon, and the violet edge would be about 2° below the red; the angular elevation of the secondary rainbow would be about 52°. The radius of the primary rainbow as a whole is always about 41° and that of the secondary bow about 52°. When, therefore, the sun is on the horizon the bows seen are the largest possible. As the position of the sun above the horizon increases, the centre of the bows gets more and more below the horizon, the arcs visible become smaller and smaller until, when the sun has an altitude of about 41°, the primary bow disappears; while the secondary bow also becomes invisible when the sun's altitude exceeds 52°. This explains why rainbows are never seen in the British Isles in the middle of the day in summer.

The optical cause of the rainbow is a little difficult to explain exactly in an elementary book, but Fig. 222 may enable the student to comprehend the general principle involved. Suppose the sun's rays to be falling in the direction indicated upon the raindrops A_1, A_2, A_3, A_4. The light falling upon A_1 passes into the raindrop, is internally reflected along the path *abc* and emerges dispersed into the spectrum colours from violet *v* to red *r*. Light entering the second drop A_2 is similarly affected. From the lower drop violet light reaches the observer's eye; and red light from the upper drop, while the various drops which may be imagined between these two contribute the intervening colours of the spectrum according to their positions. In the case of the secondary rainbow, the sun's rays undergo double internal reflection in the raindrops, as shown in A_3 and A_4. Light enters A_3, and after traversing the path *mopq* emerges, broken up into the spectrum colours from red to violet in the order indicated in the figure. Red light thus reaches the observer's eye from A_3 and violet light from A_4; and the intermediate colours are formed in similar manner.

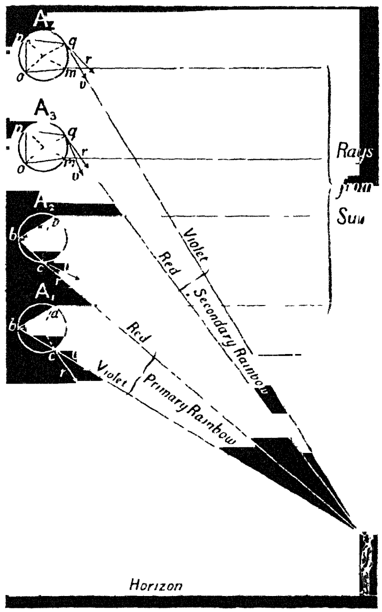


FIG. 222.—The formation of a rainbow. Adapted from the *U.S Monthly Weather Review*.

COLOUR.

Colour of transparent bodies.—The colour of transparent bodies is due to the constituents of white light transmitted by them. A blue solution through which the light from a lantern is passed is blue because, of all the colours of the spectrum, it is able to transmit easily only the blue rays; the others—green, yellow, orange, red, etc.—are absorbed by the solution. Consequently, if this trans-

mitted blue light falls upon a sheet of red glass it is, in its turn, absorbed; red glass only transmits red light, that is why it is red. So that a combination of the blue solution and a piece of red glass is quite opaque to light—none of the colours of the spectrum can pass. Similarly, pieces of red and blue glass together are, if thick enough, quite opaque. When a strip of coloured glass, or a solution in a narrow test tube, is held between a spectrum and a screen, it appears as a black shadow upon the screen in all parts of the spectrum except in the colour which it is able to transmit. Transparent bodies like glass, water, and so on, transmit all the colours of the spectrum with nearly equal facility, and therefore appear colourless when only thin layers are used. As, however, no substance transmits light of all wave-lengths equally, there is no perfectly transparent body.

EXPT. 198.—**Absorbed and transmitted rays.** (i) Make an Oxford blue liquid by adding ammonia solution to copper sulphate dissolved in water until the precipitate formed is re-dissolved. Place the blue liquid thus made in a glass cell. Focus the round hole of a lantern cap on the screen. Interpose the filled cell. Notice the pure blue colour. Now interpose red glass either before or behind the cell. Notice that no light can pass now.

(ii) Use a hollow prism containing carbon bi-sulphide to produce a long spectrum of a slit on the lantern cap. Pass the filled glass cell from the last experiment through the spectrum, and notice that it is only able to transmit blue light. Repeat the experiment with as many coloured transparent solutions as possible, *e.g.* a solution of bichromate of potash and of permanganate of potash. Notice in each case that a particular liquid is only able to transmit light of its own colour.

Colour of opaque bodies.—The colour of opaque bodies is due to the constituents of white light which they reflect. If the light from a lantern in an otherwise dark room be made to fall upon sheets of cardboard which have been painted with various brilliant colours, and the light reflected from the coloured sheets be caught on a white surface, it is at once seen that the colour of the light reflected is the same as that of the card from which it comes.

Coloured opaque bodies when passed through a spectrum only appear coloured when in that part of the spectrum which is the colour they appear to have in white light. A red substance like sealing-wax is red only when there are red rays falling upon it

which it can reflect. The sealing-wax absorbs all the other constituents of white light; and hence when it is held in blue light, or light of any other colour than red, since all the light rays of this colour are absorbed, no light is reflected and it appears black. A white opaque substance, like a sheet of paper, appears white because it reflects all the constituents of white light equally well. Similarly, when a card painted violet is passed through a spectrum it only appears violet when in the violet rays, and in all other colours it seems black, because it cannot reflect these colours.

EXPT. 199.—**Absorbed and reflected rays.** (i) Paint sheets of cardboard with various brilliant colours Send the light from a lantern in an otherwise dark room upon them, and catch the reflected light on white sheets of cardboard. Notice that the colour of the light reflected is the same as that of the card from which it is reflected.

(ii) Pass through the same spectrum various coloured opaque bodies, *e.g.* a rod of sealing-wax. Notice in this case it is only coloured when passing through the red rays. It appears a dull grey colour in most parts of the spectrum Observe that green leaves are only coloured when passing through the green part of the spectrum

Selective absorption and transmission.—In every case the colour of a body depends on selective absorption or selective transmission. Of the coloured rays of white light one portion is absorbed at the surface of the body. The body is **coloured and transparent** if the unabsorbed portion traverses it; if, on the contrary, the unabsorbed light is reflected the body is **coloured and opaque.** In both cases the colour depends upon the constituents of white light which are left to reach the eye after the other constituents have been absorbed. Bodies which reflect or transmit all colours in the proportion in which they exist in the spectrum are white; those which reflect or transmit none are black. Between these extreme limits infinite tints exist depending on the smaller or greater extent to which bodies reflect or transmit some colours and absorb others.

Bodies have no colour of their own; the colour of a body changes with the light which falls upon it. It is interesting to remember that this absorption of certain constituents of light necessitates a using up of energy. But since energy cannot be destroyed it is in these cases converted into heat. Theoretically,

a blue glass would get hotter than a red one, because the former absorbs all the red rays, and these have a greater heating effect than blue rays.

EXERCISES ON CHAPTER XXIV.

1. Describe and explain the effects observed when cards coloured bright red, green, and blue respectively, are passed from the red to the blue end of the spectrum

2. Some glass houses in which ferns are grown are constructed of green glass Describe the appearance, to an observer in such a house, of a lady in a red costume carrying a book with a bright blue cover. Give reasons for your answer.

3. How would you explain to a class of children the effect of a stained glass window upon sunlight? What simple experiments would you perform to convince them of the truth of your statements?

4. A ray of white light is passed through a glass prism; make a sketch showing how the direction of the ray is changed by its passage through the prism and the order of the colours seen when the light falls on a screen.

How would you show that when these colours are re-combined white light is produced?

5. Describe an arrangement by means of which a spectrum may be formed upon a screen.

If the light is made to fall upon a piece of red glass before reaching the screen, how and why will the spectrum be affected? What would the effect have been if blue glass had been used?

6. How can it be proved that:

(*a*) White light is a mixture of many colours? (*b*) Different colours have different degrees of refrangibility?

7. What is meant by the dispersion of light? On what fact does it depend?

8. Explain the term refrangibility as applied to a ray of light. Are rays of all colours equally refrangible?

9. It is sometimes said that "red glass colours the sunlight red," and that "blue glass colours the sunlight blue" Mention facts or experiments which show that this is not accurate. Put the statement in a more accurate form.

10. Bright sunlight falls obliquely upon the surface or the water contained in a white china basin; a penny is held near the surface of the water, and in such a position that its shadow falls upon the bottom of the basin. Parts of the shadow are found to be edged with colour. What colours may be observed? On what part of the shadow is each to be seen? How do you account for the colours?

11. What is a prism? Give a diagram showing the course of a ray of white light through a prism made of glass. Which colour is refracted most, and which least?

12. Describe the essential parts of either (*a*) a spectroscope, or (*b*) an astronomical telescope.

13. Explain the coloured fringes seen when objects are viewed through a glass lustre or prism.

14. Why does a field poppy appear red? What experiment could you arrange to make it appear black?

15. Several coloured cards (*e.g.* red, blue, green) are lying on a table in a lighted room. State how far the colours could be identified by a person seeing them through a piece of cobalt (blue) glass. Give reasons for your answer.

16. Light from a slit is allowed to fall on a prism. State and explain what may be observed when the slit is illuminated with (i) sodium flame, (ii) white light.

17. Describe a spectroscope. What will be observed in the instrument when the light passing through the slit comes from (i) a spirit lamp with a salted wick, (ii) an electric light, (iii) an electric light in a red glass globe?

18. How would you test, in a given instance, whether the light transmitted by a blue glass is homogeneous or not?

PART V.

SOUND.

CHAPTER XXV.

VIBRATORY MOTION.

Wave motion.—The circular waves set up when a stone is dropped into still water is a phenomenon familiar to all. Although the waves appear to travel outwards, yet the water itself is not travelling so; for, if a row of corks be floating on the water the corks move *up and down* only, and their distance from the centre of the waves is not increased: in fact, it is the 'disturbance' alone which travels outwards. This is an example of true wave motion, which can be defined in the following terms: **Wave motion consists in the repeated motion of a series of particles, the motion being handed on from each particle to its neighbour.**

When the stone touches the water, it causes a depression; and, like all elastic bodies which, when deformed, tend to resume their original shape, the water tends to recover its original level. Consequently, water flows into the depression; but, in so doing, the property of **inertia** (p. 103) causes the inflowing water to 'overshoot the mark,' and the depression is followed by an elevation. This is again followed by a depression, and so on. But the disturbance is not restricted to this one point; for, through the agency of the property termed cohesion (p. 57), the disturbance is handed on to neighbouring particles. In fact, the condition resembles what might be observed with a row of pendulums, the bobs of which are joined by spiral springs. A transverse motion to and fro of the first

pendulum will influence the next pendulum, imparting to it a similar motion. This will be handed on to each pendulum in turn, resulting in a kind of wave motion along the row. In this case the persistence of the motion is due to the inertia of each pendulum, and its transference along the row is due to the elasticity of the springs connecting the bobs. Subsequent paragraphs will show how a wave motion, of one type or another, may be propagated through any medium possessing these properties of inertia and elasticity.

Simple harmonic motion.—Suppose a simple pendulum to be set swinging so that its bob describes a circular path: it will traverse the path with uniform velocity. If looked at downwards in a sloping direction, the path will appear to be an ellipse (Fig. 223); but if the eye be placed at the same horizontal level it will appear to move along the straight line *Abcd*.... Suppose the circular path to be divided into 16 equal portions, AB, BC, etc.; then when viewed horizontally, the distances *Ab*, *bc*, *cd*, etc., will be the apparent distances traversed in equal time intervals. At both A and I the bob will appear to be momentarily at rest: and its velocity will appear to be a maximum at *e*. This apparent motion along the path *Abcd* is termed **Simple Harmonic Motion** (S.H.M.).

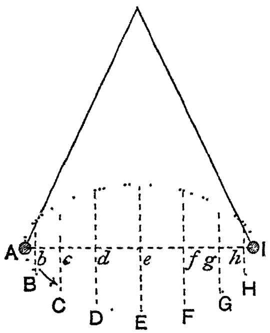


FIG 223.—A simple pendulum, describing a circular path.

The total time-interval occupied in passing from its position of rest, at *e*, until it again passes the same point in the same direction is termed the **period** of the vibrating body. The **phase** of the vibrating body at any instant is the fraction of a total period which has elapsed since passing the position of rest in a given direction, *e.g.* from left to right. Thus, the phase when passing *f* from right to left will be 7/16. The **amplitude** is the extreme distance to which the body moves from its position of rest; thus, in Fig. 223, the amplitude is *e*A.

Transverse wave motion.—In Fig. 224 (i), let A, B, C, ... represent the path of a particle moving up and down with S.H.M. The positions A, B, C, etc., occupied after equal intervals of time, are located by means of the generating circle,

the circumference of which is divided in this case into 12 equal parts.

Suppose a similar motion to be set up in a number of particles, 1, 2, 3, 4, etc., arranged along a straight line at equal distances apart; and suppose the motion to be handed on from one particle to the next. Let the amplitude of each particle be equal to AD, and let consecutive particles differ in phase by $\frac{1}{12}$ of a complete period.

If particle 1 be moving downwards through its position of rest, particle 2 will then be moving downwards at *a*; particle

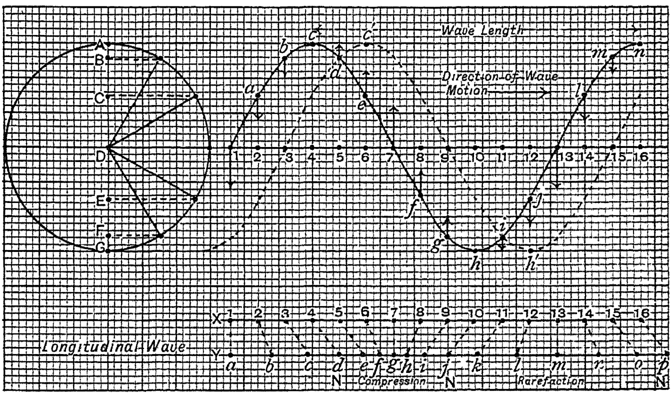


FIG. 224.—Wave motion

4 will be momentarily at rest at *c*; particles 5 and 6 will be moving upwards at *d* and *e* respectively; 7 will be passing its position of rest in an upward direction, and so on If these instantaneous positions are joined by a continuous line, a wave outline is obtained which closely resembles the outline of a water wave.

The dotted outline indicates the instantaneous positions occupied by the particles at a later moment, viz., $\frac{1}{6}$ of a period later. The elevation at *c* has now moved forward to the position *c′*; and, after one complete period has elapsed, the elevation at *c* will have travelled to the point *n*. Thus, the wave motion is propagated onwards; and, since the particles vibrate

transversely to the direction in which the waves are moving, the effect is termed a **transverse wave motion.**

The **wave length** is defined as the least distance between any two particles which are in the same phase, thus, the distances *cn*, *bm*, or *al* are equal in each case to the wave length.

The **velocity** of the wave motion is the distance through which the disturbance is propagated in unit time. The **frequency** is the number of complete waves which pass a given fixed point in unit time.

Suppose that n wave crests pass any fixed point in unit time, and that the wave length is l. Then, if V be the velocity of the wave motion, the first crest will have travelled in unit time to a distance V; hence

$$V = n \times l,$$

or velocity = frequency × wave length.

Transverse waves cannot be transmitted through gases, for the elasticity of a gas is brought into operation only when the gas is compressed or rarefied. Suppose, then, that the consecutive particles of a gas are set in vibration to and fro along, instead of transversely to, the line of disturbance. The distances apart of consecutive particles will vary—at one instant the particles being closer together than normally, and further apart at another instant. Such waves are termed **longitudinal waves**; and it may be proved that the transmission of sound through gases involves this type of wave only. Sound is transmitted also through solids and liquids by means of the same type.

Longitudinal wave motion. In Fig. 224 (ii) a row of equidistant particles 1, 2, 3,... are arranged along the straight line X. Suppose that the particles are set in S.H.M. horizontally, and that the phase difference between consecutive particles is equal to $\frac{1}{6}$ of a complete period. The wave curve *abcd*... of Fig. 224 (i) may be regarded as a *displacement curve* for determining the instantaneous positions of the particles: an upward displacement in Fig. 224 (i) corresponding to a displacement forwards in Fig. 224 (ii), and a displacement downwards to a displacement backwards. Thus, the instantaneous positions of the particles will be represented by the points *abcd*... along the line Y. It is evident that, at the instant represented, the

particles 5 to 9 are closer together, and the particles 11 to 15 are further apart, than normal; or, in other words, these are regions of compression and of rarefaction respectively. Also, the pressures in the neighbourhoods of particles 4 and 10 must be normal. A longitudinal wave may be considered, therefore, to consist of a sequence of compressions and rarefactions separated by regions of normal pressure; and this condition is transmitted onwards with definite velocity.

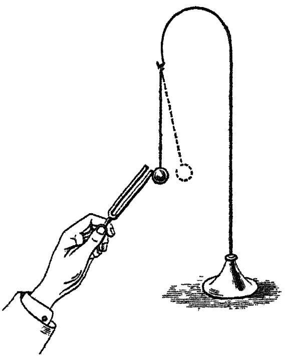


FIG. 225.—Expt 200

The production of sound. The origin of all sound is motion. Thus, when sound is derived from a stretched string, its indistinct outline shows that it is vibrating rapidly to and fro.

EXPT. 200.—**Vibration and sound.** Strike the end of one prong of a tuning-fork on the knee, or on a hard cushion. Hold the fork so that the outer side of one of the prongs touches either (i) the lip, or (ii) a pith ball suspended by thread (Fig. 225) or (iii) the surface of water contained in a beaker. The effect, in either case, demonstrates that the prongs are vibrating to and fro.

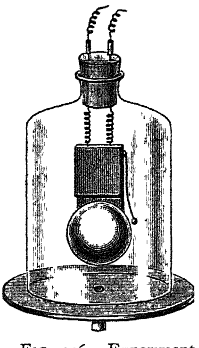


FIG 226 —Experiment to show that sound is not transmitted through a vacuum

The transmission of sound.—The fact that some definite medium is necessary for the transmission of sound may be demonstrated by means of the apparatus shown in Fig. 226. An electric bell is suspended by wire springs inside the bell jar of an air-pump. The bell is connected electrically to a voltaic cell by means of wires passing through the rubber stopper of the bell jar. When the jar is full of air the ringing of the bell can be heard distinctly; but, as the air is exhausted, the sound becomes almost inaudible.

Solid media, also, may serve for the transmission of sound: thus, if a watch be placed on one end of a table (or, in contact with one end of a long rod) the ticking is heard distinctly by an ear placed in contact with the other end of the table or rod.

The generation of sound waves by a vibrating body.—In Fig. 227 (i), the letters *a*, *b*, and *c* represent the successive positions of one prong of a vibrating tuning fork. The movement of the prong resembles closely that of a simple pendulum: when in position *a* the prong is momentarily at rest and is commencing to move towards the right; its velocity increases until it is a maximum at *b*, and then diminishes until it reaches *c*, when it is again momentarily at rest. Moreover, the movements are **isochronous**, that is, they are performed in the same time, whether they are small or large.

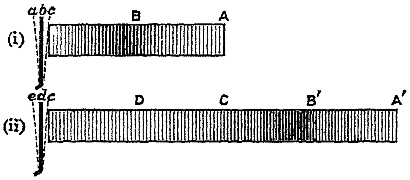


FIG. 227 —Compression wave caused by a vibrating prong of a tuning-fork

Consider the effect of this movement upon a column of air situated to the right of the prong. The first movement from position *a* causes in the air a slight compression which is propagated outwards with the velocity of sound; subsequent movement of the prong through position *b* to position *c* causes similar compression pulses, but the compression will be most pronounced at the moment when the prong has maximum velocity, *i.e.* when it is passing position *b*. When the prong reaches position *c*, the condition of the air column will resemble that shown in Fig. 227 (i); the first compression will have travelled to a position A, and the pulse of maximum compression will be situated at B. Fig. 227 (ii) represents the subsequent condition of the air column when the prong has moved back again, through position *d*, to its original position *e*, thus completing one vibration. The movement of the prong from right to left tends to leave a partial vacuum behind it, and the air is rarefied partially; this rarefaction is a maximum when the prong is moving with maximum velocity through the position *d*.

At the moment when the prong has completed one vibration, the air column will be in the condition represented. The first compression A will have travelled outwards to A′; the region of maximum condensation B will have moved to B′. The region at C will be momentarily in its normal condition; and the region of maximum rarefaction will be at D. The condition thus set up is one complete **sound wave.** After the second vibration of the prong, the first disturbance will have travelled away to a distance twice as far away as A′, and the space between A′ and the fork will be again in the same condition of condensation and rarefaction as shown in Fig. 227 (ii).

It must be borne in mind that the sound waves are not restricted to narrow columns of air, as shown in Fig. 227; in fact, the surrounding air in almost all directions is affected in a similar manner. This would be realised more fully if a point source of sound waves could be considered; and, in such a case, we could regard the point as being surrounded with spherical envelopes of compression and rarefaction spreading rapidly outwards with the velocity of sound.

The **loudness** or **intensity** of the sound depends simply upon the energy contained in that portion of the waves which strike the drum of the ear. The energy contained in any single wave remains practically constant; and, since each wave is distributed over a spherical surface which is enlarging rapidly, the energy passing through each unit area of a wave surface depends upon the distance of that surface from the source.

Since the area of a sphere varies directly as the square of the radius, the energy of the pulse when at a distance of 2 metres from the source will be distributed over an area four times as great as when the pulse was at a distance of 1 metre; and an ear placed at the former distance will experience the effect of only one-fourth the energy which it would experience when placed at the latter distance. Hence, **the loudness of the sound varies inversely as the square of the distance from the source.**

Reflection of sound.—Sound waves are reflected according to the same laws which govern the reflection of waves of light by plane or spherical mirrors, but the conditions under which these two phenomena may be observed differ on account of the wide

difference between the length of light waves and the length of sound waves and also because light waves are transmitted by the ether, whereas sound waves require a material medium for their transmission. The wave length of the lowest audible note is about 36 ft., and that of the highest audible note is about half an inch: these lengths are very great in comparison with the wave length of light. In order that the phenomenon of reflection may be observed, the surface must be large in comparison with the wave length of the vibrations falling upon it. Hence, for the reflection of sound, a surface of considerable area is required. On the other hand, long waves do not require the surface to be so smooth. Consequently, a comparatively rough surface, such as a sheet of cardboard, a wooden board, or a brick wall, serve to produce the phenomenon of reflection of sound

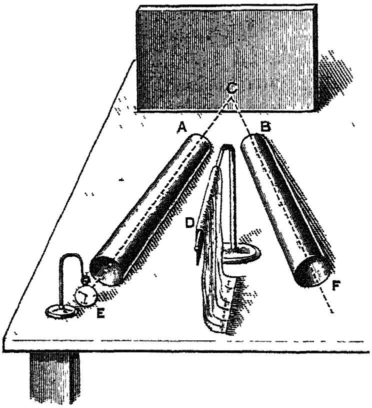


FIG. 228 —Experiment on the reflection of sound.

EXPT. 201. — **Reflection of sound.** A and B (Fig. 228) are two tin-plate tubes, about 1 yard long and 3 inches diameter. Support them horizontally in the positions shown. Suspend a watch at E near to the end of one tube. Place the ear near to the other end of the tube at F. If the sound can be heard, hang up a screen (such as a wet towel) at D so as to prevent the sound waves from passing directly to the ear Now place a flat reflector in a vertical position at C, and slowly rotate it round a vertical axis. Notice the position of the reflector when the sound can be heard; and observe also the effect of tilting the reflector forwards or backwards.

Echoes are familiar examples of the reflection of sound, and they can be observed frequently in the neighbourhood of houses, high walls, cliffs, or hillsides. If the sound be generated near the observer, it is necessary that the reflecting surface be not less than a certain distance away, otherwise the echo is confused

with the original sound, owing to the fact that the impression on the ear of a sound wave persists for at least $\frac{1}{10}$ of a second The reflecting surface must be therefore at such a distance that the sound wave will require at least this interval of time to travel to the surface and back again.

A peculiar phenomenon of echo can be observed occasionally when a single wave-pulse of sound is reflected from a series of receding steps, as in a staircase, or from a series of separate flat wooden posts along a roadside. The greater distance of consecutive reflecting surfaces causes consecutive echoes to be retarded slightly in reaching the ear. When the interval between the echoes is regular, the sequence will give rise to a note of which the pitch corresponds to the frequency of the echoes.

Masses of water vapour, such as clouds, or of any gas denser than the air, will serve as surfaces for the reflection of sound. Thus, the irregular roll of thunder which often follows a flash of lightning consists in the numerous and overlapping echoes due to direct reflection from cloud surfaces at different distances and to multiple reflection of the waves between two or more cloud surfaces, or between the clouds and the earth. The original noise is extremely brief, and its duration corresponds with that of the lightning flash.

When a source of sound, such as a watch, is placed at the focus of a concave reflecting surface, the sound waves are reflected along parallel paths, and they can be detected at a greater distance than if the 'sound-mirror' is not used. In the **speaking-tube** the waves are reflected repeatedly from side to side of the tube; and, instead of the energy of the sound waves being distributed through a rapidly increasing space, it remains more or less concentrated within the limits of the tube, and sufficiently so for the sounds to be detected by an ear placed at the distant end.

EXERCISES ON CHAPTER XXV.

1. A regular series of waves is transmitted along a flexible cord. What do you understand by the wave-length and the amplitude of the waves? Would you call the waves longitudinal or transverse? Explain the meaning of these terms.

2. An observer on the sea-shore notes that the waves are breaking at the rate of 10 per minute, and that they take two minutes to reach the shore from a rock 100 yards out at sea. Find the mean wave-length and the velocity of propagation in feet per second.

3. What is meant by saying that the vibration of a pendulum or tuning-fork is isochronous? What would be the effect observed after striking a tuning-fork if the frequency of the vibration increased in the same proportion as the amplitude diminished?

4. Explain the formation of echoes. How is it that the sharp crack accompanying a flash of lightning produces the long roll of the thunder?

5. Two observers are stationed at a distance of 1 mile and $\frac{1}{2}$ mile respectively from a point where a bugle note is sounded. If there are no reflections of the sound, how much louder will the sound appear to the second observer than to the first?

6. Describe an experiment showing that air or some other medium is necessary for the transmission of sound. What practical difficulty arises in such an experiment?

7. Describe an experiment to show that sound is not propagated through a vacuum.

How does the air move when sound waves travel through it?

8. Explain the formation of echoes. A person standing 92 yards from the foot of a cliff hears the echo half a second after clapping his hands. Deduce the velocity of sound in air.

9. Show what must be the direction of sound vibrations, relative to the direction of propagation, in order that sound may be transmitted by a gas. What characteristics of the vibrations determine the character of the sound heard?

10. Describe the action of a vibrating tuning-fork.

11. Describe two experiments which show that sound waves can be reflected. A sharp tap is sounded in front of a long flight of stairs. What impression would you get if you were standing in front of the stairs?

CHAPTER XXVI.

ELASTICITY. VELOCITY OF SOUND.

Elasticity.—The influence of elasticity upon the transmission of wave motion has been referred to previously (p. 328). All forms of matter, when subjected to external forces, undergo changes of volume or of shape; and when the forces are removed, the matter tends to recover more or less completely its original volume or shape. This power of recovery is termed **elasticity.** Thus, a bent clock-spring or a stretched steel wire are examples of bodies which possess elasticity to a high degree. Liquids and gases offer resistance to change of volume only, and not to change of form; and we may say that such substances possess considerable *volume-elasticity.*

The change of volume, or of form, which a substance undergoes, is termed the **strain**; and the force causing it is termed the **stress.** The ratio **stress/strain** is termed the **coefficient of elasticity.**

Elasticity of gases.—When a given volume of a gas, confined under an observed pressure, is subjected to an increased pressure, the volume is diminished by a definite amount, in accordance with Boyle's Law (p. 83). Thus, suppose v c.c. of a gas be measured at a pressure p dynes/cm.^{2}; and let the volume be reduced to $(v-dv)$ c.c. when the pressure is increased to $(p+dp)$ dynes/cm.^{2}, where dv and dp represent small changes in volume and pressure respectively. Then, since the strain is measured by the change in volume of each unit volume of gas, it is represented by the ratio dv/v; and the stress producing this strain is dp dynes. Hence,

$$\textbf{coefficient of volume-elasticity} = \frac{dp}{dv/v} = \frac{v\,.\,dp}{dv}.$$

It can be shown that, in the case of a gas, *this coefficient is numerically equal to the original pressure.* For, by Boyle's Law,

$$pv=(p+dp)(v-dv)$$
$$=pv-p\,.\,dv+v\,.\,dp-dp\,.\,dv.$$

But, since both dp and dv are very small, their product may be neglected;

$$\therefore\quad p\,.\,dv=v\,.\,dp$$

$$\text{or}\quad \frac{v\,.\,dp}{dv}=p.$$

Velocity of sound in gases.—Steam is seen issuing from the whistle of a distant locomotive sooner than the sound is heard: the flash of a gun, or the striking of a cricket ball with the bat, is seen before the sound reaches the observer: so also the lightning flash precedes the thunder. Such observations demonstrate that time is required for the transmission of sound from one point to another.

It was proved theoretically by Sir Isaac Newton that the velocity (V) of sound in a gas varies directly as the square root of the volume-elasticity (E) of the gas, and inversely as the square root of the density (D). Or, expressed as an equation,

$$V=\sqrt{\frac{E}{D}}.$$

For example, the density of air, at 0° C. and at a pressure of 76 cm. of mercury, is 0·001293 gm./c.c.; and, by the previous paragraph, E is measured by the pressure acting upon it. The pressure on each sq. cm., due to a column of mercury 76 cm. high, is ($76 \times 13·6 \times 981$) dynes. Therefore

$$V=\sqrt{(76\times 13·6\times 981)/0·001293}=28000 \text{ cm./sec.}$$

This value is not in agreement with that obtained by actual experiment, viz 33180 cm./sec.; and the cause of the discrepancy was not explained until a much later date, when it was proved that the numerator of the above equation should be increased by the product 1·41. In this case,

$$V=\sqrt{(1·41\times E)/D}=33170 \text{ cm /sec.}$$

Experimental determination of velocity of sound in air.—The first attempt to measure the velocity of sound in air was made in 1738 under the auspices of the French Academy of Sciences. Two groups of observers, with cannon, were stationed on hills

about 17 miles apart; one of the cannon was discharged, and the observers on the distant hill measured the time interval between the flash of the cannon and the sound of the discharge. The observation was made in both directions so as to eliminate the effect of wind. From the observations it was calculated that the velocity at 0° C. is about 332 metres per second. Taking these and more recent experiments into consideration, the mean value deduced for the velocity at 0° C. is 331·7 metres (or 1088 ft.) per sec.

Effect of various conditions on the velocity of sound.—Since, by Boyle's Law, the volume occupied by a given mass of gas is inversely proportional to the pressure, it follows that the *density of a gas must be directly proportional to the pressure.* The elasticity of a gas is also directly proportional to the pressure. A change of pressure thus affects equally both the density and the pressure of a gas. The relation of numerator to denominator in the ratio *elasticity/density* is therefore unaltered by any change in the pressure. Hence, the velocity of sound is the same at any atmospheric pressure. This conclusion has been verified by experimental determinations of the velocity at high altitudes.

An increase in temperature reduces the density of a gas. Hence, the velocity of sound increases with rise of temperature, and diminishes with fall of temperature. Based upon the known rate at which a gas expands (p. 164), and upon the fact that the density of a gas is inversely proportional to the volume occupied by a given mass of the gas, it can be proved readily that $V_t = V_0\sqrt{(1 + \alpha t)}$, where V_t and V_0 are the velocity of sound at $t°$ C. and 0° C., and α is the coefficient of expansion of a gas (viz. 0·00368). From this equation the velocity of sound at any temperature $t°$ C. is

$$(33170 + 61t) \text{ cm. per sec},$$

or

$$(1088 + 2t) \text{ ft. per sec.}$$

Damp air is a mixture of ordinary dry air and water vapour. Since the density of water vapour, at ordinary temperatures, is less than that of dry air in the ratio 0·62 : 1, the density of damp air is less than that of dry air at the same temperature and pressure. Hence the velocity of sound in damp air must be greater than in dry air.

As the velocity varies inversely as the square root of the density, other conditions being the same, its relative value in any two gases can be obtained from the densities of the gases. For example, air has a density of 14·3 compared with hydrogen, hence

$$\frac{\text{velocity of sound in hydrogen}}{\text{velocity of sound in air}} = \sqrt{\frac{14 \cdot 3}{1}}$$
$$= 3 \cdot 8.$$

Taking the velocity of sound in air to be 1088 feet per second, that of hydrogen is therefore $1088 \times 3 \cdot 8$ or 4134 feet per second. Similarly, as oxygen is 16 times denser than hydrogen, and $\sqrt{16} = 4$, the velocity of sound in oxygen is $\frac{1}{4}$ of 4134, or 1033 feet per second

Velocity in solids and liquids.—By striking with a hammer one end of a long series of connected iron pipes and determining the interval between the sound transmitted by the pipes and that conveyed by the air, the relative velocity of sound in iron and air was determined by two French investigators. The total length of the pipes was 951 metres and the temperature of the air was 11° C. Calculations showed that at this temperature the sound waves in air would take 2 8 seconds to travel 951 metres. The sound was heard through the iron 2 5 seconds before that transmitted by the air, so that it took only 0·3 second to travel along the pipes The sound was therefore transmitted by the iron about nine times (2·8/0·3) faster than by the air.

The velocity of sound in water was determined by two observers on the Lake of Geneva in 1826. Two boats were moored about eight miles apart. Suspended from one was a large bell immersed in the water and from the other a tube with a trumpet-shaped receiver to catch the sound. When the bell was struck some gunpowder was ignited, and the interval between seeing this light and hearing the sound of the bell gave the velocity of sound in the water. The value found was about 1430 metres per second These direct methods of determining the velocity of sound are chiefly of historical interest; indirect methods are now employed in most cases. Thus, the formula $V = \sqrt{E/D}$ can be applied to solids or liquids, provided that the elastic constant E is known for the direction in which the waves travel, and is expressed in suitable units. Methods of determining the velocity in bodies which can be obtained in the form of rods are described in Chapter XXVIII.

EXERCISES ON CHAPTER XXVI.

1. Explain how the velocity of sound in air may be measured by two observers stationed some distance apart. How could the measurement be made independent of the velocity of the wind between the stations?

2. A man stationed between two parallel cliffs fires a gun. He hears the first echo after two seconds, and the next after five seconds. What is his position between the cliffs, and when will he hear the third echo?

3. A rifle-bullet is fired against a target one mile distant with an average velocity of 1200 ft. per second. Does the bullet, or the sound of the firing, reach the target first? If the temperature of the air is 61° F., what is the interval of time between the two arrivals?

4. Describe a method of determining the velocity of sound in air. Will the result obtained be the same in summer and in winter? Give reasons for your answer.

5. Two sources of sound A and B are situated at distances of 100 metres and 300 metres respectively from an observer, who estimates the intensity of the sound from A to be four times as great as that from B. Compare the amplitudes of vibration of the two sound waves (i) near the observer, (ii) at equal but small distances from the respective sources.

6. Describe a method of determining the velocity of sound in the open air. How will the result be affected by wind, and how can the effect of wind be allowed for?

7. How is sound propagated? Is the velocity of sound in air constant?

8. If the velocity of sound in a gas varies directly as the square root of the pressure and inversely as the square root of the density of the gas, show what effect change of temperature has on the velocity.

9. How is the velocity of sound in air affected by changes of temperature? Is it affected by changes of pressure?

CHAPTER XXVII.

MUSICAL SOUNDS.

Distinction between a musical note and a noise.—In considering continuous sounds, as distinct from short sharp sounds such as an explosion, it is necessary to realise the physical difference between a **musical note** and a **noise.**

In the case of a musical note, the vibrations are comparatively simple and they fall upon the ear with uniform frequency; and in the case of a noise, the vibrations are complex and of irregular frequency. The sound obtained from an organ pipe and the screech of a parrot are examples of these two classes of sound.

Loudness and pitch of musical notes.—The loudness of a note depends simply upon the amplitude of vibration of the source of the sound: the greater the amplitude the louder the sound.

Observation of a vibrating tuning-fork or of a stretched string will show that as the amplitude of vibration diminishes the loudness of the note diminishes; but the pitch, which depends solely upon the frequency of vibration, remains absolutely the same.

EXPT. 202.—**Smoked-glass record of vibration.** Make a thin metal style, about 1 cm. long, from thin sheet brass, or from brass wire beaten out flat with a hammer, and fix it with wax to one prong of a tuning-fork. Blacken the surface of a glass plate by holding it over the flame of burning camphor or over a yellow gas flame. Lay the glass on a table, strike the tuning-fork, and rapidly draw it across the plate

FIG. 229.—Trace of a vibrating tuning-fork.

so that the smoked surface is just touched by the style. Notice how the amplitude of the wave-trace is greater at the beginning than at the end.

Pitch of a note.—The influence of frequency of vibration on the pitch of a note may be observed by means of **Savart's toothed wheel** (Fig. 230), which consists of a toothed wheel (A) capable of being rotated rapidly while a piece of thin board is held in a position so that the teeth strike against the edge of the board. Although the sound emitted is harsh and unmusical, yet a definite pitch is unmistakable; and, by varying the speed of rotation, it can be made evident that the pitch is raised when the frequency with which the teeth strike the board is increased.

The **disc siren** affords more complete information on the relationship of pitch to frequency of vibration It consists of a cardboard disc, pierced with several concentric rows of holes, which can be rotated rapidly on a whirling-table.

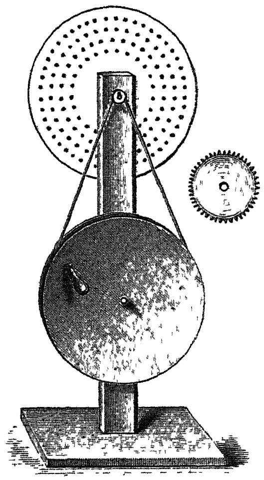


FIG 230.—Disc siren. The wheel A can be fixed upon the spindle instead of the disc.

The rows, commencing with the innermost, should have 24, 30, 36, and 48 holes respectively. When a stream of air, blown through a narrow jet such as can be made from glass-tubing, is made to impinge on either row of holes of the rotating disc a note of definite pitch is obtained. The sound arises from the fact that when a hole passes in front of the jet a momentary wave of compression is formed immediately behind the disc; and, during the interval between the passage of two consecutive holes, the inertia of the air causes a partial rarefaction to be set up behind the disc. This is followed by a compression when the next hole passes in front of the jet. Thus, we obtain a regular sequence of pulses of compression and rarefaction, which travel outwards with the velocity of sound.

If the speed be increased the same row of holes will give rise to a note of higher pitch.

If the speed remains constant, and the jet be brought successively in front of each row of holes, commencing with the inner row, the series of notes obtained will be recognised by those acquainted with musical intervals as the **third**, **fifth**, and **octave** above the lowest note.

Relation of pitch to rapidity of vibration.—If while the disc siren, described in the previous paragraph, is rotating, the jet be held near to the inner row and then to the outer row, the frequency of the impulses will be in the ratio 24 : 48, or 1 . 2, and the note of higher pitch is an octave above the lower. When the speed of rotation is increased, both notes become sharper, *but the higher note is still an octave above the lower.* When any two musical notes having this striking relationship are sounded, we can always state that the pitch, or rate of vibration, of one note is double that of the other.

Both Savart's wheel and some forms of siren often have an arrangement by which the number of teeth or holes passing a point in one second is recorded. The number of vibrations which produces a note of a particular pitch can thus be determined. Similarly, to find the vibration number of a tuning-fork, or any other body sounding a single note, a Savart's wheel or a siren having a counter connected with it is brought in unison with the note and the number of vibrations per second indicated by the dial is then observed.

A fact of everyday experience will help the student to understand how the pitch of a note depends upon the number of impulses which reach the ear per second. If the whistle of a passing express train be listened to it will be noticed that the pitch of the note while the train is approaching is distinctly higher than when it is receding. The whistle itself is sending out sound waves at exactly equal intervals of time; but, in the interval between the sending of any two consecutive waves, the train will have moved slightly forward, and the distance between any two consecutive condensations (or rarefactions) will be *less* than it would be if the train were stationary. The waves, therefore, are of shorter length, and the pitch of the note correspondingly higher. Similarly, when the train is receding, the wave-length will be correspondingly lengthened, and the pitch lowered. The explanation of this phenomenon is known as **Doppler's principle.**

Musical intervals, and the major diatonic scale.—The ratio between the rates of vibration of two notes is termed the **interval** between the notes Thus, when the disc siren is rotating at constant speed, the interval between the notes derived from the innermost row of holes and the second row is 30/24, or 5/4. This interval is termed a **major third**. The interval between the notes given by the 2nd and 3rd rows is 36/30, or 6/5: this interval is termed a **minor third**. The interval between the notes of the 1st and 3rd rows is 3/2, and is termed a **major fifth**. Since $3/2 = 5/4 \times 6/5$, it is evident that when two intervals are added together the resultant interval is expressed by their numerical product. If the three notes given by the inner rows of holes in the disc siren, or the three notes C, E, and G, of a piano, are sounded together they form a pleasing combination: it is now known as the **major chord**. The relative frequencies of these notes may be expressed by the numbers 24, 30, and 36; and these numbers have the ratio 4 : 5 : 6.

The major diatonic scale, such as is represented by the sequence of white notes of a piano, commencing with middle C, is built up in the following manner: A second major chord is obtained by starting from C′, the octave of C, and descending in the ratio 6 : 5 : 4. This gives frequencies of 48, 40, and 32; and these correspond to the notes C′, A, and F. This set of three notes is known as the **sub-dominant chord**. Finally, a third major chord is obtained by starting from G, and ascending in the ratio 4 : 5 : 6. This gives frequencies of 36, 45, and 54; and these correspond to the notes G, B, and D′. This is known as the **dominant chord**. The note D′ is above the octave of C, and its lower octave D, having a frequency 27, falls between C and E. Thus we obtain the following sequence of notes into which the octave may be divided

Notes	C	D	E	F	G	A	B	C′
Vibrations per second	256	288	320	341·3	384	426·6	480	512
Frequency	24	27	30	32	36	40	45	48
Interval (compared with C)	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

The sonometer.—The vibrations of wires or strings can be studied conveniently by means of the **sonometer** or **monochord**,

one form* of which is shown in Fig. 231. The essential parts are a sounding-box with wires stretched along it, upon one of which weights can be hung. Near the ends are fixed metal edges called 'bridges'; and similar bridges, which are movable, are required for the purpose of shortening the length of the vibrating string.

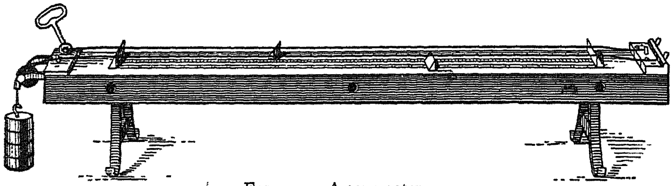


FIG. 231.—A sonometer.

When a string is vibrating the point which has greatest amplitude is termed an **antinode** or **loop**, and the points which are stationary are termed **nodes.** This simplest mode of vibration of a string occurs when there is one antinode, in the centre, and a node at each end. When vibrating in this manner the wire gives out its **fundamental note.**

It will be explained in a subsequent paragraph how, by fixing points of the wire other than the extreme ends, the wire may be made to vibrate in 2, 3, 4, or more separate portions (Fig. 233); in such cases the number of nodes and antinodes is greater than when the wire is giving its fundamental note.

Laws of vibrating strings.—The rate of vibration of a stretched wire or string depends upon the length, the stretching force, and the mass of unit length of the wire. The relationship between the rate and these several conditions is expressed in the equation

$$n = \frac{1}{2l}\sqrt{\frac{F}{m}},$$

where n is the rate of vibration of a string of length l cm., and of mass m gm. per unit length, when stretched by a force F dynes. The above equation suggests that the rate of vibration is

(i) directly proportional to the square root of the stretching force, and

* Experiments are more satisfactory if the sonometer is suspended against a wall in a vertical position, and with its lower end projecting slightly forwards so as to ensure good contact between the wire to which weights are attached and the lower bridge.

(ii) inversely proportional to the length and to the square root of the mass of unit length.

EXPT. 203 (i).—**Length of the wire.** Tune the two wires until they are in unison. Shorten by means of a movable bridge one of the wires A until it gives the octave above its original note as compared with the unaltered wire B. Its rate of vibration is twice its previous rate. Measure its length and note whether this is equal to one-half its previous length.

(ii) By means of another movable bridge tune the previously unaltered wire B until in unison with the shortened wire A. Move the bridge under A until its shorter position gives a note one octave above B, and therefore two octaves above its fundamental note Its rate of vibration is now four times as great as its initial rate. Measure its length and note whether this is equal to one-fourth of its initial length.

(iii) If two tuning-forks of known rate of vibration are available, measure the lengths of one of the wires required to give notes in unison with the two forks respectively, keeping the tension the same. Note whether the ratio of the lengths is equal to the inverse ratio of the rates of the two forks.

EXPT. 204.—**The stretching force.** Stretch a thin wire on the sonometer with a weight of one kilogram, and tune the other wire to unison. Increase the stretching force to four kilograms, and find by comparison with the other wire whether the note now given is one octave above the previous note.

Try to verify the law when the stretching forces are weights of two and of three kilograms.

EXPT. 205.—**Diameter and nature of the wire.** Select two wires (A and B) of different material, *e.g.* brass and steel, or two wires of the same material but of different diameter. Stretch one of them (A) with a known weight, and determine the length l_1 of the fixed wire C which is in unison with it. Make file marks on the wire A where it touches the bridges, unhang the weight, and cut the wire at the file marks by means of wire cutters. Weigh this length of wire, and determine the mass m_1 of unit length. Stretch wire B with the same weight as before, and determine the length l_2 of the wire C which is in unison with it. Proceed, as before, to find the mass m_2 of unit length of wire B.

If n_1 and n_2 are the frequency of vibration of the wires A and B, then $n_1/n_2=l_2/l_1$. Also, by the above equation, $n_1/n_2=\sqrt{m_2/m_1}$. Cal-

culate the values of the ratios l_2/l_1 and $\sqrt{m_2/m_1}$, and observe whether they are equal.

Beats.—When two notes very nearly in unison, and of the same quality, are sounded together, the ear is unable to hear either of them separately, as would be the case when their pitch differs considerably; but the ear can detect a throbbing effect, as though the note were being sounded alternately loudly and softly. This is due to the sequence of waves which originate from the two sources of sound alternately reinforcing and interfering with each other.

Fig. 232 represents two such sets of waves, one represented by a continuous line and the other by a dotted line: the wave-length of the former being slightly longer than that of the latter, but the amplitude is the same in both sets. At A the condensations of

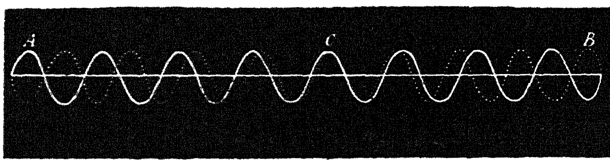


FIG. 232.—Formation of beats.

one set coincide with rarefactions of the other, and the resultant effect is silence. At C the condensations coincide, and the sound will be a maximum. At a further point B the waves again mutually interfere. If the ear be placed at B the momentary silence will be followed by maximum sound when the waves at C reach it, and this will be followed by silence when the waves at A reach it. These pulses of loudness are called **beats.**

It necessarily follows that if two forks vibrate 256 and 257 times respectively in one second their mutual interference will produce one beat in each second. **The number of beats in each second is equal to the difference in the number of vibrations per second made by the two vibrating bodies.** The difference in the number of vibrations must be small for beats to be heard. When more than 16 beats per second are formed, the ear cannot resolve them and a resultant note is heard.

EXPT. 206.—**Beats caused by vibrating wires.** Tune the two wires of a sonometer to apparent unison. Try to detect the presence of

beats. It is easier to detect them when the ear is placed in contact with the end of a wooden rod, the other end of which is pressed against the board of the sonometer. If they cannot be detected, alter *very slightly* the length of one of the wires by means of a movable bridge. Notice how the frequency of the beats increases as the previous unison is more and more disturbed.

EXPT. 207.—**Beats between vibrating wire and tuning-fork.** Adjust the length of a sonometer wire so as to be in perfect unison with a given tuning-fork, and verify the absence of beats by bringing the stem of the vibrating fork in contact with the board of the sonometer. Now load one prong of the fork with a small pellet of wax: this will have the effect of lowering the rate of vibration of the fork, and it should be possible to detect beats when the fork is sounded with the sonometer wire. Attach a larger pellet of wax and observe how the beats are still more frequent.

Harmonics, or overtones.—In previous paragraphs a string has been considered to vibrate with nodes at each end only (Fig. 233 (i)), in which case its fundamental note is produced. But, by preventing the movement of the wire at any intermediate point, a node is set up at that point and the wire vibrates in two or more short segments. Thus, when the wire is touched at its centre (Fig. 233 (ii)) and bowed or plucked at a place halfway between this point and either end, the wire vibrates in two segments. The wire now has two antinodes, A_1 and A_2, or points of maximum movement. When the wire is touched at a point one-third of its length from one end, and then bowed at the middle of the shorter portion, it will divide into three segments (Fig. 233 (iii)). Fig 233 (iv) shows how the string divides into four segments when touched at a point one-fourth of its length from one end. The positions of these nodes

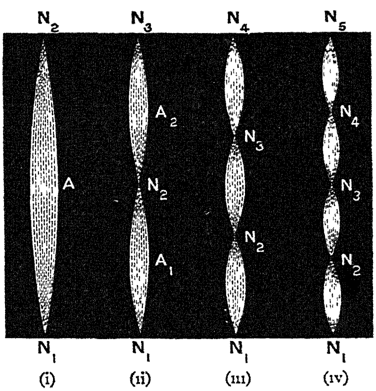


FIG. 233.—Nodes and antinodes of a vibrating string

and antinodes can be verified by placing upon the wire short narrow strips of paper, usually called riders; those at the nodes remain in their places when the wire is vibrating, but the riders at the antinodes are thrown off.

The pitch of the note emitted depends upon the length of each vibrating segment only; and since, when vibrating in the modes shown in Fig. 233 (i)–(iv), the lengths of the segments have the ratios $1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$, the pitches of the notes emitted have the ratios $1 : 2 : 3 : 4$. The lowest note is the **fundamental**; and those produced by the vibration of aliquot parts of the sounding body are termed **harmonics** or **overtones.** In the case of a string vibrating as shown in Fig. 233 (i), the fundamental note only is sounded and is said to be pure. This simple condition, however, is rare; for the vibration of the string may be a combination of the motions shown in both (i) and (ii). When this is the case, the note is not pure, but it is richer on account of the presence of the first harmonic, which is an octave above the fundamental note.

The accompanying diagram (Fig. 234) represents the positions on the musical stave of the sequence of harmonics, which may be

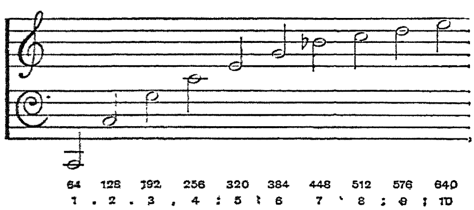



FIG. 234.—Sequence of harmonics

obtained from a wire tuned in unison with the C note below the bass clef. The **quality** or **timbre** of a note is determined by the presence of these harmonics Thus, the same note may be sung by the human voice, or played upon a violin, cornet, organ, or other instrument: but though the pitch and intensity may be alike, the timbre differs because of the different number or relative strengths of the harmonics produced when the note is sounded.

EXERCISES ON CHAPTER XXVII.

1. A stretched string 4 feet long is in unison with a tuning-fork which vibrates 256 times a second What will be the rate of vibration of the string when it has been shortened 6 inches?

2. A sonometer string is stretched with a force of 16 lb weight. What load must be attached so that the note may be an octave lower?

3. At what point must the G string of a violin be pressed by the finger of the player in order to give the note C'?

4. A fork A has a frequency of 256 When this fork and a second fork B are sounded close together, 3 beats per second can be heard. A pellet of wax is attached to one of the prongs of B, and the frequency of the beats is found to be reduced to 2 per second What is the frequency of the fork B when unloaded?

5. Determine the vibration number for each tone of a scale the key-note of which has a vibration number 260

6. A copper wire (density 8 8 gm per c c) 100 cm long and 1·8 mm in diameter is stretched by a weight of 20 kilograms Calculate the number of vibrations which it makes per second when sounding its fundamental note

7. A given note is sounded first on a piano and then on a violin. How is it that the notes can be distinguished easily though we say the same note has been sounded?

8. How does the frequency of vibration of a stretched string depend upon the length of the string, the stretching force, and any other physical property of the string? How are these laws applied in the piano?

9. If the frequency of a tuning-fork be 128, and the number of vibrations per hour of a second fork exceeds that of the first by 300, how many beats will there be in a minute if the two are sounded together?

10. Describe experiments to show that the impression of a musical interval as judged by the ear depends solely upon the ratio of the frequencies of vibration of the two notes concerned and not upon the difference of their frequencies The frequencies of vibration of two notes being 400 and 900, what is the frequency of a note that would appear to the ear to lie midway between them?

11. What is the pitch of a tuning-fork? What may be heard when two forks of nearly the same pitch are sounded together? How would you determine which of the two was vibrating the faster?

12. Describe an experiment for showing that, when a musical note is produced, the greater the frequency of the vibrations the higher is the note.

13. Describe some form of siren, and explain how you would use it to determine the frequency of a given tuning-fork.

14. How does the frequency of the note sounded by a string vibrating transversely depend on (i) the length, (ii) the tension of the string?

15. Describe a monochord and explain how to use it to compare the frequencies of two tuning-forks.

If the frequency of the middle C on a pianoforte be 256 vibrations per second, what will be the frequency of the next higher E?

16. A man standing by a railway notices that the pitch of the note due to the whistle of an engine diminishes as the engine passes him. Explain this result.

If the frequency of the whistle is 256 vibrations per second and the velocity of the engine is $\frac{1}{10}$ of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him?

17. Explain the formation of the beats heard when two tuning-forks, which are not quite in unison, are sounded together.

A standard fork A has a frequency of 256 complete vibrations per second and, when a fork B is sounded with A, there are 4 beats per second. What further observation is required for determining the frequency of B?

18. A note on a harmonium and a string of a violin have been tuned to be in unison with a given tuning-fork. How do you account for the difference in the quality of the sounds produced by the two instruments?

19. Draw diagrams to show how a stretched wire may vibrate. Upon what does the note emitted by a stretched wire depend?

20. The vibration frequency of a tuning-fork A is 256 complete vibrations per second, and the frequency of another fork X differs from that of A by 4 complete vibrations per second. Describe what will be heard when the two forks are sounded simultaneously, and explain how to determine whether the frequency of X is greater or less than that of A.

21. Explain the meaning of the 'pitch' and the 'intensity' of a musical sound.

How do they respectively depend on the nature of the sound wave which produces the note?

22. A heavy goods train was approaching a railway station. To an observer at the station the puffs of steam from the funnel appeared at a certain instant to coincide with the sound of the blasts. Presently, however, the sounds seem to precede the puffs, the difference between them continually increasing until a second coincidence was established and the process was repeated. Explain this.

CHAPTER XXVIII.

INDUCED VIBRATIONS.

Natural and impressed periods of vibration.—In order to understand clearly the phenomenon of **resonance** it is necessary to distinguish **free** vibrations from **forced** vibrations.

Every simple pendulum has a natural period of vibration when swinging freely—the period depending upon the length of the pendulum. But, by taking hold of the bob with the hand it is possible to impart to it any rate of vibration we please; in this case a forced vibration is produced.

The sound of a tuning-fork can be heard when the fork is held close to the ear. But if the handle of the vibrating fork be brought into contact with a board or table the sound can be heard at a considerable distance. The reason for this is that the vibrations are transmitted through the handle to the board, which is thus forced to vibrate at the same rate. The waves set up in the air by the vibrating board are added to those originating from the fork, and the sound seems much louder. The board is thus caused to vibrate at a rate which is not necessarily its natural rate; or, in other words, it is in a state of **forced vibration.** The tone of a violin is due, to a considerable extent, to the forced vibrations set up in the wooden case of the instrument; similarly, the tone of a piano is due to the forced vibrations set up in the sounding-board across which the wires are stretched.

Induced vibrations.—Free vibrations may be set up in a heavy pendulum, or in any other suspended body, by applying a sequence of small repeated blows, providing that the interval between the blows corresponds to the natural period of free

vibration of the suspended body. If, however, the blows come irregularly or at an incorrect frequency, they may have little or no effect in setting up vibration A regiment of soldiers crossing a suspension bridge may set up dangerous oscillations if the frequency of their step corresponds with the natural period of vibration of the bridge: for this reason the soldiers are often ordered to fall out of step when crossing such a bridge. The same effect may be observed when walking along a plank bridge; for considerable oscillation may be set up if the steps are rightly timed, but the oscillations cease when the rate of step is increased or diminished.

EXPT. 208—**Sympathetic vibrations.** (i) Select two tuning-forks of the same pitch, one of them being fixed upon a sounding-box or resonator as in Fig. 235 Strike the other fork and hold its stem upon the sounding-box for a moment; then remove it and stop its vibration. Notice that the fork upon the sounding-box has taken up the vibrations, and gives out the same note

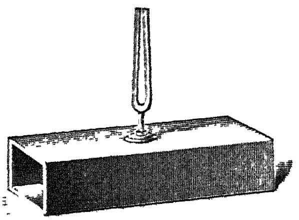


FIG. 235—Tuning-fork on a resonator

(ii) Stretch a wire upon the sonometer until it is in unison with a tuning-fork. Set the fork in vibration and hold the stem for a moment upon the sonometer. Notice that the wire has taken up the vibrations and sounds the same note.

(iii) Sing any note loudly near a piano and stop suddenly. The note in unison with it will be sounded by the piano.

It is easy to realise the process by which such waves of sound set up vibrations in a wire in unison with the note sounded. For, suppose a condensation to strike the wire; this will thrust the wire slightly aside in the direction in which the waves are travelling. During the passage of the succeeding rarefaction, the wire has time to swing back past its position of rest, when it is thrust forward again by the next condensation. Thus, a series of slight, but well-timed, impulses are given to the wire, which soon acquires a considerable amplitude of vibration. By such reasoning it is clear that a wire not in unison with the note will not be affected so readily by the waves of sound.

The familiar phenomenon of the sound obtained by blowing across the open end of a key shows that vibrations may be set up

in an air column; and an air column of definite length has a definite natural period of vibration. When a vibrating tuning-fork is held over a tall glass cylinder, into which water is poured gradually so as to vary the length of the air column, a length can be obtained which will resound loudly to the note of the tuning-fork.

The term **Resonance** is applied to all such effects of vibratory motion produced in one body by the influence of another.

Vibrations of air columns.—The circumstances in which a vibratory condition may be set up in an air column which is closed at one end may be compared to those which determine a vibratory condition in a spiral spring of which one end is fixed. Thus, suppose a weight to be suspended from a spiral spring (Fig. 236 (ii)); by imparting a succession of small taps, directed upwards, to the weight, a considerable vibration may be set up in the spring, *providing that the frequency of the taps coincides with the natural period of vibration of the spring.* In the same manner, when a succession of small taps is imparted to the open end of an air column (Fig. 236 (i)) by means of a vibrating tuning-fork, considerable vibratory motion is set up in the air column if the rate of vibration of the fork coincides with the natural period of vibration of the air column; and the sound waves originating from the fork are augmented considerably by those from the vibrating air column. It is important to notice, in this analogy, that the fixed end of the spring and the closed end of the air column are stationary, and that the opposite ends in each case are regions of maximum motion.

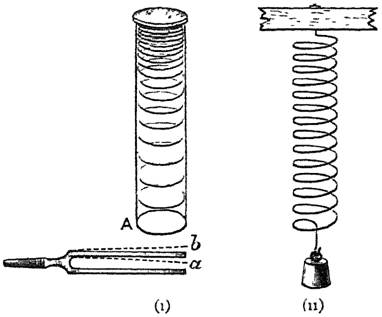


FIG 236 —Vibrations of a spring and a closed air column.

Tube closed at one end.—A **stationary** vibration is the only type which can be set up within a column of air contained in

a cylinder closed at one end. The relation between the length of the air column and the wave-length of the note given out when the air column is vibrating in its simplest manner may be derived by comparing it with a stretched string in a state of stationary vibration. In the case of the string there are points of maximum motion called antinodes, mid-way between the fixed points called nodes. In the case of the air column,

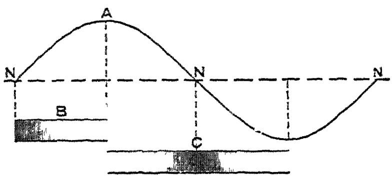


FIG. 237.—Relation between wave-length and vibrating columns of air.

the air near the closed end must be analogous to the node of the vibrating string, since the closed end prevents motion; but, at that end, rapid alternations of condensation and rarefaction are possible. The conditions at the open end are just the reverse, for the air in that position may be in a state of rapid movement; but, since it is exposed freely to the outer air, changes in density are impossible. In other words, *the closed end will be a region of maximum changes of density, and the open end will be a region of maximum motion.* The analogy between a vibrating string and a vibrating air column is shown in Fig. 237. In the former the distance between a node N and an antinode A is equal to *one-fourth* of a wave-length; similarly, the length of the air column B is equal to *one-fourth* of the wave-length of the note which it will emit. This can be verified by the following experiment.

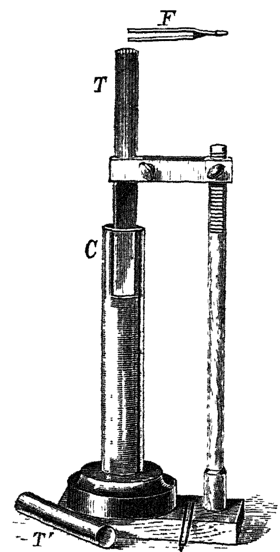


FIG. 238.—Determination of the velocity of sound in air by means of a vibrating air column.

EXPT. 209.—**Resonance of air column.** Support a glass tube T (Fig. 238), about 20 cm. by 3 cm., open at both ends, in a vertical

position with its lower end dipping into water contained in a wider cylinder C. Hold over the upper end of tube T a vibrating tuning-fork F, of which the rate of vibration is known; and adjust the position of T so that the greatest reinforcement of the sound is obtained. Measure the distance from the top of T to the water level. Put T out of adjustment, and repeat the observation at least four times, and take the mean of these results.*

If the length of the air column, as measured above, be l, the wave length of the note emitted is $4l$; and if n be the frequency of vibration of the fork, the velocity v of sound in air, at the temperature of the room, is given by the equation $v = n \times 4l$. Calculate the value of v.

Note the temperature of the room, and calculate the theoretical value from the formula $v = (33200 + 60t)$ cm.; where t is the temperature measured on the Centigrade scale.

By similar experiments the velocity of sound in any gas may be determined When the column of gas in a closed tube responds to a certain note, the wave-length of the note is four times the length of the column. Suppose the vibration-frequency of the note to be known by comparison with the note of a tuning-fork or other standard, then the velocity of sound in the gas is this number multiplied by four times the length of the vibrating column. Similarly, to find the vibration number of a tuning-fork, a column of air is adjusted as in Expt. 209 until it is in sympathetic vibration with the fork, and its length is measured. Let this be 0 22 metre. Then taking the velocity of sound to be 340 metres per second at the temperature of the air column, we have $\qquad V = nl.$

But $\qquad l = 4 \times 0{\cdot}22 = 0{\cdot}88.$

Hence $\qquad n = 340/0{\cdot}88$
$\qquad\qquad = 386.$

Tube open at both ends.—In the case of an air column enclosed in a tube open at both ends, the ends of the tube are always antinodes; and, when the fundamental note is sounded there is a node at the middle of the tube. By analogy with stationary vibrations in a string, as shown in Fig 237, C, the wave-length of the fundamental note is equal to twice the length of the tube. This can be verified by holding the same fork which was used in Expt. 209 in front of an open tube, the effective length

* Strictly speaking, the position of the antinode is slightly *outside* the end of the tube, and this distance depends on the diameter of the tube A more exact measure of the quarter wave-length is obtained by adding 0 8 of the radius of the tube to the length measured from the water surface to the top of the tube.

of which can be adjusted by means of a paper cylinder made to slide over the outside of the tube. The length which gives maximum reinforcement will be found to be twice as long as the air column closed at one end.

Organ pipes.—The conditions of vibration of air columns in closed or open tubes apply to organ pipes. In the case of reed pipes, a small tongue is set in vibration at the mouthpiece by air being forced past it; but in ordinary organ pipes, the air blasts strike against a lip, as with a boy's whistle-pipe, and vibrations are thus set up which are reinforced by sympathetic vibrations of the air column. Vibrations of many wave lengths are produced at the mouthpiece, but only those with which the air column is in unison are taken up and strengthened to form the musical note. As described for tubes, the wave length of the fundamental note given by an organ pipe closed at one end is four times the length of the pipe, but if the pipe be open at both ends, the wave-length of the fundamental note is twice the length of the pipe. The wave-length of the note of an organ pipe can thus be determined from the length of the pipe. Neither the material of which the pipe is made, nor the diameter of the pipe, provided it is small in comparison with the length, need here be considered to affect the pitch of the note produced. The pitch of the note varies directly, however, with the velocity of sound in the vibrating column, and therefore rises as the temperature increases or when a lighter gas than air fills the pipe. As the pitch of a note depends upon the frequency or number of vibrations per second, the equation $V = nl$ (p. 330) can be used to determine the velocity of sound in a gas when the vibration frequency of the note is known and the length of the pipe emitting it. The following examples illustrate the use of this relation.

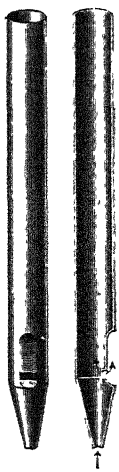


FIG. 2[illegible]3.—Organ pipes.

EXAMPLE I. An open organ pipe 0·65 metre long gives the note middle C, the vibration number of which is 256 per second. Find the velocity of sound in the air at the temperature of the tube.

The wave-length (l) of the note is twice 0·65 metre, that is, 1·30 metres. So that

$$V = 256 \times 1{\cdot}30$$
$$= 332{\cdot}8 \text{ metres per second.}$$

EXAMPLE 2. The velocity of sound in hydrogen is 1269·5 metres per second. What would be the length of a closed organ pipe which gives a note having a vibration frequency of 512 per second when blown with hydrogen?

$$1269{\cdot}5 = 512 \times l$$

Therefore
$$l = 1269{\cdot}5/512$$
$$= 2{\cdot}48.$$

The length of a closed organ pipe is $l/4$, hence the answer required is $2{\cdot}48/4 = 0{\cdot}62$ metre

Longitudinal vibrations of rods.—When a rod is clamped at one end and made to vibrate longitudinally, the wave of compression set up travels along the length of the rod in much the same way as it does in a column of air. The relative velocities of sound in the rod and in air can therefore be determined by measuring the length of a closed air column which resounds to the same note as that emitted by a rod clamped at one end and set in vibration Similarly, the velocity in rods of different material can be found by cutting the rods until they give the same note; for then the relative lengths give the relative velocities.

With a rod clamped at one end and set in longitudinal vibration the point of maximum motion (antinode) is at the free end and that of maximum compression (node) is at the clamped end. The vibration of such a rod may be compared with that of the air in a closed tube or organ pipe The wave-length of the fundamental note is, therefore, four times the length of the rod. Similarly, the longitudinal vibrations in a rod clamped at the middle are like those of the air in an open tube or organ pipe, for both ends are antinodes. The wave-length of the fundamental note of a rod clamped in this way is twice the length of the rod.

EXPT. 210.—**Longitudinal vibration.** Clamp a rod of brass loosely at its middle. Near one end hang a pellet of sealing-wax. Rub the other end with a resined leather. The rod gives out a note and the pellet is knocked away.

EXPT. 211.—**Pitch and length.** Obtain two long rods of the same kind of wood, one twice the length of the other, and clamp them

separately at the centre. Set the rods in longitudinal vibration by rubbing them with a resined leather. The longer rod will be found to give a note an octave lower than the shorter one, because a wave of compression has to travel twice as far, and consequently appears half as often.

EXPT 212.—**Relative velocities in deal and oak.** Obtain rods of equal length in deal and oak and set them in longitudinal vibration. The deal rod gives out the higher note because the waves are propagated quicker by it than by the oak. Cut down the length of the oak rod until the two rods give the same note. It will be found that a deal rod 72 in. long emits the same note as an oak rod 49 in. long. The relative velocities of sound in deal and oak are therefore as 72 is to 49

EXPT. 213.—**Determination of velocities.** Adjust a monochord string until it is in unison with a tuning-fork of known pitch. Measure the length of the string. Set a rod of mahogany clamped at the middle in longitudinal vibration, and adjust the monochord string in unison with it. Measure the length of this vibrating string. Knowing the pitch of the fork, the velocity of sound in mahogany can be found as follows:

Standard fork $C' = 543$ vibrations per second.

Length of string in unison with $C' = 60$

" " " rod $= 24$

Length of rod $= 6$ feet.

Then $\frac{24}{60} = \frac{543}{n}$

Therefore the frequency, $n = 1357 \cdot 5$.

Using the formula $V = nl$,

we have $n = 1357 \cdot 5$

and $l =$ twice the length of the rod.

Hence velocity of sound in mahogany

$= 1357 \cdot 5 \times 6 \times 2$

$= 16290$ feet per second.

Find in the same way the velocity of sound in glass, oak, and brass.

EXERCISES ON CHAPTER XXVIII.

1. A tuning-fork produces strong resonance when held over a jar 22·35 cm. long and 2 cm. radius. Find the wave-length of the note emitted. If the temperature is 15° C., calculate the rate of vibration of the fork.

2. State how the air moves in different parts of a tube 1 ft. long, open at both ends, when sounding its fundamental note. Neglecting

the correction for the width of the tube, and assuming the velocity of sound in air to be 1116 ft. per second, calculate the frequency of the note emitted

3. Find the number of vibrations per second of a fork which produces resonance in a pipe which is 10 inches long and 2 inches in diameter, and closed at one end The temperature of the air at the time of the experiment is 50° F.

4. Explain the meaning of the term 'Resonance' by reference to an experiment in which a vibrating tuning-fork is held over a column of air. Show how this experiment can be used to determine the vibration frequency of a fork, the velocity of sound in air being assumed as 1100 ft per second.

5. If there are 32 holes in the disc of a siren, which makes 1050 revolutions per minute, what is the frequency of the note emitted? What would be the length of an open organ pipe which, sounding its fundamental, emitted the same note? (Velocity of sound in air = 1120 ft. per sec)

6. The end of one of the prongs of a tuning-fork is held over the mouth of a tube which can be raised or lowered in water. When the mouth of the tube is at a given height above the water the sound of the fork appears to swell out loudly Carefully explain this. Would the height be different, if (i) the temperature of the air were higher, (ii) if the air in the tube were replaced by carbonic acid gas? Give reasons for your answer

7. A glass tube one foot long and one inch in diameter is closed at one end. Describe the motion of the air within the tube when vibrating in the simplest possible manner. How, if at all, would the pitch of the note emitted by such a column be changed by a rise of temperature, by an increase of atmospheric pressure, and by the substitution of a denser gas for the air in the tube?

8. How would you determine the number of vibrations per second executed by the prong of a tuning-fork?

9. Explain the effect of temperature upon the frequency of the note emitted by an open organ pipe Does the effect depend upon the material of which the pipe is made, or upon the nature of the gas in which the pipe is sounded? Give reasons.

10. In building an organ for use in a warm climate it is necessary, in order to produce notes of a given pitch, to make the pipes longer than if they were to be used in England. Explain why this is so.

11. How would you show that the velocity of sound is not the same in air as in carbon dioxide gas at the same temperature?

12. What is meant by resonance?
Give two illustrations of its use in acoustic experiments.

13. Explain what is meant by resonance. The length of the column of air in a tube closed at one end which gives the greatest

resonance with a tuning-fork is observed to be 32.5 cm.; find the wave-length of the note emitted by the fork

14. Describe the method of determining the velocity of sound in air by means of a resonance tube and a tuning-fork of known pitch.

How may the correction due to the 'open end' of the tube be determined experimentally?

15. You are given a tuning-fork of known frequency, a deep gas jar, and a metre scale. How would you determine the velocity of sound in air?

On what acoustic principle does this experiment depend?

16. State and explain what may be noticed when a person sings a note in front of the strings of a piano

17. How would you determine the vibration number of a tuning-fork?

18. How would you show that the velocity of sound in air is different from its velocity in carbon dioxide?

Explain how it is possible for a man to calculate roughly his distance from a cliff when the velocity of sound in air is known.

19. Why, when the handle of a vibrating tuning-fork is put in contact with a wooden board, is the amount of sound produced greatly increased? Is the time during which the fork goes on vibrating affected?

20. If you were provided with a fork of unknown pitch and the necessary apparatus, how would you determine the velocity of sound in air?

21. State how you could determine the velocity of sound in a solid which could be obtained in the form of a rod.

PART VI.

MAGNETISM.

CHAPTER XXIX.

NATURAL AND ARTIFICIAL MAGNETS

Lodestone.—A **magnet** is a solid body possessing the property of attracting iron, it has the same power of attracting a few other metals, but to a much less marked extent than in the case of iron

Stones possessing the property of attracting iron are found abundantly near Magnesia (in Asia Minor), from the name of which place the word magnet originated This stone is now termed **magnetite**; it is an oxide of iron, and contains about 72 per cent. of iron; it is distinctly heavy, and is dark-gray to black in colour. Only some specimens of magnetite possess the properties of a magnet, but all are capable of being attracted by a magnet.

When a piece of magnetite, selected as showing the properties of a magnet, is dipped into iron filings, these will cling chiefly to two regions on the surface of the magnetite These regions are termed the **poles** of the magnet, and the imaginary line joining the centres of these regions is termed the **magnetic axis** of the magnet. If this piece of magnetite be suspended so that its magnetic axis can move freely in a horizontal plane it will come to rest with its axis pointing approximately **north** and **south**. This property possessed by magnetite was known to the people of other nations at a very early date; for example, there is every reason to believe that the Chinese were aware of it in the year 2400 B.C On account of this property of setting itself in a north and south

direction like a compass needle, a piece of magnetite which behaves like a magnet is called a **lodestone** or leading-stone.

EXPT 214.—**Attractive property.** Dip a lodestone into a small heap of iron filings: observe how the filings cling to it chiefly at two points.

EXPT. 215.—**Directive property.** Suspend a lodestone by means of silk *cord* (not *twisted* sewing-silk), in such a manner that the line joining the poles may move freely in a horizontal plane. Observe that the lodestone always comes to rest in one particular position. When at rest, the axis of the lodestone lies in a magnetic north and south direction. Mark the end which points north with a spot of sealing wax or red paint.

Magnetisation.—The properties of a lodestone can be imparted to iron or steel. When an ordinary steel needle is dipped into

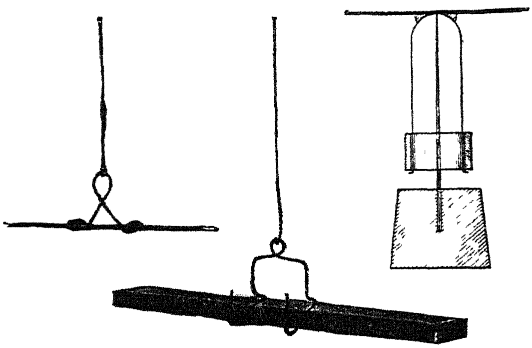


FIG. 240.—Methods of supporting magnets

iron filings it has no more effect upon them than a copper wire or a splinter of wood. Also, such a needle, like a strip of copper or wood, does not exhibit the directive properties of a lodestone, even when it is suspended so as to be free to come to rest in any direction. When, however, a needle is stroked with either pole of a lodestone, it acquires the properties of a magnet, that is, it will attract iron filings and will set itself in a north and south direction if suitably suspended. The presence or absence of magnetisation in a needle or any other iron or steel object may thus be tested by observing whether the object has the magnetic properties of a lodestone.

EXPT 216—**Magnetisation of iron by a lodestone** Support an ordinary needle horizontally in a silk fibre suspension (Fig 240) The needle may swing to and fro, but it does not indicate any tendency to come to rest pointing in any one definite direction. Dip the needle into iron filings, it does not appear to have the power of attracting the filings Stroke the needle several times in one direction with one end of a lodestone. Notice that the needle will now attract iron filings and will set itself in a magnetic north and south direction when suspended. (An alternative method of suspending needles is shown in Fig. 240 A short test-tube, 2 inches long, rests inverted on the point of a darning-needle fixed vertically in a wide cork, and a small lump of soft wax enables the needle to be attached to the closed end of the tube. In order to ensure stable equilibrium, a strip of sheet lead is cut to a length necessary to form a collar which can be slipped over the outside of the test-tube and resting on its expanded rim.)

Like and unlike magnetic poles.—That end of a suspended lodestone, or of any other magnet, which points toward the north is called the **north-seeking pole**, and the end which points toward the south is termed the **south-seeking pole.** The north-seeking end of a suspended lodestone is repelled by the north-seeking end of another lodestone brought near it Similarly, the south-seeking end of one lodestone will repel the south-seeking end of another lodestone When, however, the north-seeking end is brought near the south-seeking end, attraction takes place. These results may be expressed briefly by the words **like poles repel, unlike poles attract.**

A piece of magnetite or of iron or steel which is not a magnet, and therefore does not set itself in a magnetic north and south direction when suspended, is always attracted by a magnet brought near it; and it is never repelled. Repulsion only takes place when both the bodies under examination are magnets. This fact enables a magnetised piece of iron or steel to be distinguished easily from unmagnetised iron or steel.

It has been seen that a lodestone can impart its properties to a steel needle, that is, it can convert an unmagnetised piece of steel into a magnet having north-seeking and south-seeking poles. To magnetise a needle in this way, one end of the lodestone is used to stroke the needle several times in one direction. It is

found that the magnetic polarity generated in the end of the needle which is last touched by the lodestone is of opposite kind to that of the pole used for the process.

EXPT. 217—**Directive property of magnetised iron.** Place a needle on the table, and holding it firmly by pressing a finger on the eye of the needle, rub the marked pole of the lodestone along the needle from eye to point, lift the lodestone some distance away from the table, and bring it down again on to the eye of the needle, and repeat this operation several times Replace the needle in its support, and observe how different is its behaviour from that observed before it was magnetised.

It comes to rest with its eye pointing in the same direction as does the marked end of the lodestone.

EXPT. 218.—**Attraction and repulsion.** Bring the marked end of the lodestone near to the point of the needle attraction takes place. Bring the same end of lodestone near to the eye of the needle: repulsion is observed Repeat the observations with the other end of the lodestone; the eye of the needle is attracted now, while the point is repelled.

EXPT. 219—**Like and unlike poles.** Magnetise a second needle in exactly the same manner as described in Expt. 217, but stroke the needle with the unmarked end of the lodestone (instead of the marked end) Suspend the needle, and observe how it comes to rest with its eye pointing in the opposite direction to that obtained in Expt. 217 (when the marked end of the lodestone was used).

Magnetic substances—The terms **natural magnet** and **artificial magnet** are used frequently to distinguish the lodestone from a piece of iron or steel which has acquired the same properties by artificial means. In the experiments performed, while the lodestone is a "natural magnet," the needles which have been magnetised by mechanical processes are termed "artificial magnets." Things like iron and steel which are attracted by a magnet are termed **magnetic substances.** Nickel and cobalt are also magnetic substances, but zinc, copper, paper, wood, glass, and air are examples of non-magnetic substances The influence of a magnet can pass through any non-magnetic substance just as readily as it does through air.

EXPT. 220—**Attraction of nickel and cobalt.** Bring a bar-magnet in contact with some fragments of nickel and of cobalt; the fragments are attracted. Test, in the same manner, fragments of copper, wood, glass, etc.

EXPT 221 —**Non-magnetic substances** Suspend a magnetised needle and bring the pole of a magnet near it, so as to deflect the needle from its original position. Hold successively in front of the pole a sheet of copper foil, or zinc foil, of paper, glass or wood. In no case is the deflection of the needle affected.

Magnetisation by means of an artificial magnet.—Using a lodestone it is only possible to magnetise comparatively small pieces of steel, and even then the magnetisation is not so marked as is the case when magnets stronger than the lodestone are used It is more satisfactory therefore to dispense with the lodestone, and to use instead some form of artificial magnet, *e.g.* the long bars of magnetised steel known as **bar-magnets** (Fig. 241).

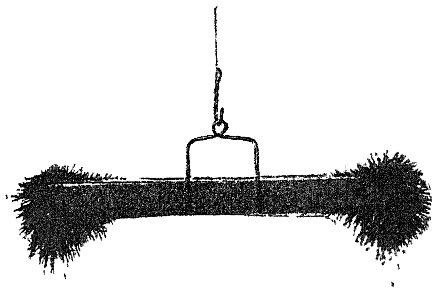


FIG 241.—A bar-magnet which has been dipped into iron filings.

Another common form of artificial magnet is the **horse-shoe magnet,** in which the steel has been bent into the form of a horse-shoe previous to magnetisation (Fig. 255) The poles of the magnet are at the ends of the horse-shoe, and are thus situated close together.

EXPT. 222.—**Magnetisation of steel.** Break off a piece of clock-spring about 5 or 6 cm. long ; hold it firmly on the table by a finger placed at one end (or, better still, fix it to the table by soft wax at the ends), and draw one pole of a magnet along the whole length of the spring, and proceed as in Expt. 217 (Fig. 242). Test the magnetisation (*a*) by means of iron filings, and (*b*) by suspending it horizontally.

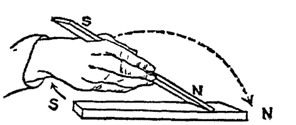


FIG 242 —Method of magnetising steel

Magnetisation by an electric current.—The strongest magnets are made by means of the electric current. If a close spiral of cotton-covered copper wire be wound round a rod of steel (Fig.

243) and a current of electricity sent through the spiral, the steel becomes a magnet and retains its magnetic properties after the current has ceased. Soft iron also is magnetised strongly when an electric current is passed round it, but it soon loses its magnetism when the current ceases. A solid rod of soft iron, which is only a magnet so long as a current of electricity continues to flow around it, is termed an **electro-magnet.**

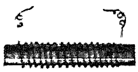


FIG. 243 —Magnetisation of a steel rod by an electric current.

EXPT. 223 —**Electro-magnetisation** Wrap a spiral of cotton-covered copper wire round a piece of thin-walled glass tubing (about 10 cm long and 0·5 cm. bore) (Fig. 244); place inside the tube a needle or piece of clock-spring, and pass a strong current through the wire for a few seconds; after stopping the current, remove the needle, and test it for magnetisation.

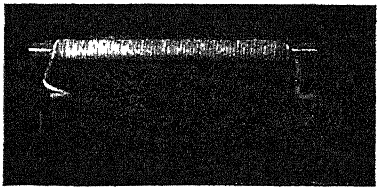


FIG. 244 —Method of magnetising a needle by an electric current

The more common form of electro-magnet is the horse-shoe, which consists of a thick core of soft iron, bent either into the form of a horse-shoe with straight limbs, or into a form resembling three sides of a rectangle. Round each limb is wound a bobbin of several layers of thick cotton-covered copper wire, the direction in which the wire is wound on the limbs being *opposite* (Fig. 245). While an electric current is passing round the bobbins, a bar of steel may be magnetised by drawing it completely from end to end several times (in one direction only) across the edge of one of the poles of the magnet. (The polarity of the electro-magnet may be determined by means of a compass-needle.)

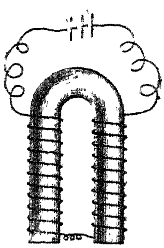


FIG 245 —An electro-magnet

Magnetic field.—The space round a natural or artificial magnet, wherein magnetic force may be detected by its influence upon a compass-needle or by other means, is termed a **magnetic field** (pp. 384-392).

Consequent poles.—Sometimes a magnet may be found which has similar poles at the ends; this may arise through faulty magnetisation, and it may be imitated artificially in a simple way. A magnet showing this peculiarity will always be found to have one or more poles of opposite kind somewhere along its length, and these may be located by dipping the magnet completely into iron filings, or by means of a compass-needle placed in a series of positions along its length.

EXPT. 224.—**Production of consequent poles.** Magnetise a long knitting-needle in four separate parts by the method of Expt 217, and so that a north-seeking pole is found at each end; another north-seeking pole is found also near the centre, and south-seeking poles at about one-quarter of the whole length from each end.

Destruction of polarity.—When a magnet is subjected to rough treatment it loses a considerable portion of its polarity; for example, if it be dropped on the floor, or struck with a hammer several times, its strength is reduced to a marked extent.

Magnets also lose their polarity when strongly heated. After a magnetised needle has been heated to bright red heat in a Bunsen (or blow-pipe) flame, and allowed to cool, it behaves like an ordinary unmagnetised piece of steel.

EXPT. 225.—**Effect of striking.** Magnetise a French wire-nail about 7 cm long, and test its magnetisation by bringing it near to a compass-needle. Strike it several times with a hammer, or drop it several times from a considerable height, and test again; it will be found to have lost a considerable portion of its polarity.

EXPT. 226.—**Effect of heating.** Hold a magnetised needle in a Bunsen flame by means of metal tongs, or by wrapping the ends of a short length of copper wire round the needle; when red hot remove it, and allow to cool; test its magnetisation by means of a compass-needle.

EXERCISES ON CHAPTER XXIX.

1. Two steel needles are supplied to you, only one of which is magnetised. (i) How would you determine, by means of a cork floating on water and a lodestone, which of the needles is magnetised? (ii) How could you distinguish the needles without the aid of a lodestone?

2. Two sewing-needles are magnetised so that the eye of each is a north-seeking pole. The needles are stuck by their points into separate bits of cork, so that when thrown into water they float upright with the eyes downwards How will they behave towards each other when floating in this way?

3. You are doubtful whether a steel rod is neutral or is slightly magnetised How could you find out by trying its action on a compass-needle? If it is found to be magnetised, how would you determine its polarity?

4. Two magnetised-needles, of equal length, are suspended from their upper ends by threads, so that they hang side by side with their lower ends at the same level. If the lower ends are both north-seeking poles, how will they act upon each other? How will the action be altered when one of the needles is reversed? Give sketches.

5. A needle is to be magnetised so that its eye acquires north-seeking polarity State fully how you would proceed to do this.

6. How would you determine experimentally whether a magnet has consequent poles or no?

7. What conclusion would you come to if a magnetised piece of steel, when suspended, does not tend to come to rest pointing in a north and south direction? If the steel be now broken into two parts, would you expect them to behave, when suspended separately, in the same manner as the unbroken piece of steel? (Give diagrams to explain your answer

8. Describe the steps you would take to magnetise a piece of clock-spring as strongly as possible.

9. An unmarked magnet, with means for its suspension, is given you. How could you determine which is the north-seeking end?

10. An unmagnetised strip of steel is balanced carefully on a needle point so that it is free to rotate in a horizontal plane. It is then taken off its pivot and magnetised. Describe how the magnetisation would be performed, and describe the behaviour of the needle when replaced on its pivot.

11. What is meant by the axis of a magnet? Where is the axis of a horse-shoe magnet? In what direction would such a magnet place itself, if placed upon a wooden board floating freely in water?

12. How would you test whether a steel bar is magnetised or not? If not magnetised, how would you proceed to magnetise it?

13. Describe the various methods of making a magnet, stating which of them makes the most powerful magnets.

CHAPTER XXX.

MAGNETIC INDUCTION

Magnetic induction.—When one magnetic pole of a lodestone is brought near either end of an unmagnetised needle freely suspended, the attraction observed may, at first sight, suggest that it is simply a case of 'Unlike Poles attracting,' and that repulsion will be found when the other end of the needle is tested. But on completing the experiment in this manner we again find 'attraction' It would seem either that an entirely new phenomenon is being brought into play, or that 'latent' magnetisation in the needle appears when the magnet is brought near one end, and reappears in a reversed direction, when the other end is tested. To decide this point, it is necessary to test the polarity of the distant end of the needle while the magnet still remains in its first position.

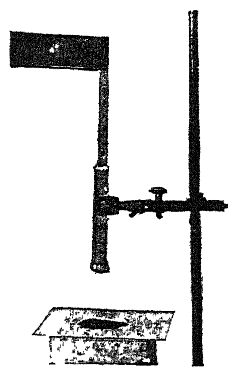


FIG 246.—Magnetisation of a strip of iron by induction.

EXPT. 227.—**Induced magnetisation.** Cut strips of the thin galvanised iron which is used in making biscuit-tins and tobacco-tins. Strips about 10 cm long by 1 cm wide are convenient. Hold a strip of galvanised iron in line with the magnet's axis, the end not quite touching the north-seeking pole; bring the distant end of the iron in contact with iron filings; some filings cling to the end (Fig. 246). Reverse the strip of iron and again test

EXPT. 228.—**Poles of an induced magnet** Attach a strip of the galvanised iron to a 'test-tube' support (Fig. 240), and arrange a

bar magnet horizontally on blocks of wood so that it is at the same level as the strip, and so that its N.-seeking pole *almost* touches the end of the strip (Fig. 247). We may anticipate that *the near end of the strip has S.-seeking polarity*; test this by bringing near to it the S.-seeking pole of another magnet, and observe any indication of repulsion. To make the effect more evident, make the latter magnet approach and recede from the strip with a frequency coinciding with the time of swing of the strip. The series of *small* impulses will set up a considerable amplitude of swing in the strip.

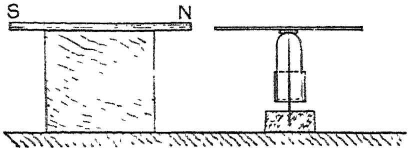


FIG. 247.—Induced polarity.

Reverse the magnet in the hand, and prove in the above manner that *the distant end of the strip has N.-seeking polarity.*

EXPT. 229.—**The induced polarity is temporary.** Remove the bar-magnet, and repeat the tests for magnetisation; we find that the strip now behaves like an unmagnetised piece of iron.

It is evident that a strip of iron actually becomes a magnet when it is near a bar-magnet, but ceases to be one as soon as the magnet is removed. We say that polarity has been temporarily induced in it, and that its behaviour is due to **magnetic induction** from the bar-magnet.

When a piece of iron or steel is magnetised by induction, the end farthest away from the inducing pole acquires polarity of the same kind, the nearer end acquires polarity of the opposite kind, to that of the nearest pole of the permanent magnet. If the piece of iron be really a magnet, then it also should be capable of inducing polarity in a second piece of iron held near to it.

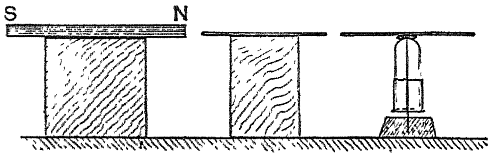


FIG. 248.—Secondary induced polarity.

EXPT. 230.—**Secondary induction.** Support on wood blocks a bar-magnet and an iron strip so that the latter is in line with the magnet's axis and quite near to it. Suspend a second strip of iron as shown in Fig. 248. Test the polarity induced in the latter.

We can understand now the cause of all the phenomena of attraction which a magnet displays towards magnetic substances. The experiments prove that **magnetic induction always precedes attraction**, and that these phenomena are all in accordance with the simple law that 'Unlike Poles attract.'

EXPT 231 —**Magnetic chain** Clamp a large bar-magnet in a vertical position, note the polarity of the lower end, and hang from this end a strip of galvanised iron. Bring into contact with the iron a number of small wire-nails, and notice how long a chain of nails can be supported. The iron and each nail are temporarily magnetised. Test the polarity of the extreme end of the chain by means of a compass-needle (Fig 249).

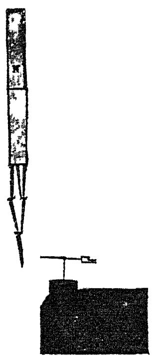


FIG 249.—Test of polarity of the end of a magnetic chain.

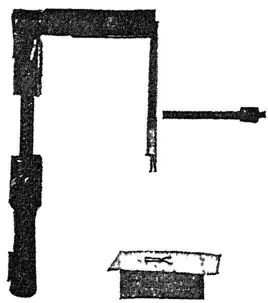


FIG 250 —Effect of bringing unlike poles near one another

EXPT 232 —**Resultant induction.** Bring near to the N -seeking end of the first magnet the south-seeking pole of a second bar-magnet. This second magnet will act inductively on the iron and nails also, but the induced polarity will be opposite to that already present The magnetisation of the iron and nails is consequently weakened, and most of the nails fall off (Fig. 250).

Replace everything as in Expt. 231. Now place the south-seeking pole of a second bar-magnet just below the end of the chain of nails We can add two or three more nails now; the induction due to the south-seeking pole tends to strengthen that originally present, and consequently the induced polarity is increased (Fig. 251). Remove the south-seeking pole, several nails will fall off; if now

the north-seeking pole of the second magnet be placed close to the end of the chain, more nails will fall (Fig. 252).

FIG 251.—Increased effect due to induction

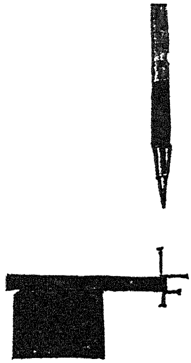


FIG. 252 —Decreased effect due to induction

EXPT 233 —**Repulsion between like induced poles.** Suspend from the pole of a vertically-clamped magnet a bunch of sewing-needles, or three or four strips of galvanised iron. Notice that the lower ends of all the needles have similar polarity, and mutually repel each other—the needles consequently bunch outwards (Fig 253)

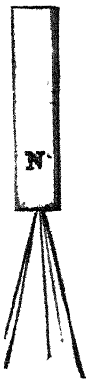


FIG. 253 —Expt. 233.

Induction in a magnet.—It has been shown that the induced polarity set up in a piece of iron by a neighbouring magnet can be either increased or diminished by means of a second magnet. Magnetic induction can also be caused in a piece of iron which is a permanent magnet. A long knitting-needle, for instance, which has been magnetised feebly may have its polarity entirely reversed by bringing a strong magnet near it. When the magnet is at some distance from the needle the induced magnetisation is weak, and its effect is hidden by that of the permanent magnetisation of the needle; but when the magnet is brought near the needle the induced magnetisation is not only sufficient to neutralise the permanent magnetisation, but completely overpowers it.

EXPT. 234.—**Effect of distance on degree of induced polarity** Feebly magnetise a long knitting-needle, and suspend it horizontally. Hold the pole of a strong bar-magnet some distance away, and observe the repulsion between similar poles. Rapidly bring the magnet to within an inch of the repelled end of the needle; instead of being repelled, the end of the needle is now attracted

This phenomenon, unless it is guarded against, often gives rise to incorrect conclusions in an experiment It is most important in all such experiments gradually to bring the iron or steel under examination from a distance near to the compass-needle, and to watch carefully for the effect. If the two ends, the polarities of which are to be compared, have unlike polarity, then the induced polarity due to the compass-needle aids the true attraction which should be observed. It is only when the polarities are alike that the true repulsion may be masked by the attraction due to magnetic induction.

Susceptibility.—The degree of induced polarity, when a piece of iron or steel of given dimensions is placed in a magnetic field, depends upon (i) the strength of the field and (ii) the nature of the iron or steel. Within certain limits, an increase in the strength of the field causes with both materials an increase in the induced polarity; but, for a given strength of field, the induced polarity set up in soft iron is always stronger than that set up in hard steel. We say that **the susceptibility of soft iron is greater than that of hard steel.**

When a piece of soft iron is brought near to the pole of a compass-needle, the permanent polarity in the latter *induces polarity* in the soft iron, and the compass-needle is attracted—the stronger the induced polarity the greater is the deflection of the needle. When a piece of hard steel of the same dimensions is substituted for the iron, the deflection of the needle is *less*, because the induced polarity in the steel is less.

EXPT. 235.—Suspend a magnetised needle just above the level of the table, and place a bar of *unmagnetised* steel horizontally with its end near to the north-seeking pole of the needle, and its length perpendicular to the needle's axis.

Now place the soft iron (of similar size to the steel) on the opposite side of the needle, and alter its distance from the pole until the needle

again points to the north (Fig 254). The soft iron completely neutralises the effect of the steel, although it is much farther away from the needle than the steel is.

FIG 254.—Expt. 235

Retentivity and coercive power.—If two similar pieces of soft iron and of steel are subjected to the same magnetising force, then, *under special conditions*, after the removal of the magnetising force, the soft iron will be found to retain almost as high a percentage of its previous polarity as does the hard steel. Both, indeed, *may* retain as much as 90 per cent. of the original magnetisation. But, as soon as the specimens are subjected to agitation, or to a magnetising force which tends to reverse the polarity, a marked difference is at once noticeable: the soft iron very readily loses nearly all, or all, of its polarity, while the steel is affected but slightly in comparison. The former property, of retaining under favourable conditions the polarity imparted, is termed **Retentivity**. The latter property, of resistance to the influence of any demagnetising force, is termed **Coercive force** or **Coercivity**. Thus, the materials in question may not differ appreciably in retentivity; but soft iron has far less coercivity than hard steel.

EXPT. 236.—**Retentivity and coercivity.** Clamp a strong bar-magnet vertically, and suspend from the pole a thin rod of hard steel and one of soft iron of exactly the same dimensions (failing more suitable specimens, the effect may be observed if two short pieces of wire of the same diameter and length are used). Very gently *slide* the rods off the magnet, and hold them successively at the same distance from the pole of a suspended magnetised needle (preferably, with a pointer attached), and note the difference of the two deflections caused. Now drop the rods several times on the floor, or otherwise subject them to rough treatment, and again observe the difference between the two deflections of the magnetised needle.

Keepers or armatures.—The use of the keeper or armature is a practical application of the phenomenon of Magnetic Induction. When a horse-shoe magnet is allowed to remain for a considerable time with its poles unprotected, its degree of magnetisation slowly

diminishes; but when a short length of soft iron is placed so as to connect the poles, and in contact with the entire length of the pole-faces, this liability to loss of magnetisation is prevented. Any piece of soft iron serving this purpose is called a **keeper**. So long as it is in contact with the poles of the magnet the soft iron is also a magnet by induction. The stronger the induced magnetisation in the keeper the better its purpose is fulfilled

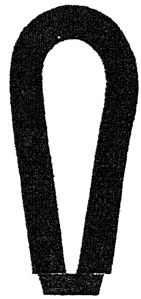


FIG 255 —Horse-shoe magnet and keeper

In Fig. 255, N induces south-seeking polarity at the near end of the keeper, and north-seeking polarity at the distant end. The pole S has the same effect, consequently N and S *help each other*, and produce a much greater degree of induced magnetisation than if they were acting separately.

The two poles of a bar-magnet cannot be connected in this simple manner, but the difficulty is overcome by keeping the bar-magnets in pairs, and placing them parallel to one another with opposite poles together. A piece of soft iron is placed at each end of the pair.

EXERCISES ON CHAPTER XXX.

1. Two similar rods of soft iron have each of them a long thread fastened to one end, by which they hang vertically side by side. On bringing near to the iron rods, from below, one pole of a strong bar-magnet, the rods separate from each other. Explain this.

2. If a compass-needle be deflected when a steel bar is brought near it, how can you find out whether the deflection is due to magnetism already possessed by the bar, or to the bar becoming magnetised by the compass-needle at the time of the experiment?

3. You have given to you two rods, one of soft iron, the other of hard steel; also a compass-needle and a bar-magnet. Describe experiments with the things provided whereby you could find out which was the iron and which was the steel rod.

If the rods are of the same size, describe how you could dispense with the bar-magnet and still distinguish the iron from the steel.

4. A bar-magnet is laid on a table with its N end projecting over the edge. A soft iron ball clings to the under side of the projecting end. State and explain what happens when the S pole of a second

magnet is brought (1) above and near to the N pole of the first, (2) below and near to the iron ball What will happen if the N pole of the second magnet is brought below and near to the iron ball?

5. A compass-needle and a straight strip of soft iron of the same length as the compass-needle are fastened together so as to be in contact with each other at both ends. Will the force which tends to make the combination point north and south be the same as that which would act on the compass-needle alone? Give reasons for your answer

6. A bar-magnet is laid upon the table, and a soft iron bar of about the same length as the magnet is hung horizontally just above it by a flexible string. What will be the effect on the soft iron bar if a second bar-magnet be laid on the table and gradually brought near the first at right angles to it, and with its north-seeking pole pointing to the middle of the first magnet?

7. Two bars of soft iron are placed to the east and west of the north pole of a compass-needle so that the needle still points north and south. If the iron to the east of the needle be replaced by a bar of hard steel of exactly the same size and shape as itself, will the direction in which the needle points be altered? If so, in which direction will it move? and why?

8. One pole of a magnet made of soft iron and only feebly magnetised is found to be repelled by the north pole of a strong magnet when the latter is some distance away, but to be attracted when the magnets are brought close together Explain this.

9. How does hard steel differ from soft iron in respect of magnetic properties? Describe two experiments to illustrate the difference. Which material would you use (1) for the core of an electromagnet, (2) for a permanent magnet? Give reasons for your answer.

10. A piece of soft iron and a piece of hard steel of the same size and shape are rubbed separately from end to end by the north pole of a strong bar-magnet How will you test their magnetic conditions, and what difference will you find between them?

CHAPTER XXXI.

MAGNETIC FORCE, AND MAGNETIC FIELDS.

Application of the principle of moments to a magnetic experiment.—When a compass-needle is acted upon by two external magnets, as in Fig. 256, it comes to rest in a position such that the moments of the two forces F and F' are equal and opposite.

The moment of

$$\mathsf{F} = \mathsf{F} \times \mathrm{OP} = \mathsf{F} \times n\mathrm{P}'.$$

The moment of

$$\mathsf{F}' = \mathsf{F}' \times \mathrm{OP}'.$$

Hence

$$(\mathsf{F} \times n\mathrm{P}') = (\mathsf{F}' \times \mathrm{OP}'),$$

or

$$\mathsf{F}' = \mathsf{F} \times \frac{n\mathrm{P}'}{\mathrm{OP}'}.$$

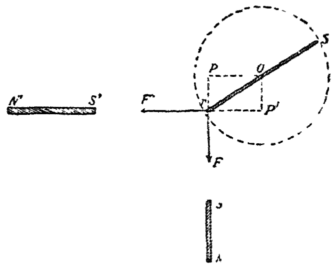


FIG 256 —Deflection of compass needle by two magnets.

In an actual experiment it is difficult to measure $n\mathrm{P}'$ and OP' separately, but the angle $n\mathrm{OP}'$ can be readily measured if a graduated circle is fixed under the needle The ratio $n\mathrm{P}'/\mathrm{OP}'$ is called the tangent of the angle $n\mathrm{OP}'$.

If F' be called the deflecting force, the above result may be stated thus:—**The deflecting force is proportional to the tangent of the angle of deflection produced.**

The law of inverse squares.—The strength of the magnetic force which a bar-magnet exerts upon a compass-needle depends upon their distance apart. This suggests a resemblance to the law

of inverse squares, which holds good with regard to gravitational forces (p. 103). To test whether this is the case, measurements are made of the deflection caused when a bar-magnet is placed at different distances from a compass made free to move in a horizontal plane. The earth's magnetic influence may be regarded as a constant force tending to pull the needle into a north and south direction; and a variable resultant force is created by placing the magnet at different distances from the compass-needle on which the forces due to the bar-magnet and the earth's magnetism act. As all magnets have two poles, it is evidently necessary to use a very long magnet, so that one of the poles is too distant to exert an appreciable disturbing effect on the needle. The apparatus with which the experiment is carried out is termed a **magnetometer.**

Fig. 257 represents a convenient form of magnetometer needle, which may be prepared from a short piece of glass tubing and

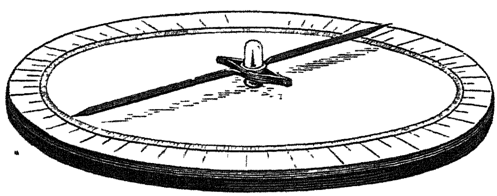


FIG. 257.—Simple magnetometer.

two pieces (2 cm. long) of clock-spring, the similar poles being bound together with copper wire. The pointer is made of thin aluminium foil, the strip on each side of the centre being bent into a vertical plane. A glass crystallising dish serves as a suitable cover for the needle.

EXPT. 237.—**The law of inverse squares.** Use as a magnet a knitting needle, or thin rod of steel, about 45 cm. long, which has been strongly magnetised. Adjust the magnetometer so that the wooden scale is horizontal and at right angles to the meridian, and lay the magnet along the scale with its near pole 15 cm. from the needle. Read the deflection of both ends of the pointer, and calculate the mean deflection Repeat the observation with the magnet at various

distances from the needle. Tabulate the results in the following manner —

Distance.	Deflection (θ)	Tan θ	(Distance)^2	Tan θ × (distance)^2
13	41°	0 87	169	147
15	33°5	0 665	225	149
20	20°6	0 3775	400	151
25	13°·6	0 2425	625	151
30	9°7	0 17	900	153
35	7°3	0 1275	1225	153
40	5°6	0 097	1600	155
45	4°3	0·075	2025	152

The experiment proves that magnetic forces do obey the Law of Inverse Squares, in other words, **the force which a magnet-pole exerts on a distant magnet-pole varies inversely as the square of the distance between them.**

The poles of a magnet.—In the previous paragraph it was assumed that the magnetic forces originated from the extreme ends of the magnet. This assumption was sufficiently correct, owing to the fact that the magnet was extremely long as compared with its width. If such a magnet be dipped into iron filings, the filings only adhere to the ends in a small compact bunch.

In the case of a comparatively short thick magnet, filings will adhere chiefly to the ends, but some adhere even at a considerable

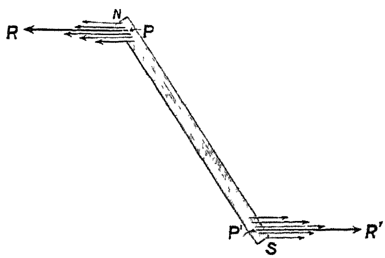


FIG. 258 —The points P and P′ are the poles of the magnet NS

distance from the ends. The pole of the magnet would therefore appear not to be a well-defined point, but a superficial area of considerable extent, each portion of which exerts magnetic force

on a neighbouring magnet. The polarity seems to be more pronounced at the ends, and to diminish gradually towards the middle of the magnet.

In Fig 258 let NS represent a bar-magnet which is suspended in a uniform magnetic field (pp 384-392). In such a field the forces acting on the several small portions of the magnet which exhibit free polarity may be regarded as parallel to each other Just as in mechanics a system of parallel forces (p. 123) may be replaced by a single force acting at a definite point, so in this case the system of parallel magnetic forces acting on the N-seeking pole may be replaced by a single force PR acting at a point P. Similarly, the forces acting on the S-seeking pole may be replaced by a single force P′R′ acting at a point P′ The points P and P′ are termed the **poles** of the magnet, and they may be defined as **the points of application of the resultant magnetic forces acting on a magnet which is situated in a uniform magnetic field.**

EXPT. 238.—**Position of the poles of a magnet.** Place a long wide bar-magnet on a sheet of paper stretched on a drawing-board, and mark its outline on the paper with a pencil. Place a sensitive compass-needle at n_1s_1 (Fig. 259), and put pencil marks in line with its

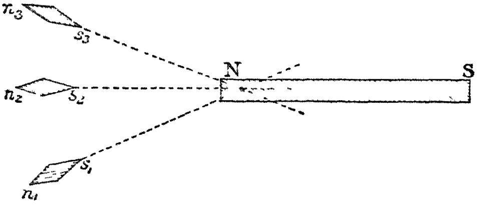


FIG 259 —Methods of locating the poles of a magnet

poles to indicate on the paper the direction in which it is pointing; repeat at other positions, such as n_2s_2 and n_3s_3. Remove the magnet, and produce by means of a straight-edge the three directions obtained These lines, if accurately drawn, should meet at a point on the axis of the magnet and near to its end

Since a slight error may be introduced by the earth's magnetic force tending to make the needle point in some direction other than that due to the magnet alone, it is advisable, before marking the direction of the needle, to turn the board round until the needle points due north.

Note what fraction of the whole length of the magnet is the distance between the position of the pole and the near end of the magnet.

In short thick magnets (about 10 cm. long) the poles are situated about 1 cm from each end When the magnet is long, and only 1 or 2 millimetres wide, the poles approximately coincide with the ends.

Magnetic forces due to both poles of a magnet.—Consider a single north-seeking pole situated at *n* near a bar-magnet NS (Fig 260); it will be repelled by N in the direction *np*, and attracted by S in the direction *nq* Since these forces are inversely proportional to the square of the distance, the force represented by *np* will be greater than that represented by *nq*, and in the ratio of $(nS)^2$ to $(nN)^2$. The resultant of these two forces is *nr*, and it is in the direction of this force that *n* will tend to move. The same method will determine the direction of the resultant force at other points, marked *n'*, in the magnetic field Similarly, an equal S-seeking pole placed at *n* would be acted upon by a force equal in magnitude to *nr*, but in the reverse direction. Hence, if a small compass-needle be placed with its centre at *n*, the forces acting on its poles will cause its magnetic axis to coincide with the direction *nr*. But, as the forces are equal and opposite, there will be no tendency for the compass to travel bodily from its initial position

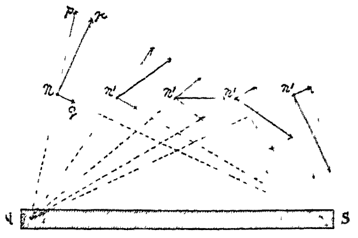


FIG 260 —Graphical method of determining the magnetic forces due to a bar magnet.

EXPT 239—**Direction of resultant force due to both poles of a magnet.** Place a long bar-magnet on a sheet of paper lying on a drawing-board. Mark its outline in pencil, and indicate the position of its poles by pencil dots. Select any point *n* (Fig. 260 about 10 cm from the magnet Join *Nn* and *Sn*, and measure their lengths Measure off, along *Nn* produced and *Sn*, lengths *np* and *nq* proportional to $(Sn)^2$ and $(Nn)^2$ respectively, the scale being such that the shorter line *nq* is not less than 4 cm long Complete the parallelogram *nprq*. The diagonal *nr* represents the direction of the resultant magnetic force acting on a single N-seeking pole placed at *n*. To verify this direction, replace the bar-magnet on the pencil outline and place a small compass-needle with its centre over the

point n. As the earth's magnetic influence may tend to make the needle point in some other direction, rotate the board until the needle points due north; in this position the earth will have no disturbing effect.

Repeat the experiment for other points near to the magnet.

The unit of magnetic pole-strength.—The unit magnet-pole is defined as that which, when placed at a distance of one centimetre from an equal pole, acts upon it with unit force (1 dyne). If one of the poles has a strength m_1 units, the force will be m_1 times as great, and if the other pole has a strength m_2 units, the force will be $(m_1 \times m_2)$ times as great. Also, if the distance apart is increased to d cm., since the force varies inversely as the square of the distance, the force (f) will be equal to

$$m_1 m_2 / d^2.$$

Example.—At what distance apart must two magnet poles, of strengths 74 units and 53 units, be placed apart so that the force of attraction or repulsion between them is equal to the weight of 1 gram?

Since $f = m_1 m_2 / d^2$,
$d^2 = m_1 m_2 / f = 74 \times 53 / 981 = 3922 / 981 = 4$ approximately.
Hence, $d = 2$ cm.

Magnetic Fields

A field of magnetic force.—When a suspended magnetised needle is allowed to swing to and fro round its point of suspension, the manner in which it swings suggests that there are invisible forces acting on the needle, and tending to bring it to rest with its magnetic axis pointing in a definite direction. Whenever these invisible magnetic forces appear to be influencing a suspended magnetised needle, it is said to be in a field of magnetic force. By observing their effects, the presence of these forces is detected, and, in addition to this, their direction also is determined by observing the direction in which the needle points when it comes to rest.

Since the needle behaves in this manner even when no other magnet is near to it, the only possible conclusion is that **the earth has a magnetic field of its own**; and if so, this must be due to a region of south-seeking polarity situated in the direction of the

north pole of the earth, and of north-seeking polarity in the direction of the south pole

If a bar-magnet be held near the swinging needle, a magnetic disturbance ensues which causes the needle to swing to and fro in the same characteristic manner, perhaps more rapidly, perhaps more slowly, and, in nearly every possible position of the magnet relatively to the needle, the needle acquires a different position of rest. Evidently the bar-magnet has a field of magnetic force of its own, the effects of which have been superposed upon those due to the earth's field The needle will come to rest in a position indicating the direction of the **resultant magnetic force,** which is due partly to the bar-magnet and partly to the earth

Again, it will have been observed that the needle swings sometimes rapidly, sometimes slowly. If it swings more rapidly, then the magnetic forces acting on it must be stronger; if it swings more slowly, the magnetic forces must be weaker. In fact, by observing the rate of vibration, it is possible to compare the strengths of the magnetic forces at two different points

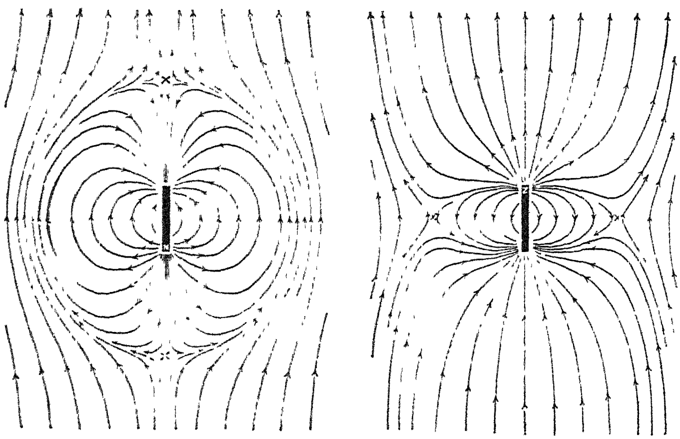
Dr. Gilbert, who was physician to Queen Elizabeth, observed these effects in 1600, and he described a lodestone or magnet as being surrounded by an "orb of virtue." About the middle of last century Faraday substituted the term **magnetic field**

The earth's magnetic field.—In order to investigate the character of any magnetic field, it is necessary to determine two factors at all parts of the field—(i) the direction of the magnetic force, and (ii) the strength of the force. Any diagram which represents the force-directions is termed a map of the magnetic field included within the area of the diagram.

If, when no other magnet is in the neighbourhood, a compass-needle be placed in a series of positions upon a sheet of paper and lines drawn indicating the direction in which it comes to rest, it will be found that the lines are parallel Such a diagram is a horizontal map of the earth's magnetic field, so far as the limits of the paper allow. Faraday, in 1837, termed the lines so obtained **lines of magnetic force,** *i.e.* lines which indicate the directions in which the magnetic forces are acting.

EXPT. 240.—**Map of the earth's magnetic field.** Fasten a square sheet of white paper (80 cms. × 60 cms.) on a table, with one edge pointing

approximately north and south Mark off one of the edges pointing east and west into spaces about 5 cms wide. Place a sensitive compass-needle so that one of its poles is just over one of the marks, and indicate by means of a pencil mark the direction in which the other pole is pointing Move the needle until its first pole is exactly over the second pencil mark, continue this process of marking the directions of the compass-needle until a series of marks have been obtained completely across the paper Join up these points by a continuous pencil line. Plot out other lines in a similar manner, in each case starting from one of the equidistant pencil marks at the edge of the paper Indicate by means of arrow-heads the direction in which the north-seeking pole of the compass-needle *tends* to move, this is called *the positive direction of the magnetic field*



(i) (ii)

FIG. 261 —Map of the combined field due to the earth and a bar-magnet. (i) N pole pointing south, (ii) N pole pointing north

The diagram indicates that the lines of magnetic force due to the earth are uniformly parallel straight lines. The earth's magnetic field is, in fact, uniform over an area much larger than is required for the carrying out of ordinary experiments in magnetism

Resultant magnetic fields.—Accurate maps of magnetic fields representing the combined magnetic forces due to a magnet and to the earth can be obtained by placing the magnet with its north pole

pointing towards the south and marking the positions taken up by a small compass-needle placed in various positions round it in a horizontal plane (Fig 261, i) The combined field will be found to be different when the magnet is placed in the reversed direction, that is, with the south-seeking pole pointing towards the south (Fig. 261, ii). In both cases regions are found in which the magnet's influence upon the needle is exactly neutralised by that of the earth, so that the needle comes to rest in any position with equal readiness · these regions are termed **neutral points.**

EXPT 241 —**Maps of resultant magnetic fields.** Fasten a sheet of paper on the table in the same manner as in Expt 240 Determine accurately the north and south line by the compass-needle, and place a bar-magnet at the centre of the paper with its axis pointing north and south. Starting from a series of equidistant points marked along the top edge, map out the lines of force in the same way as before.

(i) *With the north-seeking pole of the magnet pointing towards the south* (Fig 261, i) —Observe that the lines of force near to the magnet appear to emerge from the north-seeking pole, tracing out a curved path and re-entering the magnet at its south-seeking pole At greater distances the lines of force appear to be those due to the earth, which have been distorted by the presence of the magnet Also observe the positions (marked X) of the neutral points.

(ii) *With the south-seeking pole of the magnet pointing towards the south* (Fig 261, ii) —Observe how the lines of force due to the earth appear to be drawn together by the magnet, and how the more distant lines are distorted. The neutral points are now east and west of the magnet.

Properties of lines of force.—An instructive analogy, due to Faraday, compares the properties of the magnetic lines to the forces which would be exerted by stretched elastic threads, coinciding in direction with the lines of force, which tend to shorten themselves from end to end, but, as these would not present the bulged appearance shown by the curved magnetic lines in Fig 261, he assumed that, in addition to their tendency to contract in length, the lines of force also have the property of repelling each other sideways.

The question as to what may be the mutual action between neighbouring lines of force which travel in *opposite* directions is difficult to demonstrate; but if the hypothesis that such lines

mutually *attract* is found to afford an explanation of observed phenomena it may be accepted as true, so far as a working hypothesis allows.

Let NS (Fig 262, i) represent a bar of iron placed horizontally in a uniform magnetic field, in which AB, CD represent two lines

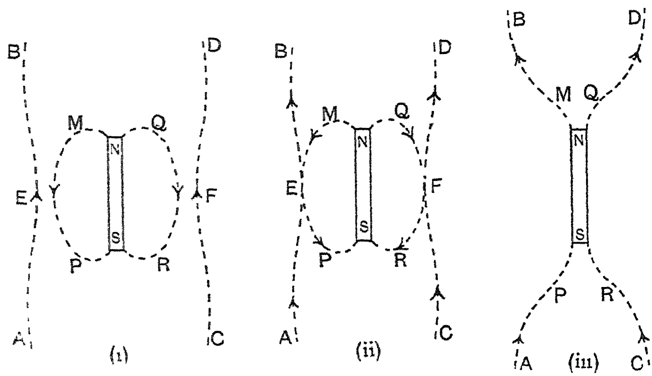


FIG. 262 —Resultant lines of force, due to the earth and to a tangent

of force. Let the bar be feebly magnetised, and let MP and QR be two lines of force associated with it. Since these, in the regions E and F, are proceeding in opposite directions to AB and CD, the latter are bent inwards. If the polarity of NS is now increased (Fig. 262, ii) the lines MP and QR will bulge out more than before, owing to new lines being formed in the space round the magnet, and the lines will actually touch at E and F. The tendency of lines of force to shorten makes this distribution unstable; the lines AB and MP break at E, the portions ME and EB joining together to form the continuous line MB (Fig. 262, iii); and the portions PE and EA combine to form the line PA. Corresponding changes take place on the other side of the magnet The final distribution closely resembles that obtained in Fig. 261, ii.

It is generally accepted that the **positive direction of a line of force is that direction in which a single north-seeking pole will tend to travel if placed at any point on that line of force.** The opposite direction is termed the **negative** direction of the line of force. Hence, a map of the magnetic field of a magnet will indicate the lines of force

as emerging from the north-seeking pole and re-entering at the south-seeking pole.

It can be shown by experiment that a north-seeking pole actually does tend to travel along a line of force in the positive direction.

EXPT 242.—**Movement along a line of force.** Support a bar-magnet, 20 cm. long, near and parallel to the edge of a large photographic dish filled with water. Magnetise a short fragment of sewing-needle, and fix it through a small piece of cork so that the needle can float freely in a vertical position. Let the north-seeking pole of the needle be uppermost. If floated near the north-seeking pole of the magnet the repulsion of the similar pole of the needle will be stronger than the attraction of the opposite pole of the needle, since the latter is more distant. The needle will travel slowly over the surface of the water, tracing out a curved path connecting the north- and south-seeking poles of the magnet

Iron-filing maps of magnetic fields.—The compass-needle method of obtaining a map of the magnetic field of the earth or of a magnet has the advantage of accuracy, and it is also capable of affording information in parts of a magnetic field which would be too weak to be mapped out by the methods now to be described The latter will give accurate maps of the field near a magnet, but will not do so for the more distant parts where the earth's magnetic field is predominant. The compass-needle method, however, cannot well be adopted in the lecture-room owing to the time required to obtain even one complete map.

The more rapid methods depend upon the principle of magnetic induction whereby a piece of soft iron, when placed in a magnetic field, becomes magnetised by induction. Soft iron filings may be used for the purpose, each fragment becomes a temporary magnet, and, if free to move, behaves in the same manner as a compass-needle. The effect is approximately the same as would be obtained if an extremely large number of compass-needles were used, and, moreover, the general contour of the whole field is visible simultaneously.

Figs. 263-7 have been reproduced mostly from permanent maps obtained by using 'paraffined paper' instead of ordinary paper.

EXPT. 243.—**Maps of magnetic fields.** Obtain maps of the magnetic fields due to the following arrangements of magnets.

(i) One bar-magnet (Fig. 263).

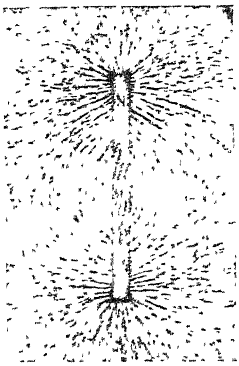


FIG. 263

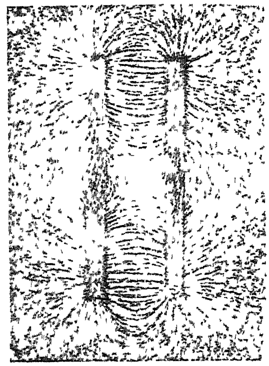


FIG. 264

(ii) Two bar-magnets side by side, with unlike poles together (Fig. 264).

(iii) Two bar-magnets side by side, with like poles together (Fig. 265).

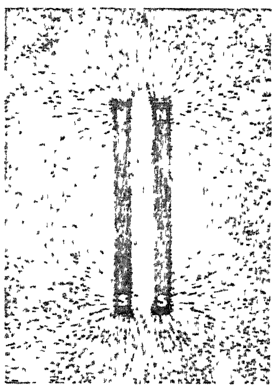


FIG. 265

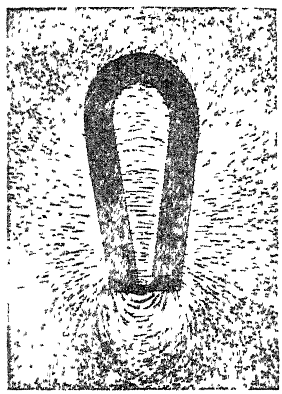


FIG. 266

(iv) Two bar-magnets, with their axes in line, and unlike poles together.

(v) Two bar-magnets, with their axes in line, and like poles together

(vi) One horse-shoe magnet, without keeper (Fig 266)

(vii) One cylindrical bar-magnet, fixed in a vertical position, and the paper supported horizontally over the upper pole (Fig 267)

Fig 267

Symmetry of the magnetic field due to a single bar-magnet.—In Fig. 263 it will be seen that lines of force emerge from and re-enter the magnet at all points (with the exception of a small portion near the centre), and the distribution of the lines is densest in parts near to the extreme ends. The map does not indicate those lines of force which pass vertically through the paper, or those which pass vertically downwards through the table: in fact, the map is really a horizontal cross-section through the magnetic field. If it were possible to obtain a vertical map by the same methods, it would be found that the arrangement of the lines of force is identical with that obtained in the horizontal maps If the magnet is turned over on to its side the lines of force which were originally in a vertical plane will now be horizontal, and a map of the field of the magnet in this position will show that their contour and general distribution is identical with that obtained when the magnet was in its original position In fact, the distribution of the lines of force is the same in all planes besides the horizontal and vertical. A bar-magnet may be imagined to be clothed completely on all sides in an invisible garment of lines of force. The lines of force in a vertical plane can be detected readily by means of a short magnetised needle supported from its centre by a silk fibre.

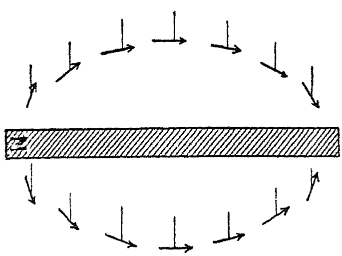


Fig 268 —The vertical field of a bar-magnet mapped out by means of a suspended magnetised needle

Expt. 244 —**Vertical magnetic field** Attach a silk fibre to a small sewing-needle and adjust the fibre so that the needle is exactly horizontal when swinging freely. Magnetise the needle by placing it inside a spiral of wire through which an electric current is passing.

Clamp a large bar-magnet in a horizontal position and support the needle vertically over and under the magnet in a series of different positions (Fig 268) It will be evident that the general contour of the vertical magnetic field is the same as the horizontal.

Intensity of a magnetic field.—The intensity of a magnetic field is expressed numerically as the force (in dynes) with which it acts on a unit magnet pole placed in the field Hence, **a magnetic field has unit intensity when the force with which it acts on a unit pole placed in the field is equal to one dyne.**

The intensity of a magnetic field is expressed graphically by the number of lines of force supposed to pass through unit area of a section of the field drawn at right angles to the direction of the magnetic lines of force. Thus, unit magnetic field would be represented by one line of force per square centimetre (Fig. 269). Similarly, a field having an intensity of 25 units would be represented by 25 lines of force passing through each square centimetre. This method of indicating the intensity of the field by means of the closeness of the lines of force is made use of even in rough hand-sketches of magnetic fields.

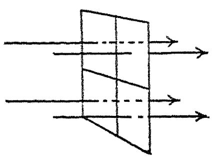


FIG 269 —A magnetic field of unit intensity

Internal magnetic fields.—So far, only the magnetic phenomena in the region round a magnet have been considered. Each line of force may, however, be regarded as being continued through the substance of the magnet, so as to form a complete loop without free ends. That this is the case may be illustrated by breaking a magnet, when it will be found that lines of force proceed from one part of the fracture to the other.

Every small fragment of a magnet is, in fact, traversed to a greater or less degree by lines of force, which enter at its south-seeking pole and emerge at its north-seeking pole (Fig. 270).

Since each small fragment is a complete magnet in itself, a bar-magnet may be regarded as consisting of a large number of minute magnets arranged with like poles pointing in the same direction. There is no theoretical reason why this breaking process should not be continued until the fragments are almost infinitely small, and each such fragment found to be still a

complete magnet. Modern theory maintains that even the smallest physical quantity—the molecule—present in a bar-magnet

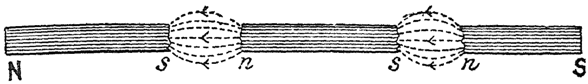


FIG 270—A broken magnet

is a minute magnet, though the bar may contain several millions of such molecules.

EXPT. 245—**Effect of breaking a magnet.** Magnetise a piece of clock-spring about 10 cm long Break it into halves. Examine the pieces by a compass-needle One half does not possess north-seeking polarity only, and the other half south-seeking polarity; each half is in itself a complete magnet, possessing two unlike poles Lay the halves on the table, in line with one another, and about 2 cm apart. Sprinkle iron filings upon a sheet of paper placed over the parts of the clock-spring, there are evidently lines of force connecting the broken ends Break the spring into still smaller fragments, and test the polarity of each. Observe that like poles of every fragment point in the same direction.

EXPT. 246—**A steel filing magnet.** Fill a glass test-tube with steel filings loosely packed; cork up the tube, and notice that it behaves towards a test-needle like an ordinary piece of iron Magnetise the tube by stroking it in one direction with one pole of a strong magnet, or better, by means of a spiral of wire and electric current (p. 368). Observe that the tube now has opposite polarities at the ends, and that the filings appear to some extent to have arranged themselves lengthwise. Each filing has been magnetised, just as small sewing-needles would have been magnetised by similar treatment Each filing has its lines of magnetic force, which come from, and afterwards pass into, neighbouring filings, and only appear at the ends of the tube where they emerge into the surrounding space. Empty the filings upon a sheet of paper, mix well together, and pour them back into the tube; again test for polarity.

Theory of magnetisation.—In a bar of unmagnetised steel or iron, each molecule may be a magnet, but the molecules are grouped into numerous independent magnetic chains, each of which may consist of two or more molecular magnets grouped in such a manner that there is no external magnetic field. Fig. 271, i. represents one of several ways in which four such magnets

may be grouped. On applying a weak magnetising force, such as an external uniform magnetic field H (Fig. 271, ii), the molecules merely turn through a slight angle, giving a slight excess of north-seeking polarity in the direction of the magnetising force, and slight excess of south-seeking polarity in the opposite direction. As the magnetising force H is increased (Figs. 271, iii and iv), the molecules turn gradually more into line. When all the molecules point exactly in the direction of the force, an increase of the latter will not produce any further effect—in fact, the magnet is saturated. This explanation is known as **Weber's theory of magnetisation.**

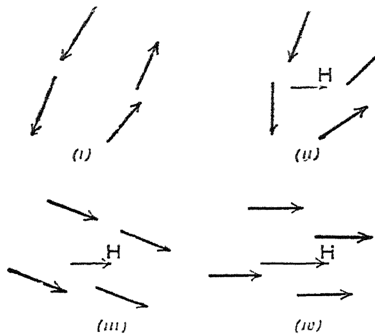


FIG. 271.—Weber's theory of magnetisation.

Behaviour of soft iron in a magnetic field.—When a bar of soft iron is placed in a magnetic field, with its length coinciding with the direction of the magnetic lines of force, the molecular magnets of the soft iron are pulled partially or completely into alignment, according to the magnitude of the magnetising force; the iron temporarily becomes magnetised by induction. At all points where lines of force enter the iron we find a region of south-seeking polarity, and a region of north-seeking polarity where they emerge. If the iron be situated so that the lines of force pass through the iron from side to side perpendicularly to its axis, then no lines of force traverse its length and no polarity is found at the ends; in such a case the polarity will be found distributed along the two sides.

Hence, in order to magnetise a bar of soft iron by induction, so as to create polarity at its ends, it is necessary to place the iron in the field so that the lines of force pass through the iron in the direction of its axis.

EXPT. 247.—**Conduction of lines of force.** Place a long strip of thin soft iron on a sheet of plain paper, its length coinciding with a north-

south line The iron should be tested previously to ensure that it is entirely free from permanent polarity By means of a compass-needle, as described in Expt. 241, map the lines of force in the region near to the iron.

The appearance of the map (Fig 272) suggests that the lines of force prefer to travel through the iron rather than the surrounding air. This idea is sometimes expressed by saying that *iron, or any other magnetic substance, conducts lines of force better than air.* The effect of iron in a magnetic field is to some extent analogous to an open gap in a hedge, across which a strong wind is blowing, more air passes through the gap than through an equal length of the hedge, since the gap offers less resistance to the passage of the air; the *stream-lines* (or lines indicating the direction of flow) of the air converge towards the gap, and diverge again on the opposite side of the hedge. The gap may be regarded as a better conductor for air than the hedge. In a magnetic field the lines (which do not indicate directions of *flow* but rather *direction of force*) converge towards the iron in a similar manner, and the degree to which this takes place depends upon the softness of the iron.

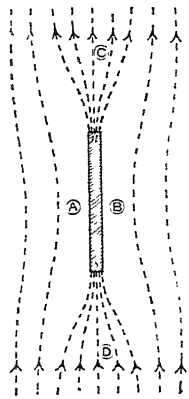


FIG 272 —A bar of soft iron in the earth's field

The end *s*, where the lines of force enter the iron, has acquired south-seeking polarity, and north-seeking polarity at *n*, where they emerge It is evident also that the *intensity* of the magnetic field on either side of the iron, *e g.* in the regions A and B, is diminished, and it is increased in the regions C and D. The change in intensity of the field in these regions might be observed by determining the rate of swing of a short magnet suspended horizontally, and by comparing the rate with that observed when the iron is removed to a distance. It can be proved that the intensity of the field is proportional to the square of the number of swings described in a given time

Since the intensity of the field is reduced in the regions on either side of the iron, the iron may be regarded as a more or less imperfect magnetic screen for these regions.

Magnetic screens.—When soft iron is placed in a magnetic field, and causes the intensity of the field to be diminished at any neighbouring point, it is said to screen that point magnetically. When a thick piece of soft iron is placed near the pole of a bar-magnet, as shown in Fig 273, many of the lines of force appear to traverse the iron from its centre towards either end. only a few lines appear to pass beyond the distant side of the iron screen The centre of the iron screen will have south-seeking polarity, and both ends will have north-seeking polarity. The deflection of a compass-needle at A will be diminished by interposing the slab of soft iron.

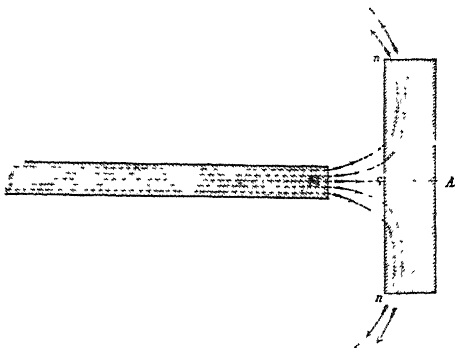


FIG 273 —The point A is magnetically screened by a slab of soft iron

This is not the only position of the iron slab which will screen the needle: in fact the point A will be more fully screened if the slab is placed on either side of the magnet and with its length parallel to the magnet's axis.

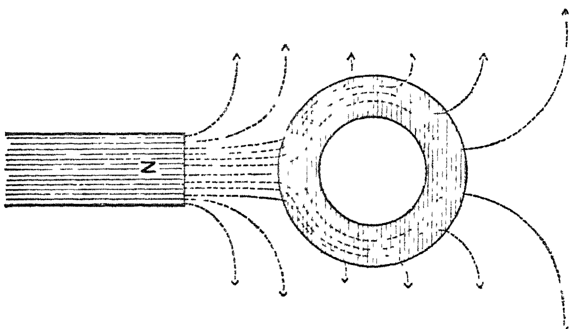


FIG 274.—The screening effect of an iron shell.

The most perfect form of magnetic screen is obtained in the case of a hollow sphere of thick soft iron. The space within the sphere may be said to be absolutely free from lines of force, all of which will pass through the substance of the iron shell (Fig. 274) The diagram indicates the general distribution of the lines of force in a section through the centre of the sphere. This principle was applied by Lord Kelvin to the screening of ship galvanometers from magnetic disturbances by placing the galvanometer inside a cylindrical shell of soft iron.

EXERCISES ON CHAPTER XXXI.

1. Three precisely similar magnets are placed vertically, with their lower ends on a horizontal table Iron filings are scattered over a plate of glass which rests on their upper ends, two of which are north poles and the third a south pole Give a diagram showing the forms of the lines of force mapped out by the filings

2. Several bar-magnets are placed on a table. How would you use a card and iron filings to determine how to place a nail, lying horizontally on the table with its centre at a given point, so that it may acquire (i) the largest, (ii) the smallest possible amount of magnetism by induction?

3. Two bar-magnets are laid on a table at right angles to each other, so that the axis of one passes through the middle point of the other. They do not touch. A sheet of cardboard over which iron filings are scattered uniformly is placed over them and tapped Draw a picture showing the appearance which the iron filings present.

4. A strong bar-magnet is placed on a table with its axis lying in the magnetic meridian, and with its north-seeking pole towards the north. State in what direction a compass-needle points (i) when placed immediately over the centre of the magnet, (ii) when gradually raised vertically upwards

5. What is meant by a line of force? Draw diagrams showing the general form of the lines of force when a small magnet is placed with its axis parallel to the lines of force of the earth's field, if the north pole of the magnet is turned towards (i) the north, and (ii) the south.

6. A short piece of soft iron is to be magnetised inductively. State how it should be placed relatively to (i) a bar-magnet, (ii) a horse-shoe magnet, in order to obtain a satisfactory result Give diagrams

7. Give a diagram of the lines of force due to a horse-shoe magnet (i) with the keeper on, (ii) with the keeper off.

8. Iron filings are scattered on a piece of cardboard which is placed over a horse-shoe magnet and tapped. What differences would be observed in the arrangement of the filings when the ends of the magnet were joined in turn by bars of (i) steel, (ii) soft iron, and (iii) copper?

9. If you were required to magnetise a circular ring of steel so that it should show no sign of magnetisation, how would you proceed? and how, being allowed to deal with the steel in any way that you pleased, would you prove that it really was magnetised?

10. An iron ball is held over a pole of a horse-shoe magnet. Will the attraction exerted on the ball be altered if the poles of the magnet are connected by a soft iron keeper, and, if so, in what way, and why?

11. A long magnet and a piece of soft iron of the same size and shape are placed parallel to each other underneath a sheet of paper on which iron filings are strewed. How will the filings arrange themselves?

12. A compass-needle is deflected by a bar-magnet placed some distance away from it. How is the deflection modified (if at all) when a bar of soft iron is placed parallel to, but not touching, the magnet? Give reasons for your answer.

13. A compass-needle is deflected 15° from the meridian when a bar-magnet is placed on the table some distance away. Will the deflection be altered if the poles of the magnet are connected by a bent iron rod? Give reasons.

14. A bar-magnet is placed with its axis in the magnetic meridian and its north pole turned towards the south. Describe and explain the behaviour of a small compass-needle as it is carried along the prolongation of the axis of the magnet, both towards the north and towards the south.

15. A bar-magnet one inch long, with its north pole pointing due east, is placed at a distance of four inches from a small compass-needle due north of the centre of the magnet. How will the needle be deflected, and how will the deflection be altered if a thick iron ring 2 inches in diameter is placed round the bar-magnet?

16. When a magnet is plunged into iron filings, the filings cling to the ends but not to the middle. Does this mean that there is no magnetism at the middle of the magnet? State the reasons for your answer.

17. A horse-shoe magnet is brought due south of a small compass-needle, the line joining the poles of the magnet being east and west, with the north pole to the west. Describe the manner in which the compass is deflected.

Describe and explain what will happen if the keeper is placed on the magnet.

18. What is meant by a *uniform magnetic field*?

A steel rod hangs vertically from the pan of a balance and its weight is observed. It is then magnetised strongly and weighed again with the north-seeking pole pointing vertically downwards. Will any change be observed?

What will be the effect upon the apparent weight of the rod, before and after magnetisation, of holding under it a thin disc of soft iron (i) with its plane faces vertical, (ii) with its plane faces horizontal?

Give reasons for each part of your answer.

19. What is meant by a *line of magnetic force*?

Draw diagrams showing the lines of force due to two equal bar-magnets, each 1 foot long, placed in line 1 foot apart with (i) opposite and (ii) like poles facing each other. Show how the lines of force are affected in each case by placing a bar of iron, 10 inches long, in line with the magnets and midway between them.

Describe the magnetic state of the iron in each case.

20. A bar-magnet 30 cms. long is placed in the magnetic meridian, and it is found that a small compass-needle, placed on the axis of the magnet produced at a distance of 30 cms. from one pole, will point in any direction. How would you explain this? State which pole of the magnet points northward.

If the strength of the earth's horizontal field is 0·18 C.G.S. unit, what is the pole strength of the magnet?

21. *AB* is a thin magnet 20 cms. long, the strength of each of its poles being 12 units. Upon *AB* as base an equilateral triangle *ABC* is constructed. Find the magnitude and direction of the force that a unit pole would experience if it were placed at *C*. Also the force upon the magnet caused by the unit pole at *C*.

CHAPTER XXXII.

TERRESTRIAL MAGNETISM.

The earth a magnet.—The characteristic manner in which a compass-needle swings to and fro and finally comes to rest pointing approximately north and south, even in the absence of any neighbouring magnet, suggests that the earth itself must be enveloped in a field of magnetic force There are lines of magnetic force originating from a region of north-seeking polarity in the neighbourhood of the south geographical pole, and traversing the earth's surface towards a region of south-seeking polarity in the neighbourhood of the north geographical pole. This suggests that a piece of soft iron will become temporarily magnetised if held with its axis pointing in the same direction as that in which a compass-needle points.

EXPT. 248 —**Magnetisation by means of the earth's field.** Hold a strip of thin galvanised iron (about 30 cm. × 2 cm) so that it is pointing approximately north and south. Tap it gently with the knuckles. Test its polarity by bringing its ends near to a compass-needle. The end pointing towards the north has acquired north-seeking polarity. Now hold the iron with its north-seeking pole pointing towards the south, and again tap it. Notice that its polarity is reversed Finally, hold the iron in an east and west position and again tap it. Notice that all polarity has disappeared

The tapping may even be dispensed with if the soft iron be simply kept in position and the compass-needle be brought near to its ends in order to detect the polarity.

EXPT 249 —**Determination of the geographical meridian.** Fix a rod upright on level ground, where the sun can shine upon it About an hour or two before mid-day mark the direction and length of the

shadow of the rod, and by means of a loop of string fitting loosely upon the rod mark out an arc of a circle with radius equal to the length of the shadow (Fig. 275 In the afternoon, when the end of the shadow again reaches the arc, and the shadow has therefore the same length as at the time of the morning observation, mark its direction. A line bisecting the angle between the directions of the two shadows of equal length is a true north and south line, or lies in the geographical meridian of the place of observation.

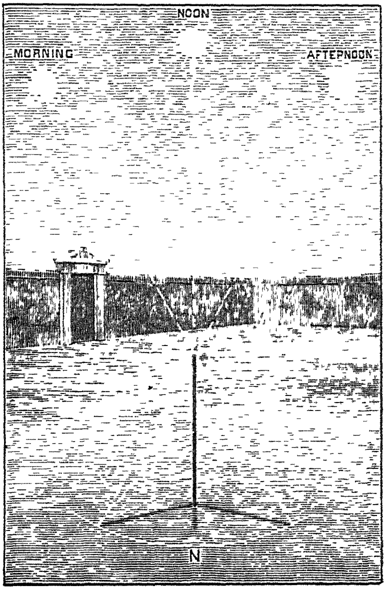


FIG 275 —Determination of the geographical meridian

Declination.—The **geographical meridian** at any point of the earth's surface is the vertical plane passing through that point and through the poles of the earth. The **magnetic meridian** at any point is the vertical plane passing through the axis of a compass-needle placed at that point. In most localities on the earth's surface these two meridians do not coincide exactly. **The angle** between the magnetic meridian and the geographical meridian at any place is called the **Declination** at that place.

The fact that the compass-needle does not point to the true north was observed first by Columbus when on a voyage in 1492. He found that, at a point near to the Azores, the compass pointed true north, but that in regions to the east of this it pointed west, and that in regions to the west it pointed east of true north.

In England, and in many other localities, the compass-needle now points to the west of true north. Elsewhere the declination is easterly, and there are comparatively few localities where the needle points due north. In India the declination is zero at all

localities having the same latitude as Pondicherri. To the north of this the declination is easterly, and gradually increases to 1° E at the latitude of Calcutta. To the south of Pondicherri it is westerly, and has a maximum value of about 2°·5 W. at the south of Ceylon.

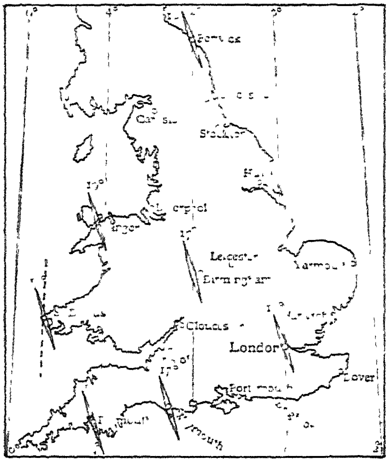


FIG. 276.—Magnetic declination is the angle between the geographical meridian and the direction in which a compass-needle points.

The magnitude of the declination in any locality is not constant, but changes slowly from year to year. The declination at Greenwich was 15° 41′ W. in 1910, and diminishes at the approximate rate of 5′·3 per annum. This **secular change**, as it is termed, was first observed in 1580 by Burroughs (comptroller in the Navy in the time of Queen Elizabeth). In that year the declination in London was 11° E.; this gradually diminished, and in 1657 the needle pointed due north. The declination then became westerly, and reached a maximum value of 24° 30′ W. in 1816. Since that date it has been slowly diminishing to its present value. It is estimated that 320 years are required for a complete cycle in the changes of the declination.

EXPT. 250.—**Determination of the magnetic meridian.** Bore circular holes through two square pieces of cardboard, and fasten silk fibres across the holes (Fig. 277). Attach these to opposite end-faces of a bar-magnet, and suspend the magnet above the table by means of a silk loop and a bundle of unspun silk fibres. Bring the magnet to rest, and by means of *brass* pins fixed vertically into the table mark the direction of the line *ab* joining the points of intersection of the silk fibres. Reverse the magnet so that the cross fibres are now underneath, and mark the direction *a′b′*. The line bisecting the angle between *ab* and *a′b′* is the magnetic meridian. Also, since the magnet comes to rest with its magnetic axis coinciding with the magnetic

meridian, a line drawn on the face of the magnet in the same vertical plane as the meridian will indicate the direction of the magnet's magnetic axis.

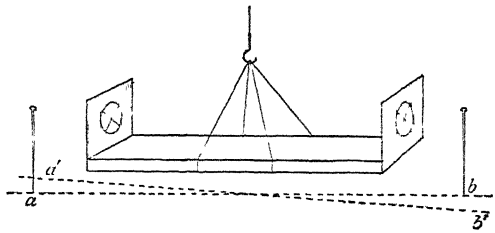


FIG 277.—Determination of the magnetic meridian

EXPT 251.—**To find the magnetic axis of a magnetised steel disc, and also the magnetic meridian.** The steel disc is supposed to be magnetised along a diameter. Mark on each face of the disc a long arrow passing through the centre and pointing in the same direction Suspend the disc horizontally, by means of silk cord, just above the table (Fig. 278). When the disc comes to rest mark on the table the direction *ab* in which the arrow is pointing, and mark this direction with an arrow head. Invert the steel disc, and mark the direction *cd* in which the arrow now points. Remove the steel disc, and bisect the angle included between the arrow heads at *b* and *d* This bisecting line is the magnetic meridian. Re-suspend the disc, and mark on its surface a line coinciding with the meridian, this line is the magnetic axis of the disc.

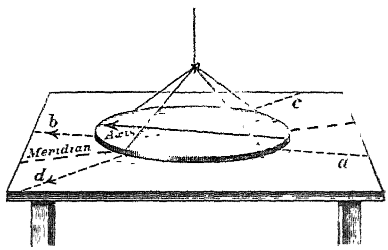


FIG 278 —The magnetic axis of a magnetised disc

Magnetic dip.—It does not follow that, because a compass-needle supported on a pivot remains horizontal, the lines of force acting upon it are also horizontal. Even if the lines of force are inclined to the horizontal, it may still be possible for them to have a directive action on the needle. In Fig 279 let **ns** represent a compass-needle, and **nI**, **sI'** the forces due to the earth's

field. The force **nI** may be regarded as the resultant of two separate forces—**nH** the **horizontal component**, and **nV** the **vertical component.** Similarly, **sI′** may be regarded as the resultant of the two forces **sH′** and **sV′**. The forces **nH** and **sH′** will pull the needle into the magnetic meridian, while **nV** and **sV′** will tend simply to tilt the needle out of the horizontal. The weight of the needle is usually sufficient to mask the effects of the latter forces. It is found that the earth's lines of force actually are inclined to the horizontal in most localities, and the tendency to tilt the needle is neutralised by making it slightly heavier at one end.

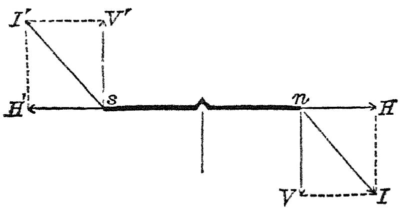


FIG. 279.—A compass-needle acted upon by magnetic forces inclined to the horizontal.

EXPT. 252.—**The dip of a magnetised needle.** Suspend a long knitting-needle by tying a silk thread to it, and adjust the thread so that the needle swings horizontally. Carefully magnetise the needle without disturbing the position of the thread. Observe that the needle now dips down with its north-seeking pole downwards. Since the needle naturally tends to take up a position along the lines of force, it follows that the latter must be inclined to the horizontal.

The dip-needle.—In order that a magnetised needle may move freely in a vertical plane, it must be supported on a rigid horizontal axle; if it is to be influenced by magnetic forces only, and to be absolutely independent of the force of gravity, the axle must coincide with the centre of the needle. The construction of an accurate dip-needle is an extremely delicate operation, and it is only possible to obtain correct measure-

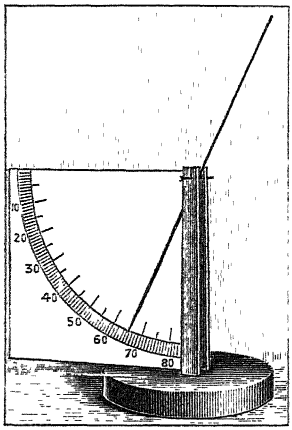


FIG. 280.—A simple form of dip-needle.

ments with a costly apparatus. Fig. 281 represents a form of dip-needle used for determinations of magnetic dip.

To make an observation, the instrument is turned until the needle is vertical, in which case its plane of movement is at right angles to the magnetic meridian. The stand is then rotated through 90° to bring the plane of movement of the needle into the magnetic meridian, and the angle which the needle then makes with the horizontal is the angle of dip.

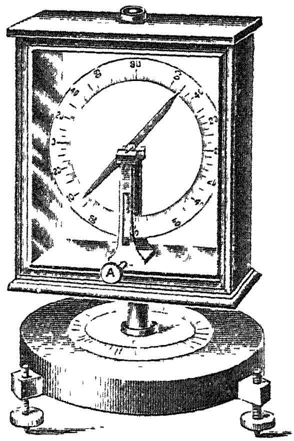


Fig 281 —A dip-needle.

The angle between the axis of a magnetised needle, which is free to move in the vertical plane of the meridian, and the horizontal line through its point of support, is called the Dip.

Angle of magnetic dip in various localities.—The **dip**, like the declination, differs in different localities, and also changes from year to year The dip in London, for the year 1898, was 67° 12′, and was 66° 52′ in the year 1910 Near the equator localities are found where the dip is nil. As the needle is conveyed northwards the dip gradually increases, and at a point in Boothia Felix (Lat. 70° 5′ N., Long. 96° 46′ W.) Sir John Ross found, in the year 1831, that the dip-needle was exactly vertical. This region must be one of south-seeking polarity; it is one of the so-called magnetic poles of the earth. When the needle is conveyed southwards from the equator, the south-seeking pole of the needle dips downwards, and the amount of dip gradually increases as the south magnetic pole is approached. The position of the south magnetic pole was found by the Shackleton Antarctic expedition in 1908 to be in latitude 72° 25′ S. and longitude 154° E. The **magnetic equator,** or line of no dip, crosses the south of India at about the latitude of Tinnevelli

The secular change in the dip is far less in magnitude than that of the declination. Thus, in the year 1576 it was 71° 50′

at London, in 1720 it was $74° 40'$, and at the present time it is diminishing at about the rate of $2'$ every year.

The directive action of the earth's field.—The action of the earth's field is directive only, and not translatory also. Consider a small magnetic needle on a cork floating in water. The forces due to either of the earth's magnetic poles acting on the two poles of the needle are opposite in direction. The distance of the earth's magnetic pole from this latitude is several thousand miles, in comparison with which the length of the needle is infinitesimal. Hence the two poles of the needle may be regarded as being equally distant from the earth's magnetic poles, and the forces acting on the needle are therefore practically equal in magnitude.

When, however, the pole of a bar-magnet is held near the needle the length of the needle is no longer small compared with the distance away of the magnet-pole. One pole of the needle will be considerably nearer to the magnet-pole than the other, one force will be greater than the other, and the floating needle will move bodily in the direction of the greater force.

EXPT 253—**Action of the earth.** Fix a magnetised sewing-needle to a flat cork with wax so that the needle is horizontal when the cork is floating on water contained in a dish. Float the cork on the water so that the needle points east and west Notice how the needle rotates into the magnetic meridian, but does not tend to move bodily towards the side of the dish

EXPT. 254.—**Action of a magnet.** Hold the pole of a bar-magnet near the needle. Notice how the needle not only points in a definite direction depending upon the position of the magnet, but also moves bodily towards the magnet.

Simple hypothesis of terrestrial magnetism.—Both declination and dip may be explained roughly as being due to a huge imaginary bar-magnet passing through the earth's centre and slightly inclined to its axis, so that one end approaches the earth's surface at Boothia Felix and the other end approaches the surface in South Victoria land. At these points the dip-needle stands vertically, and they are called the **magnetic poles** of the earth. The directions of the lines of force of such a magnet would coincide approximately with the directions in which a dip-needle is observed to point. Fig. 282 indicates the relative

positions of the north geographical and north magnetic poles, and the directions are shown of a dip-needle placed at various points on the earth's surface

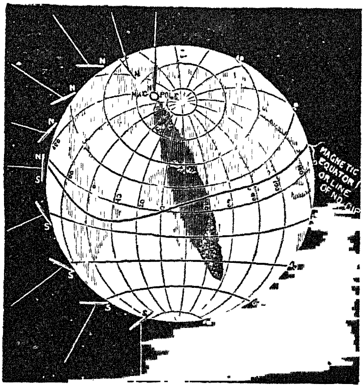


FIG 282 —Imaginary magnetic core inside the earth in explanation of magnetic dip in different latitudes.

The mariner's compass.—The simplest form of mariner's compass consists of a magnetised needle fastened underneath a circular card, the upper surface of which is divided by radii into thirty-two divisions. These divisions are called **the points of the compass.**

The form of needle suggested by Dr. Gilbert consisted of a pair of thin bent needles with their ends united. This form is adopted still in many small vessels. The needle and a card are supported on a sharp metal pivot by means of an agate cap which is fixed to the centre of the needle

In order to prevent the rolling of the ship from disturbing the compass out of the horizontal position, the circular box (made of brass or copper) containing the needle is supported on **gimbals**, the nature of which will be understood from Fig 283. The compass-box is pivoted on an axis, so as to turn freely inside a ring which is also capable of turning round a second axis at right angles to the first. This arrangement allows the compass-box to remain horizontal in spite of the rolling and pitching of the vessel.

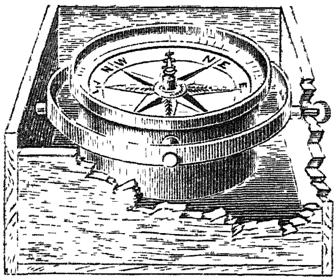
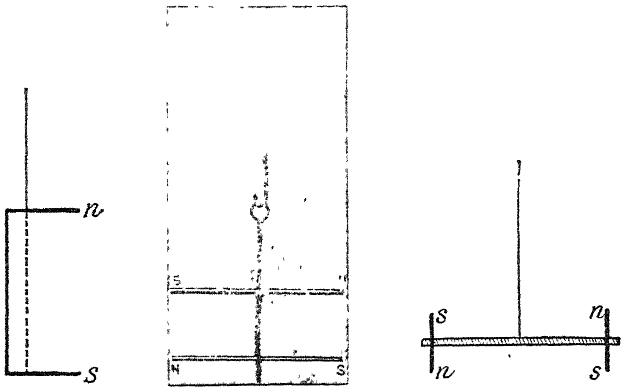


FIG 283.—Method of supporting a compass-box on gimbals.

In the modern Admiralty standard pattern the needles consist of two pairs of parallel straight bars of flat clock-spring fixed with their breadth perpendicular to

the card. The card consists of a thin disc of mica, 10 inches in diameter, with paper pasted on each side to prevent warping.



(i) (ii) (iii)

FIG. 284.—Astatic needle and astatic pairs

Astatic needles.—It is sometimes desirable to use a magnetic needle which, when suspended, is unacted upon by the earth's field. When a single magnetised needle is bent so as to form three sides of a rectangle, and is suspended as shown in Fig. 284 (i), the forces acting on *n* and *s* are equal and opposite in direction, and the needle remains at rest in any position. Such an arrangement is termed an astatic needle.

A more useful method in practice is that originally devised by Nobili. Two magnetised needles, which are as identical as possible in dimension and degree of magnetisation, are fixed together rigidly in the manner shown in Fig. 284 (ii). When freely suspended in the earth's field, the forces acting on the lower needle are neutralised by those acting on the upper needle. In practice it is almost impossible to obtain two magnets absolutely identical in every respect, but it is easy to obtain an arrangement which is sufficiently astatic for experimental purposes; usually it is termed an **astatic pair.**

Fig. 284 (iii) represents a form of astatic pair devised by Prof. S. P. Thompson.

EXERCISES ON CHAPTER XXXII.

1. A strip of steel is bent about the middle point, so that the halves are inclined to each other at a right angle. It is then magnetised so that its extremities are south poles and the angular point a north pole, and is placed on a flat piece of cork floating in a basin of water. How will it set?

2. A bar-magnet is laid on a table perpendicularly to the magnetic meridian, and so as to point to the centre of a compass-needle. Describe and explain the behaviour of the needle.

3. A rod of iron, AB, held vertical with the end B downwards, is tapped smartly with a mallet. When turned into a horizontal position and brought near to a compass-needle, the end B repels the north pole of the needle at a distance of four inches, but attracts it when the distance is reduced to one inch. Explain this.

4. A large soft iron rod lies on the table in the magnetic meridian, and a dipping needle is placed at some distance and at about the same level (i) due south, (ii) due north of it. How will the magnitude of the angle of dip be affected in each case? (Neglect any inductive action between the needle and the bar.)

5. How would you hold a rod of soft iron so that the influence of the earth's magnetic field upon it may be (i) as great as possible, (ii) as small as possible? Give reasons for your answer.

6. A tall iron mast is situated a little in front of the compass in a wooden ship. Explain the nature of the compass error when the ship is sailing in an easterly direction (i) in the northern, (ii) in the southern hemisphere.

7. A bar of soft iron is laid on a table with its length in the direction of the magnetic meridian. Give a sketch of the magnetic field in its neighbourhood in the plane of the table, and explain how you would investigate it with a compass-needle.

8. An unmagnetised bar of iron is laid on a horizontal table in a north and south direction; what is its magnetic state?

How will its magnetic state be altered if the end of the bar which is towards the north is raised until the bar is vertical?

9. A rod of iron when brought near to a compass-needle attracts one pole and repels the other. How will you ascertain whether its magnetism is permanent or is due to temporary induction from the earth?

10. An iron rod held vertically is tapped with a mallet. The upper end is found to repel the south pole, and attract the north pole of a compass-needle. The rod is inverted quietly and the same end (which is now the lower) is tested again. It is then tapped and

once more tested. State what results you would expect, and explain them.

11. Give a neat sketch showing how the lines of the earth's magnetic field in a laboratory would be distorted if the ceiling were supported on iron pillars

12. Define *dip, horizontal component of the earth's field.* If at a place A the vertical component is found to be half the horizontal component, what is the value of the dip? At what part of the earth's surface would you expect A to be situated?

13. What is meant by the statement that the declination at a place is 18° west? At such a place, how must a boat be steered so that its course may be due east?

14. A bar of soft iron is held vertically over the centre of a dip-needle, but not near enough to have magnetism induced in it by the needle. Is the dip increased or diminished by the presence of the bar, and would the result be the same in each of the two hemispheres?

15. A bar of soft iron lies on a table at right angles to the magnetic meridian, and a compass-needle is placed at some distance from the bar with its centre on the axis of the bar produced The end of the bar nearest to the needle being kept in the same position, the bar is turned round then, upon the table, until it is parallel to the magnetic meridian, the fixed end of the bar being to the south. Describe the behaviour of the compass (i) before, (ii) during the rotation of the bar.

16. A bar-magnet is carried in a horizontal circle round a compass-needle with its N-pole pointing always to the centre of the needle How will the needle be affected when the magnet is north, east, south and west, respectively, of the needle, assuming that the earth's influence on the needle is always greater than that of the magnet?

17. A wooden ball contains a bar-magnet imbedded so that the axis of the magnet lies along a diameter, but the ends do not reach the surface. Explain carefully how you would mark on the surface of the ball the points where the axis of the magnet prolonged would cut the surface.

18. The beam of a balance is made of iron. If the balance is placed so that the beam vibrates in a plane at right angles to the magnetic meridian, the beam is horizontal when equal weights are placed in the scale pans What will happen when the balance is turned so that the iron beam swings in the magnetic meridian?

19. Explain how the strength and direction of the earth's magnetic field at any given place is defined?

If the horizontal force is ·3 unit and the vertical force is ·4 unit, what is the total force?

Draw a diagram, by means of which the dip could be obtained

PART VII.

STATIC ELECTRICITY.

CHAPTER XXXIII.

ELECTRIFICATION AND THE ELECTRIC FIELD.

ELECTRIFICATION.

Electrification by friction.—The ancient Greeks observed that when amber was rubbed with wool it acquired the property of attracting light objects. This is mentioned in the writings of Thales of Miletus (B.C. 600). Our word electricity is derived from the Greek word for amber (ἤλεκτρον). Until the year 1600 A.D. it was thought that amber was the only substance capable of exhibiting these phenomena, but in that year Dr. Gilbert found that many other substances were capable of affording similar results, *e.g.* resin, sulphur, glass, etc., and these substances he called **electrics.** When a substance is rubbed with a suitable material, and is then found to possess the property of attracting light objects, it is said to be **electrified** (or to possess a charge of electricity).

In order to produce these effects of attraction actual force is required, and this force can only be due to some peculiar condition which the substance has acquired when electrified. Such forces are called electric forces. The space around the substance, extending as far as the forces are evident, is called the **electric field.**

EXPT. 255.—**Attraction of light objects.** Rub a rod of vulcanite on the coat-sleeve. Notice that the rod has acquired the peculiar

property of picking up small fragments of paper, cork, or cotton fibre when brought near to them. Also notice that actual contact is not necessary, but that the effects take place when the rod is still some distance away.

EXPT. 256.—**Attraction of balanced lath.** Balance a long wooden lath (*e.g.* a metre scale) on an inverted round-bottomed flask. Bring a piece of vulcanite which has been rubbed as in Expt. 255 near the end of the lath, and notice the attraction which takes place.

The forces of electric attraction are mutual, just in the same way that the forces of magnetic attraction between a magnet and a piece of soft iron are mutual.

EXPT. 257.—**Mutual attraction.** Rub a piece of well-dried flannel* (or brown paper) with a clothes-brush, and notice how it will cling to the walls of the room.

Two kinds of electrification.—Whenever a substance is electrified by friction, a mutual force of attraction is set up between it and unelectrified bodies. An electrified body may, however, attract or repel another electrified body. Thus, a rubbed rod of vulcanite will repel another rubbed rod of the same kind, and a rubbed rod of glass will repel another glass rod which has been rubbed with the same material, but will attract a rubbed rod of vulcanite.

EXPT. 258.—**Electrified vulcanite rods.** Suspend an electrified rod of vulcanite, and bring near one end of it another rod of vulcanite which has been similarly electrified. Notice the **repulsion** which takes place.

EXPT. 259.—**Electrified glass rods.** Repeat Expt. 258, using, instead of the vulcanite, glass rods which have been dried in the oven and rubbed with silk. Notice the **repulsion.**

EXPT. 260.—**Electrified vulcanite and glass.** Suspend an electrified rod of vulcanite. Bring near to it a rod of glass which has been rubbed with silk. Notice the **attraction.**

* **A Simple Form of Drying Oven.**—It is often expedient to dry artificially the appliances used in experiments on Static Electricity. A portable drying oven may be constructed in the following manner. Fill a shallow baking-tin (about 40 cm. × 20 cm.) with sand, and cover it with a sheet of thin iron (about 40 cm. × 35 cm.) bent into the form of a semicircle, so as to form a hood over the sand-bath. The bath is supported on tripods, and heated by Bunsen-burners placed underneath. Glass rods may be placed in the sand, and paper, flannel, silk, etc., may be spread over the hood.

The terms **vitreous** and **resinous** electricity were formerly used to express the two different kinds of electrification produced by rubbing glass and vulcanite or sealing wax respectively. It was found, however, that the kind of electrification depends upon the substance used as a rubber, for instance, glass when rubbed with fur becomes charged with resinous electricity. For this reason, the terms **positive** and **negative**, which were first suggested by Benjamin Franklin in 1747, are now adopted Using this nomenclature, the results of experiments show that·

(i) Glass rubbed with silk is charged **positively**

(ii) Vulcanite (or resin) rubbed with fur (or flannel) is charged **negatively**

(iii) Bodies with **like** charges **repel**, and bodies with **unlike** charges **attract** each other.

(iv) A charged body always attracts an uncharged body

Conductors and insulators.—Dr. Gilbert found that many substances, chiefly metals, did not show any signs of electrification when rubbed—these he called **non-electrics.** It is now known that this depends upon the manner in which the experiment is conducted.

EXPT 261 —**Loss of charge.** Suspend an electrified rod of vulcanite. Bring near to it a second electrified rod of vulcanite. Notice the repulsion. Pass the latter rod gently* through the hand, taking care that all parts of the rod are touched by the hand, and again test Attraction shows that the rod is no longer charged. Again electrify the rod, and afterwards pass it through the flame of a Bunsen burner Attraction shows that the rod has lost its charge in this case also

Objects like the hand and a flame, which can take away the charge from an electrified body, are called **conductors.** Vulcanite is evidently not a conductor, since the charge on one portion of its surface is not conveyed to the end which is held in the hand of the experimenter. Vulcanite, and all substances which Dr. Gilbert called electrics, are now termed **insulators.** If metals are conductors of electricity, then it is seen readily why Dr. Gilbert was unable to detect any electrification

* If vulcanite is passed vigorously through the hand it is –ly electrified

on the surface of a metal which had been rubbed; any charge which the metal acquired would be conducted away immediately by the hand in which the metal is held. When, however, a piece of metal is held by means of some insulating material, so that any charge produced upon it cannot escape, electrification can be produced upon it by friction. Held in this way the metal is said to be **insulated.** By adopting similar precautions it can be proved that almost all substances become electrified when rubbed with suitable material.

EXPT. 262.—**Electrification of a metal.** Fix a short brass or iron tube (or a square piece of sheet brass or zinc) on the end of a rod of vulcanite, or on the end of a piece of clean dry glass-tubing. Flick the metal with a piece of fur. Bring it near to a suspended rod of vulcanite which has been electrified. Notice the repulsion. The metal is evidently charged negatively.

Electroscopes.—Any appliance which is devised so that, by means of it, it becomes possible to detect very weak electrical forces, and also small changes in the magnitude of such forces, is termed an **electroscope.**

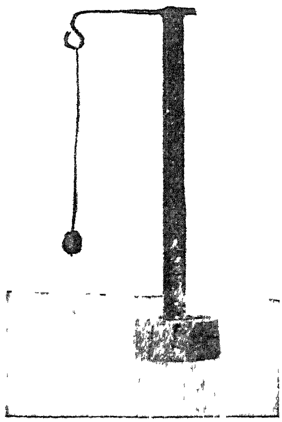


FIG. 285.—Pith ball electroscope.

A simple **pith-ball electroscope** may be made by fixing a hook in a rod of vulcanite and suspending from it a gilt pith ball by means of very thin copper wire, or cotton thread (Fig. 285). A pith-ball is satisfactorily 'gilded' by moistening the surface with weak gum and, when nearly dry, rolling it in gold-leaf (Dutch-metal leaf or aluminium leaf are satisfactory substitutes for gold-leaf).

A more usual form of instrument is the **gold-leaf electroscope** (Fig. 286) in which use is made of the fact that two similarly charged bodies repel one another. Two narrow strips of gold-leaf are suspended from the lower end of a stout wire at the top of which a metal disc is fixed. The wire is supported vertically

by a plug of insulating material (*e.g.* ebonite or sulphur), and the leaves are protected from air-currents by means of a case, the front and back of which are of glass. The sides of the case should be lined on the inside with strips of metal which are earth-connected When a charge of electricity is given to the metal disc, the leaves will diverge, and the degree of divergence depends upon the magnitude of the charge.

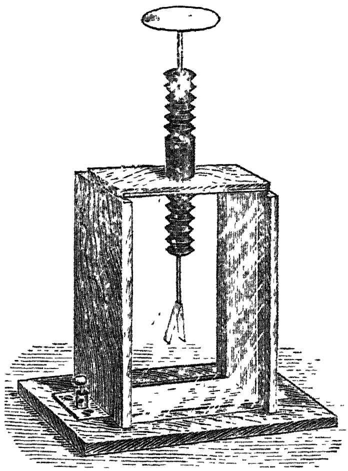


FIG 286 —Gold-leaf electroscope.

Relative power of conductivity.— It has been seen that the hand, a flame, and metals are conductors, and that sealing-wax and glass are insulators. With the aid of an electroscope the conducting or insulating power of any substance can be determined roughly.

EXPT. 263 — **Tests of conduction.** Flick the metal disc of a gold-leaf electroscope with a small piece of fur, thus imparting to the leaves a negative charge. Touch the disc with the finger, and notice the *instantaneous* collapse of the leaves. Charge the electroscope again, and touch the disc with a strip of dry paper held in the hand. Notice the *gradual* collapse of the leaves. Repeat the observations, using *dry* glass, dry and wet threads (both cotton and silk), charcoal, wood, shellac, paraffin-wax, etc

Experiments with various substances suggest the following classification :

Conductors— metals, the body, water, charcoal.

Partial conductors—paper, cotton, wood, stone.

Insulators—glass, sealing-wax, shellac, vulcanite, silk, wool, sulphur, oils

It is evident that when a conductor is required to retain a charge of electricity it is necessary to insulate the conductor

on a support of dry glass, sealing-wax, or vulcanite, or to suspend it by silk threads.

Simultaneous production of both kinds of electrification.—When glass is rubbed with fur it acquires a negative charge of electricity. Does the fur acquire any charge in the process? and if so, is it positive or negative? To answer this question by experiment, it is necessary to insulate the fur by attaching a disc of cardboard to the end of a rod of vulcanite and covering the disc with a piece of fur about the same size. A small square of glass should be mounted on a similar handle. If these precautions are taken to prevent the loss of electricity it is found that **when electrification is developed by friction the two kinds are developed in equal quantity.**

EXPT. 264.—**Equality of opposite charges.** Holding glass and fur by insulating handles, rub them together. Keeping them in contact, bring them near to an uncharged pith-ball. No effect is seen. When the fur is removed, the glass alone attracts the pith-ball. The fur alone will also attract it. Evidently both the glass and the fur are charged; but since they have no effect when together, the charge on the fur must be equal and opposite to the negative charge on the glass. To verify that the fur is charged positively, bring it near to a pith-ball charged positively and notice the repulsion.

Theories of electrification.—When two bodies are rubbed together the electricity which may be generated cannot be of the nature of a substance (solid, liquid, or gaseous), for an electrified body weighs just the same when electrified as it does when unelectrified. The difference between these two conditions may be compared more satisfactorily to the difference between a clock-spring when wound up and when run down (*i.e.* when in a condition of strain and when free from strain), or to a piece of elastic thread when stretched and when unstretched (*i.e.* when in a state of tension and when free from tension),—the difference is simply one of physical condition. But the question still remains unanswered as to where the seat of the tension or strain exists, and we may not assume that this is confined necessarily within the limits of the electrified body.

Two theories were propounded many years ago which may

be mentioned briefly here Symmer suggested a **two-fluid theory**; according to it there are two electric fluids of opposite kind present in all substances, and the process of electrification involves the complete or partial withdrawal of one of them. At a later date Franklin suggested the more feasible **one-fluid theory**, according to which all unelectrified bodies contain a normal amount of an electric fluid, the process of electrification involves either an increase or a diminution of the amount of the electric fluid present. In the former case the body was said to be charged **positively**, and in the latter case charged **negatively** If the words 'positive' and 'negative' are interchanged, Franklin's theory is in close agreement with the remarkable results which have led to the modern **electron theory**. Recent investigations on the discharge of electricity through rarefied gases have demonstrated the existence of particles which are far more minute than the smallest chemical atom, and are associated always with a negative charge These particles have been termed corpuscles. Apparently, an atom of matter under normal conditions consists of an equal number of positive and negative **electrons** or **corpuscles**; the negative electrons are expelled readily from ordinary matter by slight electrical forces, and travel through vacua with a velocity comparable with that of light. A transference of electricity may be regarded as consisting of a movement of negative electrons from a point where there is a gain of positive electricity to a point where there is a gain of negative; also, a positively charged body may be considered as one which has been deprived of some electrons or corpuscles, while a negatively charged body contains an excess. Such deductions are strikingly similar to those derived from the 'one-fluid theory' originally introduced by Franklin.

Electric Fields of Force.

Analogy of magnetic fields of force.—Experiments on Magnetism have shown that like poles repel and that unlike poles attract, that the space separating such poles is a field of force through which magnetic forces are acting in definite directions (called the lines of force), and that if we conceive these lines of force

to have properties similar to those possessed by stretched elastic threads (viz. tending to contract lengthwise and to expand crosswise), it is possible to explain all the experimental phenomena observed. It has been seen, also, that bodies with like electric charges repel, and with unlike charges attract, one another Moreover, these forces are transmitted through the intervening space in a similar manner to that observed in magnetic phenomena These analogies suggest that an electrified body must be surrounded by an electric field, at all points of which it is capable of exerting electric force upon other bodies. If such a field of force exists, then the force at any point in it must act necessarily in a definite direction, which may be regarded as the direction of the electric line of force at that point. Consequently, **an electric field may be traversed by electric lines of force, in the same way that a magnetic field is traversed by magnetic lines of force.**

Exploration of an electric field.—Unfortunately, it is difficult experimentally to map out a field of electric force either so satisfactorily or so simply as in the case of a magnetic field. Nevertheless, it is possible to construct a simple appliance which, when placed at different points in an electric field, will indicate the direction of the force at each point. In this manner, not only is the existence of the forces verified, but their general distribution in space is also proved to be comparable to that which has been found to exist in magnetic fields of force. The following device may be used:

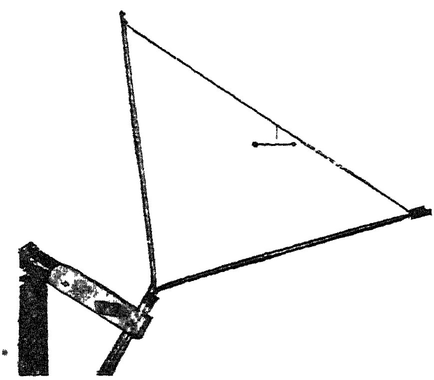


FIG. 287—Apparatus for exploring an electric field.

Fix two long pieces of glass rod in a cork, and bend the rods so that they form a large V. Bore a hole in a small cork, so that it will fit tightly on the end of one of the rods. Attach one end of a silk fibre to this cork, and the other end to the free end of the

other glass rod The fibre may be tightened by rotating the small cork. To the centre of the fibre attach another short fibre (about 2 cm. long), which carries the pointer. The pointer consists of a piece of fine copper wire (5 cm long), on the ends of which are threaded two small gilt pith-balls. Adjust the pith-balls so that the pointer hangs freely in a horizontal position (Fig 287).

It will be found that the electric forces due to bodies charged directly by friction are weak, and far more satisfactory results will be obtained by using large insulated brass spheres which are connected by wires to a Wimshurst machine (see p. 443).

EXPT. 265.—**Lines of force of a single sphere.** Charge a single insulated sphere, and hold the pointer in various positions in the surrounding space. Observe how the lines of force appear to radiate outwards from all points of the sphere's surface (Fig. 288).

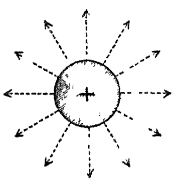


FIG 288.—Lines of force due to a positively-charged sphere.

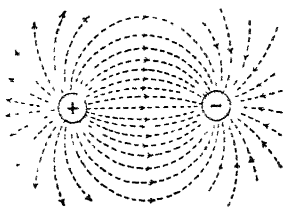


FIG 289.—Lines of force due to two electrified spheres

EXPT. 266.—**Lines of force between two spheres.** Place two insulated brass spheres about 50 cm. apart, and charge them oppositely by connecting them to the poles of a Wimshurst machine. Verify the general distribution of the lines of force as shown by dotted lines in Fig. 289.

If the electric lines of force are considered to have properties similar to those possessed by stretched elastic threads, it can be understood at once why oppositely charged bodies attract

Strength of electric field and properties of lines of force.—The more strongly two insulated brass spheres are charged with opposite kinds of electrification the stronger is the electric field which is generated; usually this is regarded as being due to an increase in the number of lines of force passing through the field, and generally is indicated so in diagrams. As +ve

and − ve electrifications are generated always in equal quantities, there will be the same number of lines leaving the + ly charged surface as there are lines entering the − ly charged surface. No line of force will end blindly in space—at opposite ends of it will always be found equal quantities of opposite electrification, whatever its path may be. This is exactly analogous to the magnetic lines of force between unlike magnetic poles.

The direction of the force in a magnetic field is chosen arbitrarily as that in which a single north-seeking pole would tend to move. **In an electric field of force the direction of the force is chosen arbitrarily as that in which a + ly charged body will tend to move** Hence the lines of electric force may be regarded as running from a + ly charged body towards a − ly charged body.

Electric potential.—In Fig 290 let A represent an insulated sphere + ly charged. Let V_1 represent a small + ly charged sphere (which we will term a test-charge) which is free to move. The force of repulsion F_1 due to A will tend to make the test-charge move further away from A. In order to move the test-charge

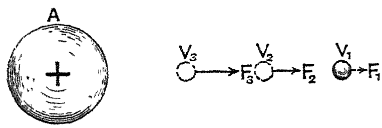


FIG 290.—The potential is greater at V_3 than at V_1

from V_1 to V_2 work must be done on it, so that its **potential energy** is greater at V_2 than at V_1. In the same way its potential energy is greater at V_3 than at V_2, and it is greatest at a point as near as possible to the surface of A. The space round A is a region of **electric potential** which gradually diminishes in value as the distance from A increases, and we say that the electric potential is greater at V_3 than at V_1.

No force will act on the test-charge when it is removed to a great distance from A, consequently the potential at a great distance from A is zero

No electric forces originate from uncharged bodies, hence the region round them (in the absence of any charged bodies) is one of zero electric potential. An uncharged body has zero potential, and since the earth may be regarded as a huge spherical conductor which is uncharged, it is usual in experimental work to take the potential of the earth as our zero or

starting-point for measurement. In the same way gravitational potential is measured from sea-level

If the charge on A is $-$ve, then work must be done in withdrawing the positive test-charge from the neighbourhood of A, and most work would be required if the test-charge is almost touching A Hence the potential is least at points near to A, and gradually increases as the distance increases, and finally becomes zero at a great distance from A The field round A in this case is said to be one of **negative potential**

From this reasoning we derive the following important rules.—(i) **A positively-charged body tends to travel from a point of higher electric potential to a point of lower electric potential.**

(ii) **Since the force acting on the body is in the direction of the lines of force at the point where the body is situated, the movement, if any, will trace out the path of the lines of force.**

(iii) **The forces acting on a negatively-charged body are opposite in direction to those acting on a positively-charged body. Hence a negatively-charged body tends to travel from points of lower potential to points of higher potential.**

Flow of electricity.—In the previous sections we have imagined the positive test-charge to be conveyed from point to point on a small insulated sphere surrounded by a non-conducting medium—air. Suppose that the small sphere containing the test-charge is fixed rigidly at some point in an electric field of force, and that a mass of conducting material (such as a metal) is brought into contact with the small sphere, then the charge will leave it, if, by so doing, it can move into a region of lower potential. The charge would subsequently be found on that portion of the conductor which is situated in the region of lowest potential. **Electricity is said to flow in a conductor from points of higher to points of lower potential.** Difference of potential and flow of electricity are allied to one another as cause and effect, but the cause will only produce the effect when the medium is a conductor No current can traverse a perfect insulator although there may be a difference of potential within it; but such a medium is thrown into a condition of strain

When a $+$ly charged insulated conductor is connected to earth by means of a wire the charge rapidly 'flows' along

the wire, and the conductor is discharged rapidly. The field of force originally surrounding the conductor has disappeared—in fact, each portion of the charge in its passage along the wire has been accompanied by the lines of force associated with it. The so-called 'flow of electricity' may therefore be regarded as a disappearance of lines of force, with the result that the surrounding medium is relieved from its condition of strain.

It might be said that the charge from a $-$ly charged sphere would flow along the wire to earth, and pass from a lower to a higher potential, thus disobeying the conclusion arrived at above. In this case, however, the electricity is regarded as flowing along the wire from the earth to the body until the conductor is raised to zero potential. This may be expressed in another manner by saying that the transference of a $-$ve charge in one direction is the same thing as the transference of a $+$ve charge in the opposite direction.

Hydrostatic and thermal analogies.—The ideas of electric potential may be grasped by means of analogies, which, although unsound in principle, are often useful in imparting the main facts of the subject. The following analogies are often adopted:

(i) The difference of potential between two charged bodies is compared to the difference of level of water in two cisterns, which are connected by means of a narrow pipe. The difference of level is termed generally the head of water. The head of water causes water to flow along the connecting-pipe from the higher to the lower level, and the flow ceases as soon as the level of water is the same in both cisterns. The equality of level in the cisterns is analogous to the equality of potential of two charged conductors connected by means of a wire.

(ii) Heat will pass from a hot body to a cold body placed in contact. The flow of heat depends upon the difference of temperature, and will cease when both bodies are at the same temperature. The difference of temperature is analogous to a difference of potential between two charged conductors.

These analogies particularly fail in not directing special attention to the field of force between the charged bodies; moreover, the analogies hold good only up to a certain point. The student is recommended not to place too much reliance upon what may, at first sight, be an easy means of understanding the principles of potential.

EXERCISES ON CHAPTER XXXIII.

1. How would you show that a brass rod is capable of being electrified? Explain why on rubbing a brass rod and a glass rod the latter only ordinarily appears to be electrified by the friction

2. How would you prove that glass and silk when rubbed together are charged equally and oppositely?

3. If you want to find out whether a body is electrified by seeing how it acts on an electrified pith-ball hung by a silk thread, why is it a surer test that the body is electrified if it repels the pith-ball than if it attracts it?

4. An electrified pith-ball is hung by a cotton thread attached to a glass rod. An electrified rod of sealing-wax is found to repel the pith-ball at first, but the repulsion gradually diminishes, and finally becomes an attraction What conclusion would you arrive at from this?

5. What is the simplest method of removing completely the charge from an electrified rod of sealing-wax? What precaution must be adopted if the hand is used for the purpose?

6. How would you charge a gold-leaf electroscope negatively by means of a piece of fur only?

7. A charged gold-leaf electroscope is required for a certain experiment, and the divergence of the leaves is observed to be greater than is required. How would you remove a portion of the charge without completely discharging the instrument?

8. Two metal spheres of equal size, standing on insulating supports, are oppositely and equally electrified, one positively, the other negatively. They are then placed near together, but not so near as to produce a spark between them. Describe the general distribution, when so placed, of the charges upon them, and of the electric lines of force in the field between them.

9. Two equal metallic spheres charged with equal quantities of electricity of the same sign are placed near together, but not in contact. Give a sketch, showing the way in which the electricity is distributed over the spheres

10. An insulated brass ball without charge is hung near a negatively-charged conductor. It is connected with the charged conductor momentarily. Is its potential altered thereby, and if so, how? It is connected momentarily with the earth. How does this affect its potential?

11. Discuss the analogies between differences of level, temperature, and electrical potential respectively.

12. A small insulated uncharged sphere has positive potential if placed near a positively-charged conductor. How would its potential

be modified if it already possessed a slight negative charge? How would the result of connecting it momentarily to the earth then depend upon the distance between the sphere and the conductor?

13. Under what conditions is it possible for a negatively-charged insulated sphere to have (i) zero potential, (ii) positive potential?

14. How is the potential of a positively-charged insulated sphere modified by bringing another positively charged body near to it?

15. A glass rod is rubbed with a silk handkerchief, and a piece of sealing wax is rubbed with flannel. Describe exactly how you would show that the state of electrification of the glass rod is different from that of the sealing wax.

CHAPTER XXXIV.

ELECTROSTATIC INDUCTION.

The proof plane.—The proof plane is a simple appliance frequently required for experiments on induction, it consists of a disc of thin copper or brass (about 2 cm. diameter) fixed to the end of an insulating handle. A halfpenny may be used as a metal disc.

EXPT 267*.—**Induced charge on a cylinder** Support on an insulating stand a cylinder of wood, with rounded ends, and coated with tinfoil or black-lead. A suitable stand may be made from a rod of unpolished vulcanite, which is fixed vertically in a hole bored in a wooden base Hold a glass rod, which has been rubbed with silk, near one end of the cylinder (Fig. 279). Hold a proof plane with its flat side in contact with the end A of the cylinder. Convey the proof plane to a pith-ball electroscope which is charged −ly, and observe that the proof plane also is charged −ly. While holding the glass rod in the same position as before, touch the distant end of the cylinder with the proof plane, and test the charge on the latter by means of a −ly charged electroscope. Observe that the proof plane is charged +ly. When the proof plane touches the cylinder it becomes part of one and the

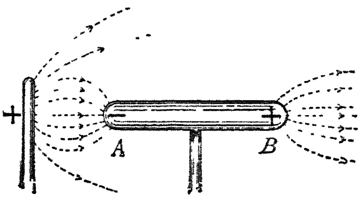


FIG 291 —EXPT. 267.

* The student is reminded that the terms 'positive' and 'negative' continue to be used, not because we assume that there are two different kinds of electricity, but because it is accepted generally as the only convenient form of nomenclature Whenever the term *negatively charged* is used it must be assumed tacitly that this implies less than the normal amount of electricity, while *positively charged* implies more than the normal amount

same conductor, and therefore acquires a portion of the electrification which may be present at the ends of the cylinder.

Induced charges.—When a positively charged rod is held near one end A of an insulated conducting cylinder, it is found that the cylinder is charged –ly at A and +ly at B. Fig. 291 represents the distribution of the lines of force in the experiment The end A is nearer to the glass rod than B, and is consequently at a higher potential. The cylinder is a conductor, therefore electricity flows from A towards B, and the flow continues until the potential of the cylinder is uniform. Lines of electric force proceed from the +ve charge on the glass rod to the –ve charge on A; lines of force also proceed from the +ve charge at B towards the walls of the room. Notice how the lines of force appear to converge towards A, and to diverge outwards from B, and how this suggests the idea that the cylinder is a better conductor than the surrounding air for the lines of force It is instructive to compare this with the flow of magnetic lines of force through soft iron. We say that **the charges on the cylinder are due to induction from the electrified glass rod**

When the glass rod is removed the electric field is conveyed away with it, and no lines of force remain to influence the cylinder. The +ve and –ve charges at A and B have become distributed over the entire cylinder, and have neutralised each other exactly, and therefore must have been present in equal quantity

EXPT. 268 —**Uncharged cylinder.** Remove the charged glass rod to a distance, and again test with the proof plane the electrification of the insulated cylinder. No charge is found on the cylinder.

EXPT 269 —**Induced opposite charge.** Again hold a charged glass rod near A. Touch the cylinder with the finger, and again test the electrification at A and B. A is charged –ly, and B is uncharged.

Fig 292 indicates the result of touching an insulated cylinder while under the influence of a charged glass rod. The potential is reduced to zero, consequently no lines of force pass from the end B to the walls of the room, and the +ve charge formerly distributed over the end B has disappeared The few lines of force from the glass rod which formerly traversed the greater distance to the walls of the room (or beyond), to terminate there in their equivalent –ve charge, are able now to find this equivalent charge by traversing

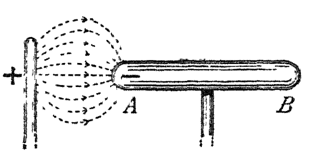


FIG 292 —Expt. 269.

the less distance to the end A of the earth-connected cylinder, this preference for a shorter path is not a new property, but simply a result of the fundamental tendency to shorten shown by all lines of force. We consequently find that rather more lines of force now terminate on the end A than was the case before connecting to earth, resulting in the −ve charge at A being slightly greater after being earthed than it was before being earthed

The fact that the cylinder can be at zero potential while still in a region of +ve potential may be understood more clearly if it is remembered that the cylinder itself has a −ve charge, which would, in the absence of any charged body, give it a −ve potential. The external field, however, tends to give the cylinder a +ve potential. The two effects are equal and opposite, thus giving to the cylinder an apparent zero potential

EXPT. 270—**Charging negatively by induction.** Bring a charged glass rod near one end of an insulated cylinder; while it is there touch the cylinder for a moment, and then remove the glass rod to a distance. Test the charges on the ends A and B. Both are charged −ly. So also are all parts of the cylinder's surface **The cylinder has been charged −ly by induction.*** The lines of force (which essentially coexist with the −ve charge) now proceed from all directions towards the cylinder, and they must originate from an equivalent +ve charge, they cannot come from the glass rod, because this has been removed to a distance, and we shall be able to prove in a subsequent experiment that they come from the boundaries of the room (Fig 293)

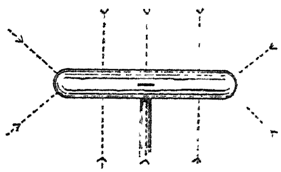


FIG 293—Expt 270

EXPT. 271—**Charging positively by induction.** Repeat Expt 270, using electrified vulcanite instead of the glass rod. Prove that A is electrified +ly, and B electrified −ly, and verify the results shown in Fig. 294

In Fig 294 (i) the point B is in a region of higher potential than A, consequently electricity flows from B to A. In Fig 294 (ii) B has been connected to earth, and electricity has passed up

* **Free and Bound Charges.**—These terms are sometimes used in order to distinguish the charge which disappears on touching with the finger from that which still remains on the insulated conductor Thus, in Expt 267, the +ve charge on the end B would be termed the free charge, and the −ve charge on the end A would be termed the bound charge

into the cylinder until its potential has been raised to zero; the lines of force entering B have been destroyed. In Fig. 294 (iii) the vulcanite has been removed, the +ve charge formerly at A is distributed all over the cylinder now, which is consequently charged +ly by induction

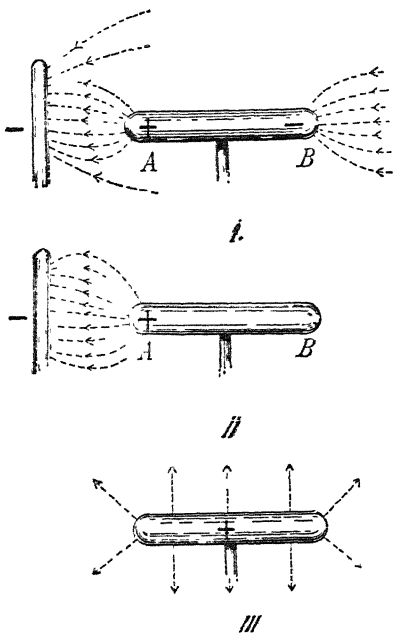


Fig. 294.—Charging an insulated conductor –ly by induction

Attraction of uncharged bodies due to induction.—When a cylinder or a lath is supported so that it can turn freely, and a charged rod is brought near the right or left of one end the lines of force would cause the cylinder to approach the rod; for the same reason, if the rod be held above (or below) one end, then the latter tends to rise (or fall). Experimental results similar to this have been obtained already with a long wooden lath (Expt. 256), and by using a proof plane it is easy to verify the induced charges at the ends of the lath.

The attraction of light objects (Expt. 255) is due to the same effect. Each fragment is acted upon inductively before attraction takes place. But if the fragments are lying on the table they are earth-connected, so that the field of force is analogous to that of a cylinder which has been touched when a charged rod is held near it (Expt 269).

Each stage of the action of a charged glass rod on a pith-ball electroscope is represented in Fig. 295. (i) shows attraction of the pith-ball. (ii) shows the pith-ball drawn up into contact with the rod, thus destroying the lines of force between the

rod and the near side of the ball. The lines of force from the distant side of the ball now tend to pull it away from the rod, with the result shown in (iii). This is a simple case of repulsion

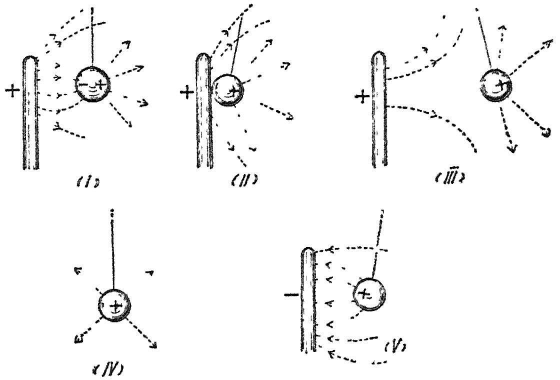


FIG. 295 —Action of a charged glass rod on a pith-ball electroscope

between similarly charged bodies (iv) represents the ball charged +ly after the rod has been removed to a distance (v) represents the effect produced by bringing a −ly charged rod near to the +ly charged pith-ball.

Theory of the gold-leaf electroscope.—The theory of Expts 270 and 271 is directly applicable to the gold leaf electroscope. Instead of the insulated cylinder we have an insulated conductor, with a flat metal disc at its upper end, and a pair of metal leaves at its lower end

EXPT. 272.—**Charging an electroscope by induction.** (1) Hold a −ly charged rod of vulcanite over the disc. The leaves are at a higher potential than the disc, consequently electricity passes from the leaves to the disc, giving the former a −ve and the latter a +ve charge The charge on the leaves induces a +ve charge on the tinfoil Lines of force (see Fig. 296 (1)) proceed across from each tinfoil strip to the nearest metal leaf. resulting in the leaves being pulled apart. The same number of lines of force also pass from the disc to the vulcanite. The degree of divergence will depend upon the number of lines of force passing between the leaves and the tinfoil.

(ii) Hold the vulcanite still in same position, and touch the disc with the finger. The potential of the leaves is raised to zero, the lines of force between the tinfoil and the leaves disappear, and the leaves collapse (Fig. 296 (ii)).

(iii) Remove the vulcanite to a distance. The +ve charge distributes itself uniformly over the conductor, a portion going into the leaves and inducing a −ve charge on the tinfoil The lines of force thus brought into play cause the leaves to diverge (Fig. 296, (iii)). *The electroscope has been charged +ly by induction.*

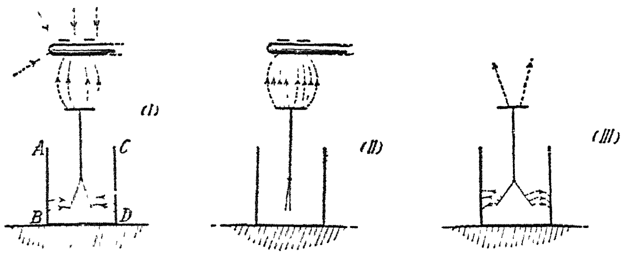


FIG. 296 —Stages in the action of a charged rod on a pith-ball electroscope.

(iv) Hold a +ly charged glass rod over the disc. The potential of the disc is raised above that of the leaves. More electricity enters the leaves and the increased number of lines of force *causes the leaves to diverge more.*

(v) Hold a −ly charged rod of vulcanite over the disc. The potential of the disc is lower than that of the leaves Electricity passes from the leaves to the disc, thus *diminishing* the number of lines of force between the leaves and the tinfoil, and therefore the divergence.

(vi) Repeat Expt. 272 (i)-(iii), using a +ly charged body instead of the charged vulcanite rod. Explain how, after touching the disc and removing the external +ve charge, the instrument has acquired a *negative* charge.

Hold in succession above the disc a +ly charged and a −ly charged body, and observe that the divergence of the leaves is diminished and increased respectively.

(vii) Hold the hand, or any other earth-connected conductor, immediately over the disc of a charged electroscope Notice and explain the change produced in the divergence of the leaves.

It is evident that a charged electroscope may be used to determine the sign of the charge on any body held over the disc. The rules to observe are as follows

Electroscope charged +ly { *Increased divergence implies +ve charge.* *Diminished divergence implies −ve charge (or an earth-connected conductor)*

Electroscope charged −ly { *Increased divergence implies −ve charge.* *Diminished divergence implies +ve charge (or an earth-connected conductor)*

The electrophorus.—This is a convenient appliance for obtaining larger charges of electricity than can be obtained from electrified glass rods or vulcanite rods. It was devised by Volta in 1775. The instrument consists essentially of a circular slab of vulcanite, sealing-wax or shellac, and a flat metal disc with an insulating handle. (Fig 297)

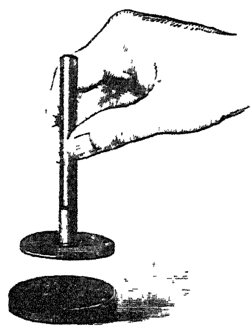


FIG. 297 —An electrophorus

EXPT. 273 —**Use of an electrophorus.** Charge the electrophorus plate −ly by rubbing with fur or flannel Place the metal disc resting on the top of the plate. Touch the disc Raise the disc away from the plate. Test the charge on the disc by holding it over the disc of a +ly charged gold-leaf electroscope; an increased divergence shows that the electrophorus disc is +ly charged. Bring the finger near the disc: when sufficiently near, a small spark is seen to pass from the disc to the hand. Completely discharge the disc by touching it with the hand Again place it on the plate, and repeat the experiment. The disc may be charged many times without it being necessary to re-charge the plate.

Potential of a conductor.—That the potential is the same at all points of a conductor may be verified by deduction from the fundamental facts of static electricity. When two points on the surface of a conductor are at different potentials, electricity will continue to pass between them until they are at the same

potential Hence, in an electric field which is not changing, all points of a conductor must be at the same potential. The same conclusion may be arrived at experimentally in the following manner:

EXPT. 274.—**Equality of potential.** Charge the insulated cylinder (as used in Expt 267) by means of the electrophorus Connect the disc of a proof plane to the disc of a gold-leaf electroscope by means of a thin copper wire. (It is convenient to bore two small holes through the discs) Holding the proof plane by its insulating handle, bring it into contact with the cylinder and observe the divergence produced Fig. 298). The degree of divergence is a measure of the potential of the point on the cylinder which is in contact with the proof plane. Move the proof plane to other points of the cylinder, and notice that the divergence remains unaltered.

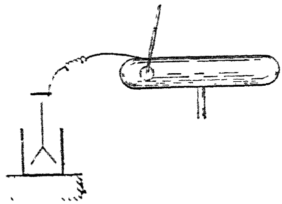


FIG 298—Expt 274

Hollow conductors.—We have now to consider whether lines of force are present inside, as well as outside a charged body. One way to test this in the case of a hollow conductor is to introduce a proof plane within the conductor, with the object of removing a portion of any charge which may be present. When this is done no charge is found to be taken by the proof plane, thus indicating that electrification is absent

EXPT 275.—**Absence of charge inside a hollow conductor.** Place a coffee-tin (or calorimeter) on an insulating stand. Charge the tin by means of an electrophorus. Touch the outside of the tin with a proof plane, and verify the charge with a gold-leaf electroscope. Discharge the proof plane, and with it touch the inside of the tin; carefully remove the proof plane without touching the edge or outer surface of the tin. Test it by means of the electroscope; it is uncharged Hence there is no charge inside a conductor (solid or hollow).

If a charged hollow vessel could be turned inside out, would the outer surface be charged still although now inside the vessel? or, would the charge leave it and pass to the surface which is now outside? These questions may be answered experimentally by using a cotton net, supported on an insulated handle, and

capable of being turned inside out by means of a long silk thread attached to the end of the net (Fig. 299). This arrangement is known as **Faraday's butterfly net.** When such a net is charged,

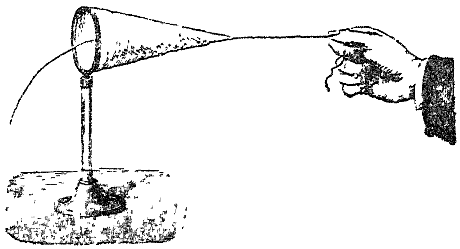


FIG 299 —Faraday's butterfly net.

the electrification is found on the outside surface even after the net has been turned inside out. It is, in fact, a fundamental property of an electric charge to distribute itself on the outer surface only of a conducting body.

EXPT 276 —**Faraday's butterfly net** Charge a Faraday butterfly net by means of an electrophorus Test for the charge outside and inside by means of a proof plane The charge is entirely on the outside. Now turn the net inside out, taking care not to touch the cotton net with the hand Again test the inner and outer surfaces. The charge is found again on the outside only.

Distribution of a charge on the surface of a conductor.—Although the potential of a conductor is the same at all points, it does not follow necessarily that the charge is distributed uniformly over the surface. The quantity of electricity on each square centimetre of the surface of a conductor is not necessarily the same. Usually this quantity is termed the density of the charge. Hence, **although the potential of a charged conductor is uniform, the electric density is not necessarily so, but depends upon the shape of the conductor**

EXPT 277.—**Sphere.** Charge a large insulated sphere. Touch the surface with the flat side of a proof plane, and bring the proof plane into contact with the disc of an uncharged electroscope Notice the degree of divergence. Discharge the proof plane and electroscope. Test other portions of the sphere's surface in the same way The

divergence is the same in each case When the proof plane is touching the surface of the sphere it becomes a portion of the sphere's surface (so far as the distribution of the electricity is concerned, because the charge is entirely on the outer surface); the removal of

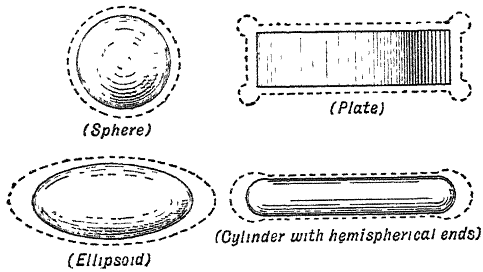


FIG. 300 —Distribution of charge on conductors

the proof plane is equivalent to removing a portion of the sphere's surface equal in size to the proof plane, and the divergence of the leaves therefore measures the quantity of electricity on such a portion.

The distribution of electricity on the surface of a sphere is uniform (Fig 300).

EXPT. 278.—**Cylinder.** Repeat this experiment with a large insulated cylinder with hemispherical ends. The greatest divergence is obtained when the proof plane has touched either end, and the least divergence when the proof plane has touched the straight sides (Fig 300).

EXPT. 279.—**Plate.** Repeat the experiment, using a charged flat metal plate More electricity is obtained from the edge than from the sides (Fig 300).

EXERCISES ON CHAPTER XXXIV.

1. If you were given a negatively electrified stick of sealing-wax and two metal balls mounted on insulating supports, how would you, with this apparatus, charge the balls with opposite kinds of electricity? How could you find out afterwards whether you had charged the balls as you intended, and whether their charges were equal or unequal?

2. Describe how to arrange an experiment so that a conductor charged all over with negative electricity may nevertheless receive a further charge of negative electricity on being connected with the ground by a conducting wire.

3. An insulated conductor, A, is brought near the cap of a gold-leaf electroscope which has been charged positively. State and explain what will happen (i) if A is unelectrified; (ii) if it is charged positively; (iii) if it is charged negatively.

4. Describe an experiment to prove that two parts of the same conductor may be electrified differently although they are at the same potential.

5. Two equal insulated uncharged spheres, B and C, are placed on opposite sides of, and at equal distances from, a charged sphere A What is the electrical state of B and C, and what will happen if the part of B nearest to A is connected by a fine wire with the part of C farthest from A?

6. The cap of a gold-leaf electroscope, resting on an insulating stool, is joined by a wire to the gas-pipes How will the leaves be affected when a charged glass rod is brought near the electroscope? Give reasons for your answer.

7. Describe an experiment to prove that the charge on an electrified conductor lies wholly on the surface.

8. An insulated conductor A is charged with electricity Another conductor, B, earth-connected, is placed near A Is the induced charge on B greater than, equal to, or less than the charge on A? Give reasons for your answer.

9. The caps of two gold-leaf electroscopes, A and B, are connected by a long wire, and a positively charged sphere is brought near A What will be the indications of the electroscopes, and how will they alter if either A or B is touched?

10. Describe a proof plane and its use

A positively charged sphere is held a few inches above a table, and the table is then tested for electricity with a proof plane. Would you expect to find any? If so, which kind?

11. A positively charged sphere is held a few inches from another insulated sphere previously uncharged. An observer's knuckle is now brought so as nearly to touch this second sphere, and a spark passes.

Will the character and strength of the spark depend on the part of the sphere to which the knuckle is presented?

12. An ebonite rod rubbed with flannel is brought successively near

(*a*) sawdust, scraps of iron, iron filings.
(*b*) a wooden ruler lying on the bench.
(*c*) a wooden ruler balanced on a round flask.
(*d*) an ebonite rod rubbed with flannel mounted in a stirrup.
(*e*) a glass rod rubbed with silk similarly mounted.

Say what happens in each case, and give reasons

13. Two insulated spheres, A and B, are placed near together, and A is charged positively. How is the potential of B affected by the presence of A, and how will it be modified if B is touched with the finger, and A is then removed?

14. How would you prove that positive and negative electricities are produced in equal quantities (i) by friction, (ii) by induction?

CHAPTER XXXV.

CONDENSERS. ELECTRIC MACHINES.

Capacity of a conductor.—When two insulated conductors, one of which is charged, are brought into contact, the charge it has been seen spreads over both conductors. The uncharged conductor becomes charged, but as yet we have not described what fraction of the original charge has been transferred to it. The amount transferred depends evidently upon the size of the uncharged conductor—a larger conductor receiving a larger fraction of the original charge than is the case when the conductor is small.

The potential of the two conductors becomes the same as soon as they are brought into contact, but it does not follow that the quantity of electricity is the same on each. The final potential would be expected, however, to be less than that of the charged body before contact was made, since the same number of lines of force which formerly originated from the charged conductor will be distributed now over a larger area.

EXPT. 280.—**Capacity and size.** Obtain two or three metal spheres of different sizes, each mounted on an insulating support. (Instead of the spheres, bottles of different sizes and coated with tin-foil may be used.) Place a hollow can on the top of the electroscope disc. Charge one of the spheres by means of an electrophorus, and touch it with an uncharged sphere. Both spheres are now charged to the same potential. Convey the larger sphere to the electroscope, lower it into the can, and allow it to touch the inner surface. The whole charge is now transferred to the can and electroscope. Withdraw the sphere, and observe the divergence of the leaves. Discharge the electroscope. Proceed in the same manner with the smaller sphere. Notice that the divergence is much less. Hence

the larger portion of the charge was on the larger sphere. We say that the spheres have not the same **capacity for electricity.**

The capacity of a conductor evidently depends upon its size, therefore a larger conductor requires more electricity to raise it to a given potential than a smaller conductor.

The capacity of a conductor is measured by the quantity of electricity which must be given to it in order to raise its potential to a given amount.

$$\text{Or, } \textbf{Capacity} = \frac{\textbf{Quantity of electricity (Q)}}{\textbf{Potential to which it is raised by Q}}.$$

From this definition it is seen that if the capacity of a conductor increases while the quantity of electricity on it remains constant, its potential will become less.

EXPT. 281.—**Quantity and potential.** Connect a large insulated sphere to the electroscope by means of a long thin wire Give a small charge to the sphere by means of the electrophorus. Notice the divergence of the leaves. Bring an insulated uncharged sphere into contact with the charged sphere. Observe the diminution of the divergence, showing that, although the total quantity of electricity is the same, the potential is less Repeat the experiment, using a larger uncharged sphere, and observe the greater diminution of divergence.

Capacity affected by neighbouring conductors—So far only the relationship between the capacity and the size of a conductor have been considered. The capacity is increased by the presence of neighbouring conductors, either insulated or earth-connected. If the hand or any other conductor be brought near to a charged electroscope, the divergence of the leaves is decreased The conductor (*i.e.* the disc and leaves of the electroscope) does not change in size, nor does the quantity of electricity on the conductor alter, yet the potential is reduced. Evidently the 'capacity' of the conductor is increased by holding the hand over the disc. The diminution of potential is explained by remembering that the +ve charge on the electroscope induces a −ve charge on the under-surface of the hand. This induced −ve charge creates a region of −ve potential in its neighbourhood, thus causing a reduction of the +ve potential of the electroscope.

EXPT 282.—**Action of neighbouring conductor** Charge an electroscope +ly Observe the divergence of the leaves. Hold the hand just above the disc, and observe how the divergence is less than before. When the hand is removed the divergence increases to its original value

The lines of force tend to accumulate on the side of a charged conductor facing an earth-connected conductor

EXPT. 283—Charge an insulated sphere; the density of the charge is uniform Hold a metal plate in the hand and near to the sphere. Touch the near side of the sphere with a proof plane, and test the density of the charge by means of an electroscope. Observe the divergence, and discharge the electroscope. Test the distant side of the sphere in the same manner, and observe that the density is much less. Hence the charge has become accumulated on the side facing the earth-connected conductor.

This tendency of a charge to accumulate (or to become piled up) owing to the presence of a neighbouring earth-connected conductor is termed the condensing of electricity. **Any arrangement by which the capacity of a conductor is increased artificially is termed a condenser** The capacity of a condenser depends directly upon the area of surface of the two conductors and is inversely proportional to the distance separating them. It also depends largely upon the medium through which the lines of force pass. This medium is generally termed the **dielectric** because the electrical forces between the conductors are transmitted through it.

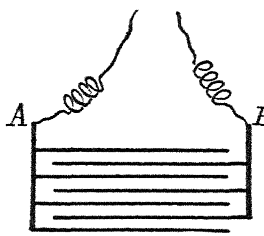


FIG 301 —Diagram of ordinary form of condenser

Usual form of condenser.—The most usual form of condenser consists of a large number of sheets of tinfoil separated from each other by sheets of paraffin paper; alternate sheets of the foil are connected together, so that the area of surface of the two conductors is many times greater than that of a simple two-plate condenser (Fig. 301).

The Leyden jar is a simple form of condenser which derives its name from the fact that it was first used by Van

Musschenbroek, a professor at Leyden, in Holland. It consists of a glass jar, coated outside and inside with tinfoil to within a short distance of the top. It may therefore be regarded as a condenser consisting of two parallel plates separated by a glass dielectric. A brass rod, terminating above in a brass knob stands in a collar fixed to a tripod foot in contact with the inner lining. The tinfoil lining serves as the insulated conductor, which may be charged conveniently through the knob; the jar is placed either on a table or held in the hand, so that the outer coating is consequently earth-connected (Fig. 302).

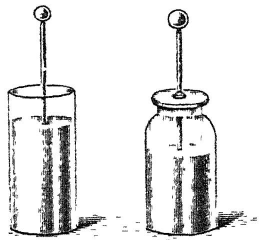


FIG. 302.—Two forms of Leyden jar.

EXPT 284.—**Charge and discharge of Leyden jar.** Place a Leyden jar on the table. Bring the charged disc of an electrophorus into contact with the knob; repeat this five or six times. The jar is charged now. Hold the knuckle near to the knob, and observe the slight shock which is felt when the spark passes.

As a rule it is advisable not to discharge a Leyden jar through the body by touching the knob, since a powerful discharge may have serious consequences. A safe method of discharging the jar is afforded by **discharging tongs**, which consist of a jointed brass rod with brass balls at each end, and provided with glass handles. To use the tongs, one knob is placed in contact with the outer coating, and the other knob is brought towards the knob of the jar.

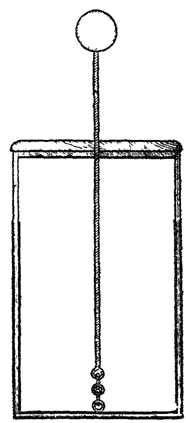


FIG. 303.—Section of a Leyden jar.

A much simpler method of charging a Leyden jar than by using an electrophorus is to bring the knob into contact with a terminal of a Wimshurst machine (see Fig. 305), the other terminal of which is connected to the nearest gas or water pipe.

ELECTRIC MACHINES.

An electric machine—It has been seen that a body may become electrified either by friction or by induction Any mechanical appliance designed to produce these effects on a large scale is termed an electric machine. The electrophorus may be regarded as a simple example of an electrical machine, depending for its action upon the principles of Static Induction, but it is unsuited for the generation of large electrical charges.

The earliest forms of machines were mere elaborations of the simple experiment in which a rod of sulphur or resin is charged $-$ly when rubbed with the dry hand, at a later date glass was substituted for the sulphur, and suitable rubbers were used instead of the hand. Such machines may be termed **frictional electrical machines,** as distinct from **induction (or influence) machines,** which for all experimental purposes have replaced almost entirely the former type.

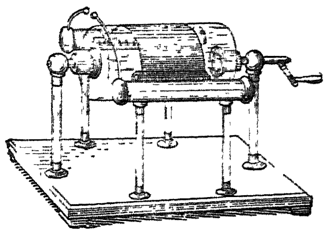


FIG. 304.—Cylinder electric machine

The glass cylinder machine (Fig. 304) consists of a glass cylinder mounted on a horizontal axis which can be rotated by means of a handle The cylinder, when rotated, is electrified $+$ly by rubbing against a pad of silk. In the earlier types of machine the electric charge was collected from the surface of the glass by a metal chain which hung against the cylinder on the opposite side to the pad. Franklin replaced the chain by a metal comb, the teeth of which point towards and nearly touch the surface of the glass. The comb is acted upon inductively by the charged glass, and the surface density of the induced $-$ve charge on the teeth is naturally very high—so much so that, as Franklin discovered, a stream of air electrically charged $-$ly is driven from the points towards the glass. The $-$ly charged air impinges on the surface of the glass and neutralises the charge on its surface; on passing the rubber a second time the glass is electrified again.

The metal comb is connected usually to an insulated metal cylinder, over which the induced +ve charge is distributed: thus the metal cylinder is raised to a high +ve potential, and will yield a constant succession of sparks when the knuckle is held near to it, since its potential is maintained by the freshly-charged glass cylinder.

Since the glass acquires its +ve charge from the rubber, the machine only continues to supply +ve electricity when the

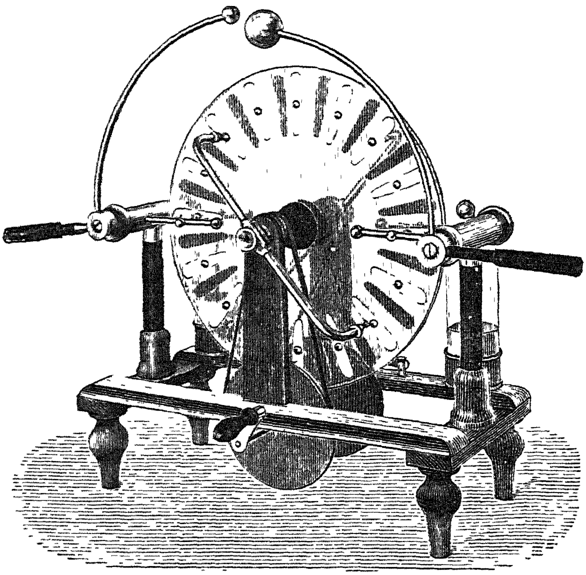


FIG 305.—Wimshurst influence machine

rubber is earth-connected by means of a wire or metal chain. When the machine is required to give −ve electricity the comb is connected to earth, and the rubber is mounted on an insulating support and provided with a convenient metal knob for removing the charge. When both comb and rubber are insulated and connected together by a metal wire, a continuous passage of +ve electricity takes place along the wire from the comb to the rubber—so that a current of electricity then passes through the wire.

The cylinder machine only works satisfactorily in a dry atmosphere and is consequently not altogether trustworthy, for this reason the modern influence machine has superseded it almost entirely

The Wimshurst influence machine (Fig. 305).—Although numerous types of influence machines have been devised, yet the Wimshurst machine is adopted so frequently in this country that a description of it alone will suffice to exemplify the class to which it belongs It consists of two circular plates of varnished glass, placed as close together as possible, and geared so as to rotate in opposite directions. On the outer surface of each plate are fastened an even number of thin metal sectors, which serve both as **inductors** and **carriers.** Across the front is fixed a diagonal conductor terminating in metal brushes which touch the sectors as they pass; a similar diagonal conductor is fixed across the back plate, but sloping in the opposite direction. The insulated collecting combs are placed at opposite ends of the horizontal diameter, and each comb has teeth projecting towards the sectors on both front and back plates. Two Leyden jars are supported often on the base-board of the machine, with their knobs connected to the collecting-combs by movable wires. The combs are supplied with adjustable discharging knobs which are placed above the machine.

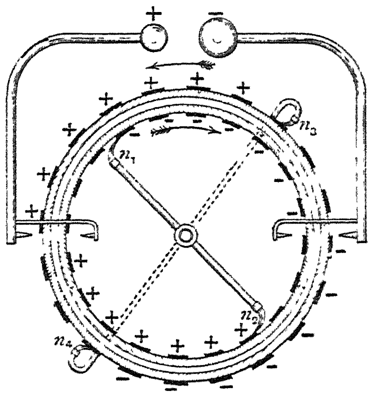


Fig 306.—Action of Wimshurst influence machine.

The action of the machine is explained best by mean's of a diagram (Fig. 306), in which the two plates are represented as though they were two cylinders of glass rotating opposite ways as shown by the arrows. The neutralising brushes are represented by n_1n_2 and n_3n_4. In order to start the machine it is sufficient if one of the sectors has a slightly different potential from that of the others; as a rule this is the case, and the machine

is then self-starting. Imagine that one of the back sectors at the top of the diagram has a slight +ve charge When it comes opposite the brush n_1 it acts inductively on the sector touching n_1, giving to it a slight −ve charge, and simultaneously giving a +ve charge to the sector touching n_2. These sectors, with their induced charges, leave the brushes and rotate into positions opposite the brushes n_3 and n_4; the sectors touching n_3 and n_4 now receive induced +ve and −ve charges respectively, which they retain after leaving the brushes. Thus, after one or two revolutions, all sectors approaching the left-hand comb have +ve charges, and all sectors approaching the right-hand comb have −ve charges. The sectors are neutralised by the combs, the knobs connected to which acquire +ve and −ve charges respectively.

If the machine is found not to be self-starting it is sufficient to hold a piece of electrified vulcanite near the front plate opposite the brush n_3.

THE ELECTRIC DISCHARGE.

Action of points.—When a needle is connected with the charged conductor of an electric machine the surface-density at the point of the needle becomes so great that the air in contact with the point becomes charged with similar electrification, and is repelled forcibly away from the needle. This action continues until the conductor is discharged.

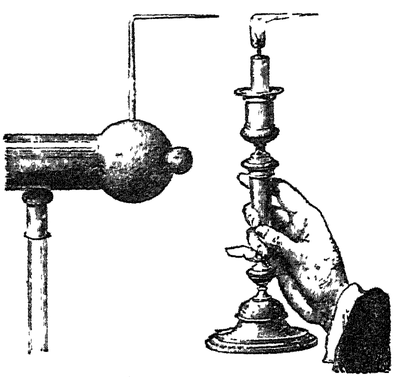


FIG 307.—Expt. 285.

EXPT 285.—**Discharge from points.** (1) Attach a sewing needle or a piece of copper wire with sharpened end to the terminal of a Wimshurst machine by means of soft wax, taking care that the needle is in metallic connection with the terminal. Connect the other terminal to earth. On turning the machine, hold the hand near to the point of the needle, and notice the current of air which appears to be driven from the point.

Hold a candle flame near to the point, and observe how it is blown aside (Fig. 307).

(ii) Transfer the needle to the other terminal, and earth-connect the terminal which carried the needle in Expt (i) Observe that the phenomena observed with the −ve terminal are the same as with the +ve terminal

(iii) Allow the current of air from the point to impinge on a small insulated metal plate or sphere. Verify by means of an electroscope that the plate is charged with the same kind of electricity as that of the terminal to which the point is attached Verify this statement by transferring the needle to the other terminal and testing the charge acquired by the metal plate. Evidently the stream of air which is repelled from the point is charged electrically.

When the point of a needle is held near the charged conductor of an electric machine it becomes charged with opposite electrification by induction, and produces similar effects to those observed when a needle is connected directly with the machine This action illustrates the use of lightning conductors. During a thunderstorm the clouds are charged electrically and induce an opposite charge on the earth's surface immediately beneath the cloud, when the potential difference is sufficiently great a spark discharge (in the form of lightning) takes place between the cloud and any conductor projecting above the earth's surface. By fixing an earth-connected metal point (*i.e.* a lightning conductor) over the building to be protected, any +ly charged cloud will cause an induced −ve charge on the metal point, resulting in the partial or complete neutralisation of the cloud's charge.

EXPT. 286.—**Principle of lightning conductor.** (i) Hold a needle in the hand, with its point towards the terminal of the machine Interpose a candle flame between them, and observe how the flame is blown away from the point.

(ii) Hold an insulated metal plate between the point and the terminal, and verify that the plate is charged now with the opposite electrification to that which is found on the terminal.

Spark discharge.—When the knobs of an electric machine are not far apart, the sparks pass between them quickly and almost in straight lines. Upon separating the knobs, however, the sparks are less frequent and follow irregular paths. The

diminished frequency of the sparks with increased distance of the knobs apart is due to the fact that a greater difference of potential is necessary in order to overcome the dielectric strength of the air, and a greater interval of time is required to charge the knobs to the required potential. The discharge always follows the path of least resistance, and dust particles floating in the air are sufficient to divert the discharge from a straight path into a variable and zigzag path.

EXPT. 287.—**Character.** Notice the sharp intermittent sparks which pass almost in a straight line between the knobs of the machine. Separate the knobs still farther apart, and notice that the sparks are less frequent and trace out a zigzag path (Fig. 308).

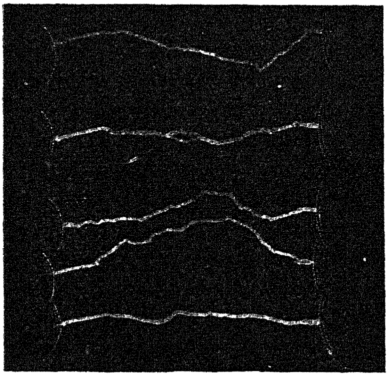


FIG. 308 – Photographs of the electric spark.

EXPT. 288.—**Quantity.** Connect the two Leyden jars to the terminals of the machine, and notice that the sparks are less frequent but far more violent than before. The capacity of the knobs is increased considerably now by being connected to the Leyden jars, and a far greater quantity of electricity must be collected in order to raise the potential of the knobs to a sufficient degree to cause a discharge between the knobs. Carefully bring the knobs together before completing the experiment.

EXPT. 289.—**Duration.** What seems to the eye to be a single electric spark is really a succession of discharges from one knob to the other, yet the total duration is extremely brief, being about the twenty-four thousandth part of a second.

Attach a few small pieces of gummed paper to the surface of one of the glass discs of the machine, as near to the edge as possible. Darken the room, and observe the scraps of paper when illuminated by successive sparks between the knobs. Notice that the paper seems to be absolutely at rest although they are really revolving at a high speed. The duration of the spark is so brief

that the discs do not move appreciably during the passage of a spark discharge.

The spark discharge has considerable penetrative power, and is capable of piercing holes through solid dielectrics.

EXPT 290.—**Penetrative effect.** Hold a sheet of cardboard between the discharging knobs, and observe that each spark pierces a small hole through the cardboard ; notice also that each hole appears to have a slight burr on both sides, as though the discharge had simultaneously passed in both directions.

Discharge through conductors.—We see that the electric field of force between the terminals of a Wimshurst machine when in action may be destroyed rapidly by means of the spark discharge A sequence of sparks is accompanied by the equally rapid destruction and remaking of the electric field of force. The energy used up in this process is derived from the mechanical work done in turning the machine.

The field of force may be destroyed also by connecting the terminals together by means of a conductor. When a good conductor (such as copper wire) is used, the field is destroyed almost instantaneously, even before it acquires any considerable intensity. In fact, we have two opposing tendencies, (i) the machine tending to create a field of force, and (ii) the conductor tending to destroy it, with the result that there is a steady 'flow' of electricity along the wire, which continues so long as any potential difference is maintained between the ends of the conductor.

When the machine is in action there is a gradual fall of potential between consecutive points of the wire, and the end in contact with the +ve terminal has the highest potential. But copper is such a good conductor that the charges are not able to accumulate in the terminals sufficiently to make the potential differences at all great. When a poor conductor, such as string or cotton, is used instead of copper, the discharge is sufficiently slow to enable the machine to maintain a considerable potential difference between the terminals. The potential at various points along the string might be compared by connecting the points momentarily with a gold-leaf electroscope

and observing the divergence of the leaves, but this instrument is too sensitive for such high potentials. The experiment may be conducted more satisfactorily by using several pairs of pith-balls suspended by cotton threads from various points of the string; the degree of repulsion of the pith-balls will indicate the potential of that point of the string to which they are attached.

EXPT. 291.—**Change of potential.** (1) Stretch a piece of thin string AE (1 metre long) between two vertical glass rods (40 cm. high).

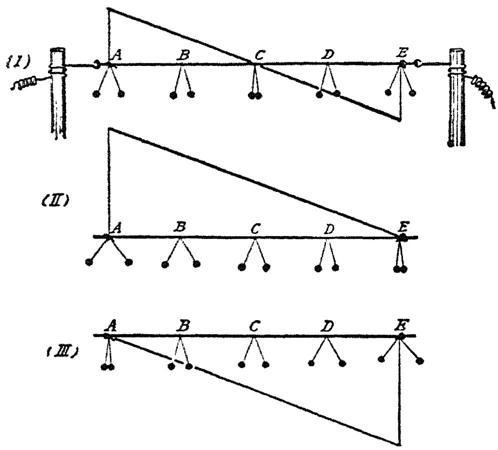


FIG. 309—Difference of potential along an electrified string.

Connect the ends of the string to the terminals of a Wimshurst machine by means of copper wires. Suspend five pairs of pith-balls (on cotton threads) from equidistant points of the string. When the machine is in action notice how the greatest divergence is at A and E, less at B and D, and nil at C (Fig. 309 (i)) Verify that the pith-balls at A are charged +ly by bringing near to them the charged plate of an electrophorus, and that the pith-balls at E are −ly charged by holding near to them an electrified rod of sealing-wax. The sloping line indicates the gradual fall of potential along the string. Place the finger on the string at C. The divergence of the pith-ball is in no case altered, since C is already at zero potential.

Place the finger at E. The pith-balls at E collapse, those at C now diverge, and the divergences at A and B increase. This would be

anticipated from the change in position of the potential line (Fig. 309 (ii)). The potential at E has been raised to zero, and the potential at all other points will be raised to a corresponding degree, since the machine will maintain the same potential difference between the terminals quite independently of the actual values of the potentials.

Place the finger at A. The result is represented in Fig 309 (iii) The potential at A is reduced now to zero, and the potential at all other points is reduced to a corresponding degree.

While the actual potentials are modified by connecting a point on a conductor to earth, yet the potential differences and the consequent 'flow' are not altered in any way. But this ceases to hold good if two different points of an electrified string are connected simultaneously to earth (*e.g.* the two ends); in this case the two ends are both at the same potential, and the flow does not take place along the conductor, but is diverted through the hands and arms (assuming, of course, that the human body is a much better conductor than that which is connecting the terminals; an assumption which holds good if cotton or string, and not metals, are used).

Chemical, heating, and magnetic effects of an electric discharge—The mechanical effect of an electric discharge has been illustrated already (Expt. 290). The discharge is also able to produce chemical, thermal, and magnetic effects.

When an electrical machine is working, the characteristic smell of ozone is often evident; the chemical action by which oxygen is converted into ozone being due to the electric discharge.

If a piece of white filter paper, previously soaked in an emulsion of starch and potassium iodide, be laid on a sheet of glass and fixed just below the terminals of an electric machine, patches of blue coloration will be developed where the discharge strikes the paper, owing to the liberation of iodine from the potassium iodide. The same chemical change is produced when the paper is touched by the terminals of a voltaic battery, but in this case the coloration is only evident round the positive terminal.

The heat effect of the discharge may be shown by connecting two insulated metal balls by a short length of very fine wire; on discharging a Leyden jar battery through the wire it is volatilised with explosive violence. If the discharge be made to pass through a small heap of gunpowder, it is scattered simply, but not ignited; this is owing to the discharge

being so brief that the mechanical scattering takes place before the powder is heated to its temperature of ignition. But if the discharge be slowed-down by including a poor conductor —*e.g* a piece of wet string—in the circuit the powder will be ignited.

The discharge from a Leyden jar will ignite ether. Also, if the experimenter stands on an insulating stand with one hand on a terminal of an electrical machine, the other terminal being earth-connected, and holds a finger near to a gas burner from which coal-gas is escaping, the gas will be ignited by the spark passing from the finger to the burner.

A steel sewing-needle may be magnetised by placing it inside an insulated open spiral of thick guttapercha-covered copper wire through which a discharge is passed from a Leyden jar battery.

Such effects as those now described can be obtained only by **electricity in motion**, and none of them can be obtained when an electric charge is stationary. An electrically charged body may indeed indicate a force of attraction upon either pole of a suspended magnet, but this is not due to any magnetic phenomenon, for the same effect could be observed if the magnet were replaced by a strip of any metal or other material.

As will be seen in subsequent chapters, similar chemical, heating, and magnetic effects can be observed when an **electric current**, generated by means of a voltaic cell, is passing along a metal wire The only difference is that in the case of a so-called electric discharge the passage of electricity is either momentary or intermittent, while in the case of an electric current it is steady and continuous.

EXERCISES ON CHAPTER XXXV.

1. A Leyden jar is held in the hand by its outer coating, and the knob is presented to the prime conductor of an electrical machine in action. Describe the resulting charged condition of the jar, and explain why it is safe to put the charged jar down on the table. Explain why you receive a shock on touching the knob when the jar is standing on the table, but not when you or the jar stand on a dry cake of resin.

2. An electrified drop of water, supported by a non-conductor, evaporates. Assuming that the vapour is not electrified, what changes will the potential of the drop undergo?

3. Two similar vertical insulated plates, A and B, are placed parallel to each other and about an inch apart Each is connected to the cap of a separate gold-leaf electroscope State and explain the indications of the electroscope when (i) a positive charge is given to A, and afterwards (ii) B is touched.

4. A sheet of tinfoil is suspended by a dry silk thread and charged as highly as possible by an electrical machine, but on discharging it a slight spark only is obtained. If the tinfoil is placed on a sheet of dry glass lying on the table, a bright spark can be obtained after the tinfoil has been charged by the machine. Explain the cause of the difference.

5. How do you explain the fact that a Leyden jar cannot be charged highly unless its outer coating be earth-connected?

6. How could you show that electricity gathers most at points and corners of a conductor? Give two practical applications of this property.

7. An orange, into which a sewing-needle has been stuck, point outwards, is suspended by a dry silk thread. A charged body is brought near to it (i) opposite the point of the needle, (ii) opposite the side remote from the needle. State and explain the electrical effect in each case

8. A sharp point attached to a conductor A is held near an insulated charged conductor B. What will be the effect on B if A is (1) insulated, (2) uninsulated?

9. Two gold-leaf electroscopes, similar in all respects except that a needle projects from the cap of one of them, are placed at equal distances from an electrical machine When the machine is worked both pairs of leaves diverge. When it ceases to work one pair of leaves collapses rather quickly and the other pair very slowly. Explain this difference in their behaviour.

10. Describe and sketch a simple frictional electrical machine.

11. Describe the construction of an electrophorus and explain its action.

12. Devise an experiment to imitate on a small scale the action of a lightning-conductor.

PART VIII.

VOLTAIC ELECTRICITY.

CHAPTER XXXVI.

VOLTAIC CELLS. MAGNETIC EFFECTS OF AN ELECTRIC CURRENT.

Chemical action.—The energy represented by an electric current obtained by means of a voltaic cell is derived from the chemical action proceeding within the cell. It is therefore essential that the phenomenon of chemical action should be understood clearly. The following experiments are typical, and closely allied to the changes which go on in several types of cells.

EXPT. 292 (i).—**Chemical change.** Hold the end of a strip of thin sheet-zinc in a hot gas-flame (such as that obtained with a mouth-blowpipe). Notice how the metal burns with a bright blue-green flame, and becomes changed to a white powder. This powder is oxide of zinc, and has been formed by the chemical combination of zinc and oxygen.

(ii.) Try to obtain the same effect with a strip of sheet copper. The metal does *not* burn, but it becomes coated with a black film (oxide of copper).

(iii.) When a strip of platinum is treated in the same manner, no change can be perceived.

EXPT. 293 (i).—**Action of metal on acid.** Drop a small strip of commercial zinc into a test-tube containing dilute sulphuric acid (1 in 8). Notice that bubbles of a gas are given off from the surface of the zinc. Close the end of the test-tube with the thumb for a few moments, so as to prevent the gas from escaping. Remove

the thumb and hold the open end of the tube at the side of a gas flame. The gas in the test-tube burns with a dull blue flame. The gas obtained by this means is called hydrogen. At the same time observe that the zinc disappears gradually.

(ii) Repeat the above experiment with copper, and notice that the dilute acid has no action upon it, even if the acid is heated. Even with *strong* sulphuric acid no action is perceived, unless the acid is heated.

(iii) When platinum is tested in the same manner no action takes place even with hot strong sulphuric acid.

A simple voltaic cell.—When metals are placed in dilute sulphuric acid, they are not acted upon chemically with equal readiness; and, of the metals zinc, copper, and platinum, zinc undergoes change most readily and platinum least readily. Any pair of these metals may be used for the construction of a simple voltaic cell, but since copper and zinc are the most available, and for other reasons, these metals are selected generally for the purpose.

EXPT 294.—**Electric current.** Cut two rectangular pieces of sheet-copper and sheet-zinc (10 × 4 cm); solder a short piece of thick copper wire to the upper edge of each. Fig. 310 represents a convenient method of supporting the plates in a beaker of dilute sulphuric acid. Connect the terminals by means of a fairly long thin copper wire. Place a compass-needle on the table and hold just over it a straight piece of the connecting wire, the wire being so adjusted that it lies in the magnetic meridian. Notice how the needle is deflected. This experiment is a simple test for the presence of an electric current, and the theory of it will be explained in a subsequent paragraph.

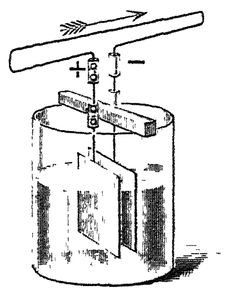


FIG. 310.—A simple voltaic cell.

A wire through which an electric current is passing not only has a magnetic action upon a compass-needle, but also it is capable of magnetising a piece of steel. When such a wire (providing that it is cotton-covered) is wound in a close spiral round a piece of narrow glass-tubing inside which a steel sewing-needle is supported, the needle is found to acquire slight permanent magnetisation.[1]

[1] See p. 368.

Local action.—When plates of copper and zinc are placed in dilute sulphuric acid, and are connected by means of a wire outside the acid, bubbles of gas can be seen escaping from the surface of both plates. Upon removing the connecting wire, however, the bubbles cease to form on the surface of the copper, but continue to appear on the surface of the zinc, thus showing that chemical action is going on between the acid and the zinc. As no electric current is being produced, the zinc is being wasted, and the equivalent amount of chemical energy is being lost. This action occurs only when commercial zinc is used, and does not take place when pure zinc is placed in dilute acid. It is due to the presence of impurities—chiefly iron and carbon—in commercial zinc.

When commercial zinc is dipped into dilute acid, each speck of iron and carbon on its surface forms a minute voltaic cell, which eats away the surrounding zinc, and hydrogen is liberated from the specks of iron or carbon. This effect is known as **Local Action.** By rubbing a drop of mercury over a clean surface of commercial zinc, an amalgam of the two metals is formed which effectually prevents local action. This is because mercury will dissolve zinc but will not dissolve iron and carbon; the film of mercury on the surface therefore supplies pure zinc to the dilute acid, but serves as a protecting layer to the particles of iron and carbon, which would cause local action if allowed to come into contact with the acid.

EXPT. 295.—**Pure zinc in acid.** Place a fragment of *pure* granulated zinc in a test-tube and add dilute sulphuric acid. No chemical action can be observed.

EXPT. 296.—**Amalgamation.** (i) Place a small piece of commercial zinc in a test-tube, add some dilute acid, and observe the rapid chemical action. Now add a *small* drop of mercury, and thoroughly shake the tube. The surface of the zinc soon becomes completely amalgamated by the mercury, and the chemical action ceases

(ii) Amalgamate the zinc plate used in Expt. 294, by dipping it into dilute sulphuric acid for a few moments in order to clean the surface, and then rubbing a drop of mercury over the surface with a piece of cotton wool or cloth. Connect up the cell as in Expt. 294, and observe that bubbles of gas no longer escape from the zinc, but that they continue to form on the surface of the copper.

EXPT. 297.—**Voltaic actions.** (i) Add some dilute sulphuric acid to a fragment of *pure* zinc in a test-tube. No chemical action takes place Add a few small pieces of copper turnings. Vigorous chemical action immediately proceeds. Notice that bubbles of gas are being liberated from the copper, *but not from the zinc.* The phenomenon is really a repetition of Expt 296 (ii) on a small scale. What is the gas which is being given off? Close the end of the tube with the thumb for a few moments, and verify that it is the combustible gas hydrogen. We really have, in this case, a simple voltaic cell. The connecting wire is dispensed with, since the copper and zinc are already connected together beneath the surface of the acid

(ii) Repeat Expt (i), but add a few iron filings instead of copper turnings. The phenomena observed are identical in both cases

(iii) Repeat Expt. (i), adding some finely divided charcoal instead of iron or copper. Well shake the tube, and observe that the phenomena are again the same.

It can be shown readily that the consumption of the zinc in a simple voltaic cell is the source of the energy represented by the current passing along the connecting wire. Thus, when a zinc plate is dried carefully and weighed, connected up as shown in Fig. 310, and, after the current has passed for some time, again dried and weighed, it is found to weigh less than before, the loss in weight being proportional approximately to the time during which the current has passed. In the case of the copper plate its weight is found to remain practically constant.

The difference of potential between the terminals of a voltaic cell.—It is necessary to consider briefly the **cause** of the electric current observed in the wire connecting the plates of the simple voltaic cell.

A mass, falling under the action of the force of gravity, moves from a position where its potential energy (p. 116) is greater to a position where it is less. The two positions may be termed points of higher and of lower **gravitational potential** respectively. Similarly, electricity tends to flow from a point where the **electric potential** is higher to one where it is less; and the difference of electric potential (or P.D.) is related to the flow as cause and effect. But the flow takes place only if the two points are connected by a medium which is capable of conveying electricity; such a medium is termed a **conductor.** Thus, in Expt. 294, the copper wire connecting the copper and zinc plates of the voltaic

cell is a conductor. The combination of the cell and the wire may be termed an **electric circuit.**

Hydrostatic analogues.—The electrical conditions in an electric circuit may be compared to the hydrostatic conditions in the case of two cisterns, connected together underneath by a pipe which may be closed by means of a tap. If the level of the water in the first cistern be much higher than that in the second, then, when the tap is opened, water will flow from the first cistern to the other until the level of the water is the same in both cisterns. The rate of flow (or "current") will gradually diminish as the difference of level becomes less, and it may be made to cease at any instant by closing the tap, when the pipe ceases to be a conductor of water from one level to another. This is analogous to breaking the electric circuit of the simple voltaic cell by disconnecting the copper wire from either of the terminals of the cell.

The rate of flow of water along the pipe could be kept uniform only if some kind of pump were arranged to pump water from the second cistern to the first at a rate exactly equal to that at which the water is flowing along the pipe; thus the original difference of level would be maintained by the energy consumed in working the pump. In the simple voltaic circuit the initial potential difference (P.D.) between the plates is maintained by the chemical action between the zinc and the acid As soon as all of the zinc, or of the acid, is used up, the electric current ceases.

In the simple voltaic cell the copper plate has a higher electric potential than the zinc: it is analogous to the water cistern in which the water is at a higher level. The copper and zinc plates are termed the **positive** and **negative** terminals of the cell; and the current is said to flow along the connecting wire *from* the copper *to* the zinc.

In all types of voltaic cell, to be described subsequently, the zinc plate is always the negative terminal of the cell.

Electro-motive force.—The force which serves to maintain the potential difference between the metal plates is called the **electro-motive force** of the cell (usually written E.M.F.). Since the degree of potential difference between the plates is dependent

upon the degree of E.M.F. inside the cell, it follows that the numerical value of either expresses at the same time the value of the other, and it is customary to refer to the potential difference of the metal plates as the E.M.F. of the cell.

The E.M.F. of cells is expressed usually in terms of a unit which is called the **volt** (p. 493). The student may realise the magnitude of this unit by the fact that the E.M.F. of a simple voltaic cell, such as described on p. 454, is approximately 1 volt. The E.M.F. of a Daniell cell (p. 460) is about 1·07 volts, and that of a Grove cell (p. 461) is about 1·95 volts.

Polarisation.—Attention has been directed already to the fact that, when a simple voltaic cell is in action, bubbles of gas (hydrogen) collect on the surface of the copper plate. Each small portion of the copper plate to which a bubble of hydrogen adheres is protected from the acid, and thereby the effective area of the copper plate is reduced. The accumulation of hydrogen is injurious for another reason. Hydrogen is oxidised readily, and behaves, in this sense, similarly to zinc; if present in a voltaic cell it behaves somewhat similarly to a zinc plate and tends to send a current through the acid from the copper to the zinc. In this manner the E.M.F. of the cell is reduced more or less by the opposing E.M.F. (or "back" E.M.F.) set up by the liberated hydrogen. The current passing through the cell and that in the connecting wire is reduced consequently. This effect, due to the hydrogen accumulating on the copper plate, is termed **polarisation** of the cell.

Removal of the hydrogen by mechanical means is not convenient, but it is possible to prevent its accumulation by chemical means, *e.g.* by oxidising it. It cannot be burnt in the air in this case, but other substances besides air afford a supply of oxygen available for this purpose (*e.g.* permanganate of potash, manganese dioxide, or bichromate of potash). These substances contain much oxygen, which they readily give up when dissolved in water; they are termed **oxidising agents.**

Other chemical methods are available for preventing polarisation; and of the many modifications of the simple cell which have been devised, the chief differences are due to the various methods adopted for preventing polarisation.

The bichromate cell.—In this cell potassium bichromate is used as a depolariser, and is mixed with the dilute sulphuric acid Carbon plates are used instead of copper, since the latter would be attacked by the mixture of acid and bichromate. A simple form of the cell is shown in Fig. 311, in which a carbon plate is placed on each side of the zinc. The carbon plates are joined together at the top. The zinc plate is supported by a metal rod which slides through the lid of the cell in order that the zinc plate may be raised out of the liquid when the cell is not in use. A solution of suitable strength for this cell may be made by mixing together the following quantities by weight :—100 parts water, 10 parts bichromate of potash, 30 parts sulphuric acid; in order to keep the zinc plate amalgamated it is well to add 0·25 part of mercurous sulphate.

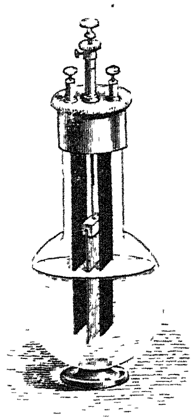


FIG 311 —A bichromate cell

When in action the chromium trioxide, contained in the bichromate, is reduced by the hydrogen to chromium sesquioxide (Cr_2O_3), which is then dissolved in the sulphuric acid with which it is mixed, forming chromium sulphate. The change is accompanied by an alteration in colour from orange-red to dark green-blue.

The Leclanché cell.—In this cell, known by the name of the physicist who devised it, the materials are zinc, carbon, and a concentrated solution of ammonium chloride (sal-ammoniac). Manganese dioxide is used as a depolarising agent. The carbon plate (C, Fig. 312) is placed in the centre of a cylindrical porous pot which is packed closely with a mixture of carbon and manganese dioxide. The zinc rod Z dips into the solution of sal-ammoniac contained in the glass jar. When the cell is in action, ammonia and hydrogen are produced; the ammonia gas, being very soluble in water, does not tend to produce polarisation. The manganese

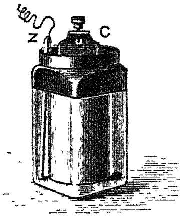


FIG. 312 —A Leclanché cell.

dioxide is but a very slow oxidising agent, and the cell consequently soon becomes polarised if used continuously, it becomes depolarised, however, if allowed to remain unused for a short time. A Leclanché cell has the advantage of requiring very little attention, and is for this reason very generally used in telegraphic work, in houses for working electric bells, and in any work where current is required only occasionally. The cell may not require renewing for months or years, and its failure to act is often due simply to evaporation of the water from the solution of sal-ammoniac, which can be remedied by the addition of more water.

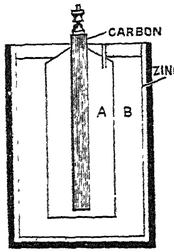


FIG 313 —A dry cell

Dry cells.—Owing to the want of portability of ordinary cells containing fluids, it is often preferable to use *dry* cells, nearly all types of which are modifications of the Leclanché cell. In reality the cells are by no means dry, and their success in working largely depends upon the contents being kept moist. A central plate of hard carbon (Fig. 313) is surrounded by a thick layer A made of a mixture of manganese dioxide, carbon, sal-ammoniac, zinc chloride, and gum. This is surrounded by a paste B made of plaster of Paris, flour, sal-ammoniac, and zinc chloride. The whole is contained in an outer zinc vessel covered with millboard. The contents are kept in position by a layer of pitch, through which is fixed a small tube for the escape of any gases generated.

The Daniell cell.—In a Daniell cell copper and zinc are used as the metals, and sulphate of copper (blue vitriol) is used as a depolariser. The outer vessel (Fig. 314) is of copper, and serves as the copper plate. The porous pot is surrounded by a strong solution of sulphate of copper, the strength of which is maintained by placing crystals of the sulphate on a perforated copper shelf near the top of the outer vessel. The zinc rod and dilute sulphuric acid are contained by the porous pot.


FIG 314 —A Daniell cell

When the cell is in use the hydrogen generated by the action of the zinc on the sulphuric acid passes through the porous pot, and, instead of appearing on the surface of the copper, displaces copper from the copper sulphate. The result is, that pure copper, and not hydrogen, is deposited on the copper plate. Hydrogen may be supposed to act on the copper sulphate to form sulphuric acid and copper; or, expressed as a chemical equation,

$$H_2 + CuSO_4 = H_2SO_4 + Cu.$$

When the cell is left standing for a long time, some of the copper sulphate passes through the porous pot and is decomposed by the zinc forming zinc sulphate and copper, the latter being deposited on the zinc rod. This effect reduces the power of the cell, and it is consequently necessary to remove the liquids to separate bottles as soon as the experiments are completed.

Bunsen's and Grove's cells.—The only difference between these two kinds of voltaic cells is that, whereas, in the former a piece of hard carbon replaces the copper plate, in the latter there is a plate of platinum. Owing to the cheapness of the carbon, Bunsen's cell is more commonly used.

In Bunsen's cell there are two separate vessels: the inner smaller one alone is porous and is filled with strong nitric acid, into which a rod of carbon dips. The outer vessel contains dilute sulphuric acid, and in it is placed a zinc plate, which is usually made cylindrical in shape. The arrangement of the parts is understood easily by a reference to Fig. 315.

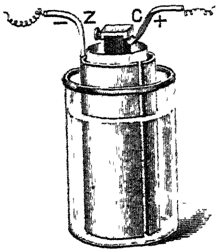


FIG. 315.—A Bunsen cell.

The hydrogen-destroying agent in both these cells is nitric acid. As soon as hydrogen is formed, instead of adhering to the platinum or carbon plate, it reacts with the nitric acid, giving rise to poisonous red fumes which escape into the air.

Cells in series and in parallel.—In many experiments it is necessary to use a stronger current than would be obtained from a single cell. Several cells may be connected together **in series, in parallel,** or in a combination of

these two systems. Fig. 316 (i) represents four Bunsen cells connected together *in series*, the zinc plate of one cell being joined to the carbon plate of the next, and so on; a long thin line represents a carbon plate and a short thick line a zinc plate. The potential difference between the carbon plate C at one end of the battery and the zinc plate Z at the other end will be four times as great as that which would be obtained by using only one cell.

Fig. 316 (ii) represents four cells joined together *in parallel*. All the zinc plates are connected together, and also all the carbons are connected together, thick copper wire being used for the purpose. The potential difference between the two terminals of the battery will be the same as if one cell only had been used, in fact, the arrangement is exactly the same as if one large cell, with plates four times the size of those in either of the constituent cells, had been used. The same potential difference would, indeed, be obtained if a minute cell, no larger than a sewing thimble, were used, but a larger cell has other advantages, which will be explained in a subsequent chapter. The potential difference depends only upon what metals and acid are used in the construction of the cell, and is quite independent of the size of the cell.

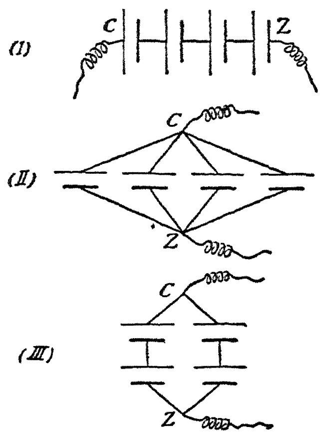


FIG 316 —Arrangement of four cells, (i) in series, (ii) in parallel, (iii) in multiple arc.

Fig. 316 (iii) represents the four cells joined together in two rows of two cells each. The potential difference between the terminals is equal to twice that obtained from a single cell—in other words, the potential difference is the same as that obtained by joining two cells in series. The advantage of using four cells in this manner, instead of two single cells, depends upon the fact that the arrangement is equivalent to two large cells, each twice the size of a single cell.

The commutator, or current reverser.—It is advisable to have a simple appliance for reversing the direction of the current in a wire without interchanging the connections of the wire to the battery. Fig. 317 represents a simple form of **commutator**. It

consists of a square block of wood, with a circular hole bored near each corner to serve as mercury cups. The cups are connected diagonally by thick copper wires. Two thick copper wires dip into two of the cups on one side of the block and serve as terminals to which the ends of the circuit are connected. The swinging arm consists of two pieces of copper wire which are insulated from each other by means of a short piece of glass tubing, which also serves as a handle. The arm carries two pieces of thick wire bent into an arc, which can be made to dip into either pair of mercury cups by swinging the arm over in the required direction. The poles of the battery are connected to the ends of the arm by means of binding-screws. The various parts can be fixed in position by means of wire staples. When the arm is in a vertical position the circuit is broken and no current flows along the wire. When the arm is swung to the left the current will traverse the wire in the reverse direction to that which is traversed when the arm is swung to the right.

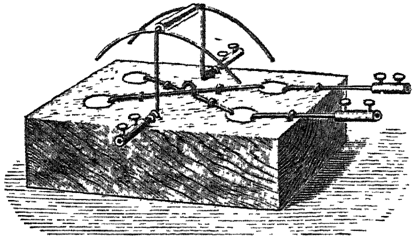


FIG. 317.—A commutator.

MAGNETIC EFFECTS OF AN ELECTRIC CURRENT.

Oersted's experiment.—The effect which an electric current has upon a neighbouring compass-needle has been used already (Expt. 294) as a means of detecting the current. This effect was first observed by Oersted, of Copenhagen, in 1819.

EXPT. 298.—**Action of the electric current on a magnetic needle.** Connect the poles of a Grove or Bunsen cell to a commutator, the other terminals of which (Fig. 317) are connected by means of a long thin wire. Stretch out a length of the wire so that it lies horizontally in the magnetic meridian. Place a compass-needle underneath the wire and complete the circuit by swinging over the arm of the commutator so as to allow the current to pass along the wire. Observe how the needle is deflected. Break the circuit by moving the commutator arm into the vertical position, and

observe that the needle swings back into the meridian. Reverse the direction of the current by swinging the commutator arm over in the opposite direction. Observe that the needle is deflected again, but in the opposite direction. Also place the needle above the wire, and observe the direction of the deflection with the current direct and reversed. Verify the results tabulated below:

Current passing from	Needle above (or below) wire	North-seeking pole deflected towards
South to North	below	West
" "	above	East
North to South	below	East
" "	above	West

Ampère's rule.—The following rule was suggested by Ampère to describe the effect of an electric current upon a magnetic needle:—**Suppose a man to be swimming in the wire in the same direction as the current, and with his face towards the needle; the north-seeking pole is deflected towards his left hand.**

It is important to notice that when the current ceases to pass along the wire the deflection of the needle simultaneously ceases. Hence the magnetic field is dependent for its maintenance upon the flow of the electric current.

As there is a magnetic field below the wire we should expect a field to be present above the wire, and on each side of it—we should, in fact, expect the field to be distributed symmetrically round the wire, as is really the case.

EXPT. 299.—**Lines of force due to a current.** Lay a sheet of paraffined paper on a sheet of cardboard, and bore a small circular hole through the centre of both sheets. Clamp the cardboard and paper in a horizontal position, and thread a straight piece of thick copper wire (40 cm. long) vertically through the circular hole. Clamp the wire in this position, and sprinkle iron filings on the paper. As a strong current is necessary, a battery of several large cells must be used. Complete the circuit, and tap the cardboard. Break the circuit, and notice how the filings have arranged themselves in circles concentric with the wire (Fig. 318).

Which is the positive direction of the circular lines of force around a wire through which an electric current is passing? In other words, would a single north-seeking pole appear, to an observer looking down on the experiment, to travel round the wire in the same direction as the hands of a clock or in the opposite direction? It can be shown by experiment that **the positive direction of the lines of force appears to be clockwise to an observer looking along a wire which is conveying a current away from him.**

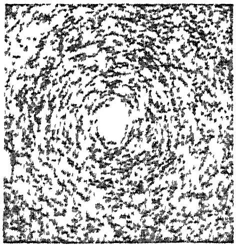


FIG. 318.—Map of the magnetic field perpendicular to a wire conveying a current.

EXPT. 300.—**Directions of current and compass needle** Place a compass-needle on the paraffined paper and near to the wire. Complete the circuit and observe the direction in which the needle points when placed to the north, south, east, and west of the wire. Reverse the direction of the current and notice that, in each position, the direction in which the needle points is reversed Adjust the connections so that the current is passing down the wire, observe the direction of the needle, and verify the rule expressed above

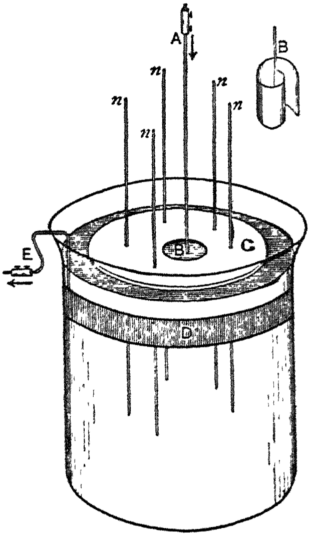


FIG 319.—Experiment to demonstrate the rotation of a magnet-pole round a wire conveying a current.

EXPT. 301.—**Rotation of a magnet-pole round a wire conveying a current** In Fig. 319, C is a thick disc of hardwood, about 8 cm in diameter, with a central hole 1 cm. diameter. The disc carries 5 or 6 pieces of strongly magnetised knitting-needle, each about 15 cm long, with similar poles pointing in the same direction. At least one-half of each needle projects below the surface of the disc. The disc and magnets should be protected with a coat of varnish. AB is a straight piece of thick copper wire terminating below in a spirally bent

strip of thick sheet copper, as shown in the inset. D is a circular strip of thick sheet copper, which just fits inside a beaker, the diameter of which is 2 or 3 cm greater than that of the disc. A thick copper wire is soldered to D, and bent round the edge of the beaker so as to serve as a support. The disc floats on a strong solution of copper sulphate (with 5 per cent. sulphuric acid added), the surface of which is just above the upper edge of D. On passing a current (about 5 amperes) down AB, through the liquid and out at E, the action of the current in AB on the upper poles causes the disc to rotate, and the speed is increased by increasing the current. On reversing the current the direction of rotation is reversed. As the south-seeking poles are immersed to a considerable depth within the liquid, the magnetic force acting on them is small in comparison. The action of the current on the south-seeking poles may be observed by inverting the disc.

Magnetic field due to a current in a circular wire.—When a current is sent through a wire bent into a form of a circle the space enclosed by the wire is traversed by lines of force all travelling in the same direction. A horizontal cross-section through the centre of the circle is similar to Fig. 320, which represents the current passing down through the paper at A, and returning up through the paper at B. The lines of force shown in Fig. 320 are those due to short lengths of the wire near to A and B, and they are all in the direction from right to left. Outside the wire circle the direction of the lines are from left to right. The lines of force due to all other portions of the wire proceed in the same direction—in fact, Fig. 320 may be regarded just as readily either as a vertical or as an inclined cross-section of the wire.

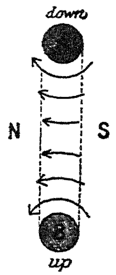


FIG 320.—Magnetic field due to a current in a circular wire

The magnetic field of the wire circle closely resembles that of a magnetised disc of steel, of which the thickness is equal to the diameter of the copper wire, the diameter equal to that of the wire circle, and magnetised so that the opposite faces of the disc have opposite polarity.

This suggests that the wire circle with its current should resemble

a magnet in other respects, *e.g.* that the right-hand face should have south-seeking polarity, and the left-hand face north-seeking polarity This can be readily verified by means of De la Rive's floating battery, which consists of a simple voltaic cell made to float in water and with the terminals connected by a coil of wire. Such a coil, free to turn with the cell in any direction, sets with its plane perpendicular to the magnetic meridian; and the two faces exhibit magnetic polarity. If the coil be held so that its face is perpendicular to the line of sight, and if the current appears to pass round the coil in a clockwise direction, then that face will have south-seeking polarity. If the direction be anti-clockwise, then the face will have north-seeking polarity (Fig. 321)

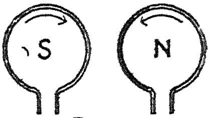


FIG. 321.

EXPT 302—**De la Rive's floating battery.** (1) Use the zinc and copper plates which were made for the simple voltaic cell. Pass the copper wires attached to these plates through holes in a flat cork, and protect the soldered joints with sealing-wax or varnish Make a circular coil (about 5 cm diameter, and of four or five turns of thin cotton-covered copper wire, and bind the turns together with cotton

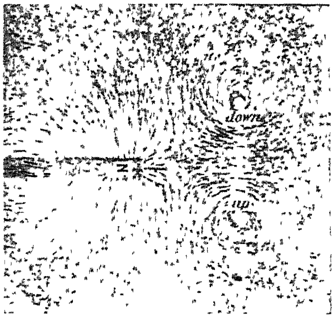


FIG 322.

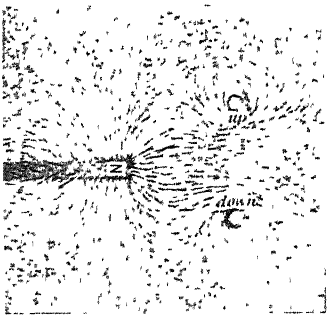


FIG 323

Combined magnetic fields due to a magnet and a coil of wire conveying a current

Fasten the free ends of the coil by means of binding-screws to the ends of the thick wires attached to the plates, and arrange the coil so that it is vertical when the cork is floating on dilute sulphuric acid contained in a large beaker or deep dish. Notice how the coil sets with its plane

perpendicular to the magnetic meridian Evidently the two faces of the coil exhibit magnetic polarity. Trace out the direction in which the current is passing, and verify the rule given above.

(ii Hold a pole of a bar-magnet near the coil, and observe how the latter is either attracted or repelled, according to which face of the coil is directed towards the magnet. From the results obtained verify the clock-face rule stated above Notice how the coil threads itself along the magnet if the latter is held at a convenient height, and comes to rest opposite the centre of the magnet.

The results of the foregoing experiments with a coil in which a current is passing may be more readily understood by examining the distribution of the lines of force. *Repulsion* is shown in Fig. 322, and *attraction* in Fig. 323 In the latter case it is evident that the tension of the lines of force will tend to move the coil up to the centre of the magnet. It is possible to show this action by a simple experiment.

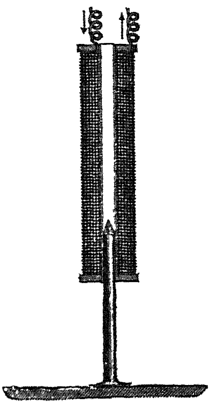


FIG. 324.—To illustrate Expt. 303

EXPT. 303.—**Mutual action between a circular current and a magnet-pole.** Wind several layers of thin cotton-covered copper wire on a narrow glass tube, to each end of which a cork disc has been fixed. Select a large wire nail which will slide easily within the tube (Fig. 324). Fix the coil vertically above the table, and with the nail projecting just within the tube. Note the effect when a fairly strong current is passed through the coil, and when the circuit is broken. Explain fully the effect observed.

Magnetic field due to a spiral of wire conveying a current.—As a single turn of wire conveying a current behaves like a magnetised disc, then several turns of wire placed face to face, and each turn conveying the same current in the same direction, would be expected to show magnetic properties similar to a row of magnetised discs placed with faces of opposite polarity in contact—in other words, a spiral of wire conveying a current should resemble an ordinary bar-magnet.

EXPT. 304.—**Magnetic properties of a spiral carrying a current.** Wind a close spiral of cotton-covered copper wire on a cardboard tube (5 cm.

diameter and 20 cm. long). Support a sheet of paraffined paper horizontally so that its plane coincides with the axis of the tube, having previously cut away portions of the paper so as to fit symmetrically round the tube. Sprinkle iron filings over the paper, and obtain a map of the field due to a current passing through the spiral (Fig. 325).

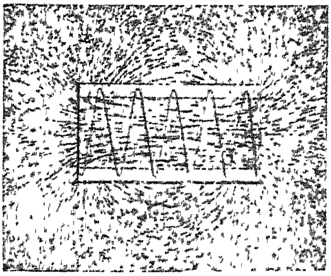


FIG. 325.—Magnetic field due to a spiral conveying a current.

Observe how closely this magnetic field resembles that of a bar-magnet. The hollow spiral enables us to obtain a map of the complete magnetic circuit, and the map indicates that the lines of force inside the spiral are approximately parallel to the axis.

Electro-magnets.—The student has already learnt that a piece of soft iron becomes temporarily magnetised when placed in a magnetic field, and that the degree of magnetisation acquired is (within certain limits) proportional to the strength of the magnetic field. When a rod of soft iron is inserted inside a spiral of wire (see Fig. 325) conveying an electric current, the magnetic field within the spiral acts inductively upon the iron so that the magnetic polarity of the spiral itself is augmented by the polarity acquired by the iron. As soon as the current ceases, the polarity of both the spiral and the soft iron disappears. Such an arrangement is termed an **Electro-magnet.** By using a sufficiently strong current and soft iron it is possible to obtain electro-magnets of great strength.

If the iron bar and spiral are bent into the form of a horse-shoe we have a *horse-shoe electro-magnet* (see Fig. 245). If they are bent so that the two ends are brought completely together so as to form a solid ring, all the lines of force within the spiral become *closed magnetic chains*, and no external magnetic field can be detected.

EXPT. 305.—**Effect of iron within coil conveying a current.** (1) Select a cardboard or glass tube of sufficient diameter to allow a rod of soft iron to be inserted. Wind two or three layers of cotton-covered copper wire round the tube. Adjust the magnetometer as described

in Expt. 237, and place the coil of copper wire on the scale about 20 cm distant from the needle with its axis perpendicular to the meridian. Connect the ends of the coil to a single cell of constant E M F Note the deflection of the needle Insert the soft iron core into the coil, and note the largely increased deflection Break the circuit, and observe how the needle returns more or less completely to zero. The experiment may be made more instructive by substituting lengths of soft iron wire for the iron rod, and gradually increasing the number of wires inside the coil

(ii) Dip the electro-magnet used in Expt (i) into a heap of wire nails. Notice the great lifting-power which it has. Break the circuit, and notice how all the nails fall off If the iron is not very soft it will retain a little permanent magnetism, and a few nails will remain attached to it

The electric bell.—The electric bell (Fig 326) is a simple application of the electro-magnet. The principal parts are a horse-shoe electro-magnet M with a soft iron armature A, which is carried by a steel spring S. The distant end of the armature carries the hammer H. The freedom of movement of A is regulated by the side screw and spring at C. A current entering at T passes through C and S, round the coils of M, and to T′ The armature is attracted and the circuit broken at C, the spring S brings the armature back and remakes the circuit. Each approach of the armature causes the hammer to strike the bell. The whole process is repeated continuously while the bell-push is depressed

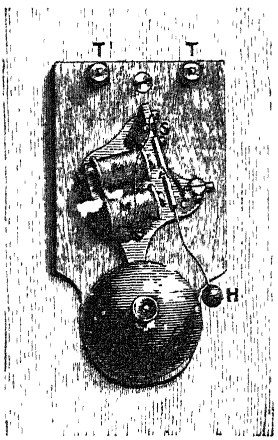


FIG. 326.—An electric bell.

Application of Oersted's experiment to telegraphy.—The instrument represented in Fig. 327 is frequently to be seen in provincial post-offices and in telegraph offices attached to railway stations. In front of the disc of the instrument a vertical pointer is moving rapidly to and fro, and a tinkling noise is heard as long as the pointer continues to vibrate. This is the single-needle telegraph instrument, first introduced by

Cooke and Wheatstone in the year 1837, and it is used for the purpose of transmitting messages between distant localities.

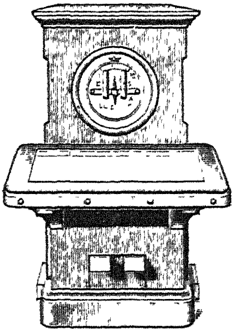


FIG 327 —A single needle telegraph instrument

This instrument is similar in principle to the astatic galvanometer (p. 484), but the coil of wire and the magnetised needle are fixed vertically instead of horizontally. The coil and magnetised needle are fixed inside the instrument; the end of the axle on which the needle is mounted passes through the front of the instrument and carries the pointer

One end of the coil is connected to a metal plate buried in the earth, and the other end is joined to a long insulated wire supported on telegraph poles leading to the distant post-office, where there is a battery and a commutator. One terminal of the commutator is connected to the telegraph wire, and the other terminal is connected to a metal plate buried in the earth (Fig. 328). The two metal plates are always at the same potential (zero), and since the earth is a conductor it serves the same purpose as a very thick copper wire (shown by dotted line), the expense of which is saved thereby. By making use of the earth's conductivity in this manner a single wire only is needed in order to connect two offices together telegraphically

A special form of commutator is used which consists of two strips of metal (L and R) which are capable of moving up and

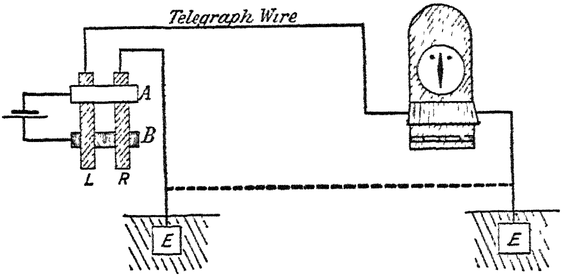


FIG 328 —Diagram of a simple telegraphic system.

down like the notes of a piano. In their upper position they are both in contact with a cross-piece of metal (A) connected to the

– ve terminal of the battery If L be pressed down its contact with A is broken and contact with B is made, thus connecting the – ve terminal of the battery to earth; B (and therefore L) will have + ve potential, and a current will flow along the telegraph wire towards the distant end, causing the needle in the instrument to deflect in a certain direction If L be released and R pressed down into contact with B a current in the reverse direction is obtained, and the needle will be deflected in the opposite direction. A recognised code of signals is adopted whereby the letters of the alphabet are represented by various combinations of left and right motions of the needle; thus a single swing to the left represents the letter *e*, a single swing to the right represents *t*, a swing to right followed by a swing to the left represents the letter *n*

In order that the telegraph operator may be able to interpret messages by ear as well as by sight, two small pieces of tinplate are fixed on either side of one end of the pointer, the movement of which will cause the familiar tinkling sound when the instrument is working. The pieces of metal are cut to slightly different sizes in order that the deflections in opposite directions may be readily distinguished by the sound emitted

The Morse system.—The **Morse sounder**, shown on the right of Fig. 329, consists of an electro-magnet *m* with a soft iron

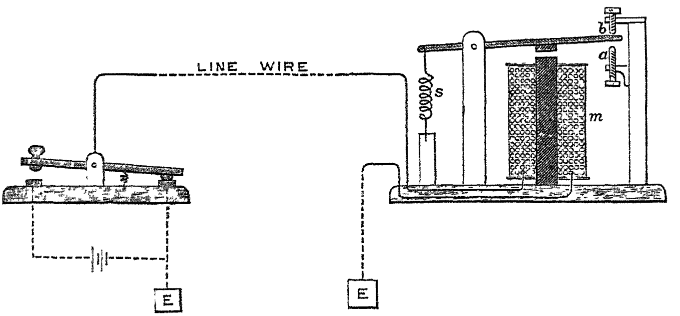


FIG 329.—The Morse sounder and key.

armature fixed on a pivoted lever which has freedom of movement between two adjustable stops *a* and *b*. When no current is passing through the instrument a spring *s* keeps the lever in contact with the upper stop *b*. When a current passes, the lever

is pulled down into contact with the stop *a* The signals are based upon the duration of the interval which elapses between the striking of the stop *a* and of the stop *b*, and this interval depends solely upon the duration of the current. The two signals, short and long, are usually termed 'dot' and 'dash' respectively, and the recognised relationship between these intervals is that the latter is three times as long as the former.

The dot-and-dash signals of the Morse alphabet correspond to a movement to the left and to the right respectively of the needle instrument.

A single dot represents the letter *e*, and a single dash represents the letter *t*. A dot before each of these represents the letters *i* and *a*; while a dash before each represents the letters *n* and *m*. A dot before each of these represents the letters *s*, *u*, *r* and *w*; and a dash before each represents *d*, *k*, *g* and *o*. Thus ·

$$\left.\begin{array}{ll} \cdot & e \\ - & t \end{array}\right\}\ \begin{array}{ll} \cdot\,\cdot & i \\ \cdot\,- & a \end{array}\quad \left.\begin{array}{ll} -\,\cdot & n \\ -\,- & m \end{array}\right\}$$

$$\begin{array}{ll} \cdot\,\cdot\,\cdot & s \\ \cdot\,\cdot\,- & u \end{array}\quad \left.\begin{array}{ll} \cdot\,-\,\cdot & r \\ \cdot\,-\,- & w \end{array}\right\}\ \begin{array}{ll} -\,\cdot\,\cdot & d \\ -\,\cdot\,- & k \end{array}\quad \left.\begin{array}{ll} -\,-\,\cdot & g \\ -\,-\,- & o \end{array}\right\}$$

By placing either a dot or a dash before each of the last eight letters a distinctive signal is given to all other necessary letters. Numerals are represented by combinations or groupings of five signals each.

In this system the operator receives the message *by ear*. More rapid transmission is obtained by a mechanical method in which a small disc, revolving in an inking fluid, is attached to the left-hand end of the lever. The depression of the lever brings the wheel into contact with a strip of paper which is moved by clockwork at a constant speed In this manner the dots and dashes are permanently recorded on the paper

The signals are transmitted by a **key**, represented on the left of Fig. 329. This consists of a metal lever mounted on a wooden stand. The line wire is connected to the middle of the lever. When not in action, a spring keeps the line wire in communication with the earth; when the front end of the lever is depressed a battery circuit is closed, and a current passes along the line wire to the sounder at the receiving end

When the line wire is extremely long the current may not be strong enough to operate the sounder, unless excessive battery power is used. In such cases a **relay** (R, Fig. 330) is inserted in

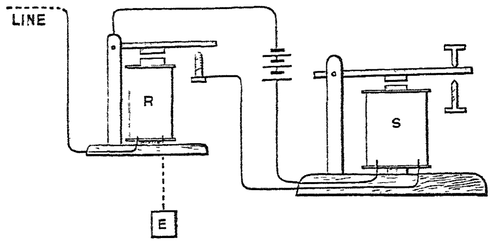


Fig 330—A relay

the circuit near to the sounder S. The weak current passing along the line wire is passed through the relay, which is simply an electro-magnet with an armature attached to a lever. The depression of this lever puts into circuit a local battery which is strong enough to operate the sounder efficiently

Action of Magnets on Currents.

Behaviour of a linear current in a magnetic field.—If A (Fig. 331, i) represents the cross-section of a wire conveying a current *down* through the paper, and *n* a single north-seeking pole, the latter will tend to move round A in a clockwise direction to *n′*. But if *n* be fixed and A free to move, then A will move in such a manner that it will subsequently occupy the same relative position with respect to *n* as would be the case if A were fixed and *n* were free to move; hence A will move towards A′ (Fig. 331, ii). This effect, being continuous so long as the current lasts, will cause A to rotate round *n*, and it can be experimentally verified with the apparatus shown in Fig. 332.

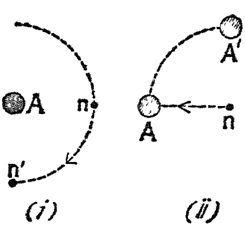


Fig. 331.—*A*, cross-section of wire, *n*, north-seeking pole

EXPT. 306 —**Rotation of current in a magnetic field.** G is a glass tube (20 cm. × 4 cm.) closed at both ends with corks. A cylindrical bar-magnet is fixed through the centre of the lower cork with its north-seeking pole uppermost and projecting a short distance into the tube, a copper wire is also fixed through the same cork. A thick wire bent into the form of a hook is passed through the centre of the upper cork, and supports a thin wire, the lower end of which dips into the mercury (H). (Special precautions should be taken that the surface of the mercury is quite clean.) Pass a strong current *down* the wire, and observe the direction of rotation. Reverse the direction of the current and observe how the rotation is reversed

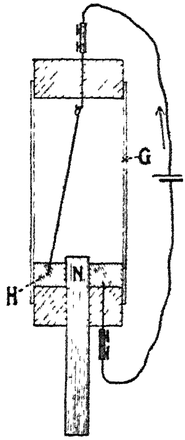


FIG 332.—Apparatus for showing rotation of a linear current round a magnet-pole.

The movement of the current in the foregoing experiment is due to the field of the magnet, and at any instant the direction of motion is perpendicular to the direction of the lines of magnetic force and also to the direction of the current. The following rule, due to Prof. Fleming, will be found useful in determining the direction of motion of a linear current when placed in a magnetic field: **Hold the thumb and first finger of the left hand as fully extended as possible, and bend the second finger at right angles to the palm If the first finger represents the direction of the lines of force, and the second finger that of the current, then the thumb will indicate the direction of motion** (Fig. 333). By means of this rule verify the direction of rotation of the current in Fig. 331, ii.

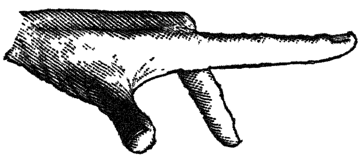


FIG 333

The same phenomenon can be demonstrated by passing a current along a long piece of very thin copper wire (or, preferably, tinsel) which hangs near a strong bar-magnet held vertically in a clamp. As soon as the current passes, the tinsel wraps itself spirally round the magnet (Fig. 334), the direction in which it

winds itself round the magnet depending upon the direction of the current and the polarity of the magnet.

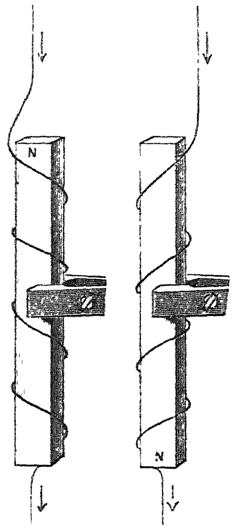
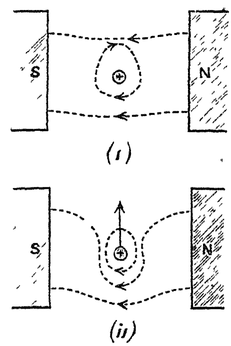


FIG 334.—Tinsel along which a current is passing winding itself around a magnet.

The movements of a conductor conveying a current when placed in a magnetic field, may be explained by applying the hypothetical properties of lines of force (p. 387) to the resultant magnetic field Thus, in Fig. 335, i, suppose a conductor, which conveys a current perpendicularly through the paper and away from the observer, to be placed in the uniform magnetic field obtained between the two flat pole-faces of an electro-magnet. For simplicity, two lines only of the latter field are shown. Since lines of force proceeding in opposite directions attract each other, and those proceeding in the same direction repel, the distribution of the lines of force in the resultant field will resemble that shown in the diagram. Suppose now that the current is increased slightly; the line of force associated with the current will expand and touch the line of force due to the magnet in the region where these lines are proceeding in opposite directions; they will coalesce, and form a single line bent round the conductor, as shown in Fig. 335, ii. The tension along this line of force will cause a force to act upon the conductor in the direction of the arrow. The diagram indicates also, near the conductor, a new line of force due to the increased current. Fig. 335, iii, represents the resultant magnetic field in fuller detail.



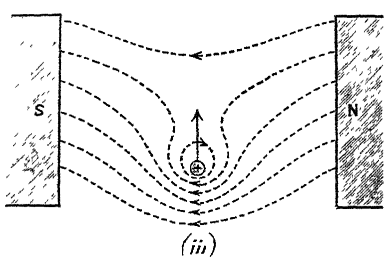


FIG 335 —Resultant Magnetic Fields.

A consideration of the diagrams provides another rule by which the direction of the force acting on the conductor may be remembered: **The force acts from that side of the conductor where the two fields reinforce each other to that side where the fields oppose each other.**

Motion of a linear current in the field due to another linear current.—Let AB (Fig. 336) be a fixed wire conveying a current from A to B. The direction of the magnetic force at P, due to the current in AB, will be downwards and at right angles to the paper. If a wire CD, which is free to move and conveying a current from C to D, passes through P and is parallel to AB, then the *left-hand rule* deduced above indicates that CD will move towards AB; in other words, CD will be *attracted* by AB. If the current in CD is reversed, then *repulsion* will take place. Hence, according to theory, **two parallel wires conveying currents in the same direction attract each other, and if the currents are in opposite directions they repel each other.**

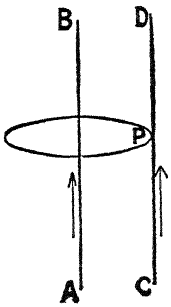


FIG 336

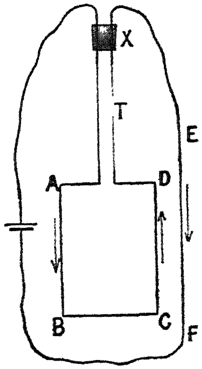


FIG 337.—Apparatus to show attraction and repulsion of linear currents

EXPT. 307 —**Attraction and repulsion between wires conveying currents.** Bend a copper wire into a rectangular form ABCD (Fig. 337) and solder the ends to two lengths of tinsel, the upper ends of which are soldered to two copper wires passing through the cork X. Clamp the cork at a convenient height. Connect the wires to the terminals of a battery, and include in the circuit a length of free wire EF. Hold EF near to and parallel to the sides of the suspended rectangle, and verify attraction in one case and repulsion in the other.

EXERCISES ON CHAPTER XXXVI.

1. A plate of pure zinc, and a plate of copper, are dipped into dilute sulphuric acid, and then connected by copper wire. What changes take place in the plates, wire, and acid, when the circuit is complete?

2. When a galvanic cell, consisting of zinc and copper plates immersed in dilute sulphuric acid, has its terminals joined by a wire, the E.M.F. rapidly decreases. How do you account for this? Describe a cell designed to prevent this decrease in E.M.F., and explain how it acts.

3. A glass cell is divided into two compartments by a porous partition. One compartment contains a strong solution of copper sulphate, and the other dilute sulphuric acid. A copper plate and a zinc plate, which dip into these respectively, are joined to the terminals of a galvanometer, the needle of which is deflected. State and explain how the deflection will be altered if the copper sulphate is replaced by dilute sulphuric acid.

4. Two galvanic cells are made of exactly the same materials, but in one cell the plates are much larger than in the other. What would be the effect of introducing both into a circuit so that they tend to send currents in opposite directions? Give reasons for your answer.

5. Two galvanic cells are made by dipping (i) plates of zinc and platinum into a beaker of dilute sulphuric acid, and (ii) plates of zinc and copper into another beaker containing the same liquid. The plates can be connected by copper wires. Explain with diagram how the two cells may be connected in series so as to (i) strengthen, (ii) weaken, the current produced by one of them.

6. Explain the cause of *Local Action.* Why is it objectionable? and how may it be remedied?

7. Explain the cause of *Polarisation*, and describe the chief methods of preventing it.

8. Describe the Daniell cell, and explain the functions of each part of the cell and the action that takes place when the poles are connected by a conducting wire.

What advantages does this form of cell possess over a simple voltaic cell consisting of plates of copper and zinc immersed in dilute acid?

9. A long straight wire is stretched on a table in the direction of the magnetic meridian, and a dip circle, with its plane parallel to the magnetic meridian, is placed on the table near to the wire and on the west side of it. Will the dip of the needle be altered when an electric current is passed along the wire from south to north, and, if so, how? Give reasons.

10. A straight horizontal wire is placed near and parallel to a compass-needle, and in the same horizontal plane with it. If a current is then passed through the wire, what effect is produced on the needle, and what occurs if the wire is (i) slightly raised, (ii) slightly lowered?

11. A strong electric current flows through a copper wire which passes through the centre of an iron ring, and is at right angles to the plane of the ring. Describe the magnetic state of the ring.

12. A current is flowing through a rigid copper rod. How would you place a small piece of iron wire with respect to it so that the iron may be magnetised in the direction of its length? Assuming the direction of the current, state which end of the iron will be a north pole.

13. Two long wires are placed parallel to each other in the same horizontal plane and in the magnetic meridian. A magnetic needle capable of turning in any direction about its point of suspension is placed exactly half-way between them. How will it behave if the same electric current flows through the easterly wire from south to north, and through the westerly wire from north to south? (The action of the earth on the magnetic needle may be neglected.)

14. A wire lies east and west (magnetic) immediately over a compass-needle. How is the direction in which the needle points affected when a *strong* current flows through the wire (1) from west to east, (2) from east to west?

15. Draw a plan showing how the current must circulate in the coils of a horse-shoe electro-magnet, to make the poles (*a*) both north, (*b*) one north and the other south.

16. A current flows down a vertical wire, and is of such strength that at a distance of one foot from it its magnetic field is equal to the horizontal field of the earth. Indicate in a diagram the directions in which a freely suspended compass needle would set if carried round the wire at a distance of one foot from it, when the needle is N., N.E., E., S.E., S., S.W., W., and N.W. of the wire.

17. Represent in a diagram the arrangement of the parts of an ordinary *electric bell*, and explain its action.

18. A magnet is placed at the centre of a circular coil of wire through which a current is passed. What is the direction of the force acting on the north pole of the magnet, and how does the force depend on the direction of the current?

19. A small compass needle is suspended at the centre of a vertical copper ring through which a current is passed. How is the needle affected by the current (i) when the ring is in the magnetic meridian, and (ii) when it is at right angles to the magnetic meridian?
What are the forces acting on the needle in each case?

20. A wire conveying a current is placed in a magnetic field, the direction of the lines of force of which is known. Explain how the direction in which the wire will tend to move may be deduced from theory.

21. State the law of attraction and repulsion of straight wires conveying currents, and describe an experiment by which the law may be verified

22. An electric current is flowing along a wire. You are given a pivoted compass needle, and are required to find out by its aid which way the current is flowing. How would you proceed (*a*) if the wire in question lies horizontally ; (*b*) if the wire runs vertically , (*c*) if the wire is coiled up in a circular coil or open hank ?

23. A road in the northern hemisphere runs magnetic north and south. At one point an insulated conductor passes beneath it in which an electric current flows from east to west. How will the indications of a dip circle be affected at points near to the conductor?

24. A wire is stretched from east to west (magnetic). How, without breaking it, can you test whether, and in what direction, an electric current is passing through it ?

25. Describe experiments which show the mutual action between magnets and electric currents.

26. What is the advantage of a two-fluid cell over a one-fluid cell ? Sketch a Leclanché cell. Describe the chemical action that takes place, and state the purpose for which it is suitable

27. Describe the nature of the influence that a current flowing in a long straight wire exerts (i) upon a magnetic pole, (ii) upon a small magnet capable of turning in any direction, in its neighbourhood

Explain the fact that iron filings cling round a wire traversed by a strong current.

CHAPTER XXXVII.

GALVANOSCOPES AND GALVANOMETERS. THE UNIT OF CURRENT.

Detection and measurement of electric current.—The action on a neighbouring magnet of the magnetic field created by a current of electricity passing along a wire may be used as a means of detecting currents of electricity. Moreover, since the strength of the field depends upon the strength of the current, it is also possible to apply the same principle to the comparison of the strengths of various currents. An instrument for detecting an electric current according to this principle is termed a **Galvanoscope,** and instruments for measuring the strength of a current are termed **Galvanometers.**

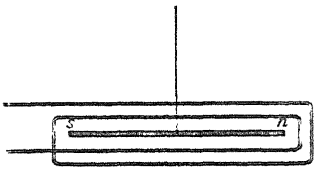


FIG. 338.—The principle of a galvanoscope.

A simple galvanoscope (Fig. 338) consists of a freely suspended magnetised needle surrounded by several turns of wire, the plane of which coincides with that of the magnetic meridian. By applying Ampère's rule it is evident that the portions of the coil above and below the needle both tend to produce a deflection in the same direction. The magnetic field due to the coil is the resultant of the fields due to each individual turn of the coil: hence by increasing the number of turns of wire it is possible to detect extremely weak currents. The lines of force due to the current are perpendicular to the plane of the coil, and therefore tend to set the needle at right angles to the meridian, but at the same time the earth's field tends to pull the needle

back into the meridian, the deflection therefore depends upon the relative magnitude of these two forces, which may be termed the deflecting force and the controlling force. By rigidly attaching to the needle a horizontal pointer below which is fixed a circular scale the amount of deflection may be observed accurately.

EXPT 308—**A simple galvanoscope** Clamp a narrow strip of cardboard horizontally so as to serve as a support for a compass needle. Connect the poles of a voltaic cell by means of a long thin copper wire (cotton-covered). Hold a portion of the wire so that its length is in the meridian, and bring it just over the needle and as near to it as possible Observe the deflection Still keeping the portion of wire above the needle in position, wrap the wire round under the instrument, so that a portion of the wire is just underneath the needle. Observe that the deflection is now greater than before. Wrap the wire once more round the needle, and observe the still greater deflection. Wrap several more turns round the needle, and note the deflection during the process

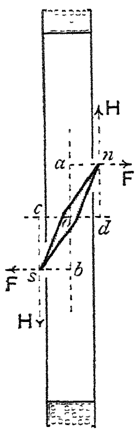


FIG. 339—The principle of a galvanometer

The amount of deflection obtained is determined by the relative strengths of the magnetic forces due to the current and to the earth's magnetic field. The former tends to pull the needle into a position at right angles to the meridian, while the latter tends to pull the needle back into the meridian.

Fig. 339 represents a horizontal cross-section through the centre of a circular coil of wire, at the centre of which a short magnetised needle *ns* is pivoted. If H and F are the strengths of the magnetic fields due to the earth and to the current respectively, and if m is the magnetic pole strength of the needle, then the forces acting on both n and s will be $(m \times \mathrm{H})$ and $(m \times \mathrm{F})$. Each pair of forces tends to rotate the needle in opposite directions, and the needle finally comes to rest in such a position that the moments (p 122) of these forces round the centre of the needle are equal and opposite.

$$\text{Moment of force } m\mathrm{H} = \text{moment of force } m\mathrm{F},$$

or,

$$m\mathrm{H} \times od = m\mathrm{F} \times ao.$$

Hence

$$mF = mH \times \frac{od}{oa} = mH \times \frac{an}{oa} = mH \times \text{tangent of angle } aon:$$

or, the tangent of the angle of deflection $= \frac{mF}{mH} = \frac{F}{H}$ *

This formula assumes that the magnetic field due to the coil is uniform, but in reality it is only uniform in a very small region round the centre of the coil, and therefore the formula would hold good only if the magnet were very short.

The sensibility of a galvanometer.—The sensibility of a galvanometer may be defined as the amount of deflection obtained with a given current of electricity. The sensibility is great if a considerable deflection is obtained with an extremely weak current.

It is evident from Fig. 339 that the sensibility may be increased by diminishing the controlling force due to the earth's magnetic field. This may be done by placing a bar-magnet in a suitable position near the instrument.

By referring to Fig. 261 it will be clear that if the needle of the instrument occupies the position of one of the neutral points in the resultant field due to the earth and the magnet, it will come to rest in any direction If the magnet be moved a little farther away from the instrument it will obey the forces due to the earth, but these will be much weaker than if the magnet were withdrawn. Fig. 261 (ii) explains how this result may be obtained by placing the magnet vertically over or under, or in front of or behind, the instrument. Fig 261 (i) explains how the magnet may be placed with its axis in line with the needle's axis, and with its south-seeking pole directed towards the north.

The astatic galvanometer.—One method of increasing the sensibility is to use an astatic pair of needles (Fig. 284 (ii)) instead of a single needle; and, when this device is used, the instrument is termed an astatic galvanometer. If the two magnets are of exactly equal strength and size, the force tending to make one magnet set in the magnetic meridian is exactly neutralised by the force acting on the other magnet, and the astatic pair comes to

* It will be observed that the deflection is independent of the pole-strength of the needle.

rest in any position. In practice it is impossible to obtain two magnets so identically alike, and the pair comes to rest in the meridian in obedience to the force due to the stronger magnet The astatic pair will behave just like a single magnet, the pole strength of which is equal to the difference between the pole strengths of the two separate magnets. Hence, the controlling force due to the earth is very small. Moreover, by placing the coil so that its upper layer lies between the needles (Fig. 340), the presence of the upper needle tends to increase the deflection of the lower needle, since, by Ampère's Rule, the deflection of the upper needle due to a current in the upper layer of the coil will be in the same direction as that of a reversed needle placed below the upper layer of the coil.

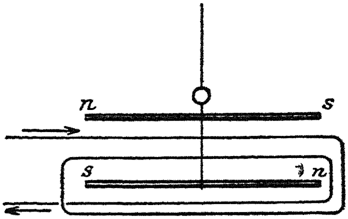


FIG 340.—Principle of an astatic galvanometer

The mirror galvanometer.—In principle this instrument is the same as the galvanoscope, but a far more accurate method of reading deflections is used than that afforded by the pointer and circular scale of the latter type. A small circular mirror of silvered glass is attached to the needle, and a beam of light is directed on to the mirror and reflected back to a horizontal paper scale placed at some distance from the instrument. A scarcely perceptible deflection of the needle causes a considerable movement of the reflected beam of light on the scale. This type of instrument (Fig. 341) consists of a circular coil of many turns of thin silk-covered copper wire, in the centre of which a circular mirror is supported by means of a silk fibre. To the back of the mirror are attached three or four short lengths of

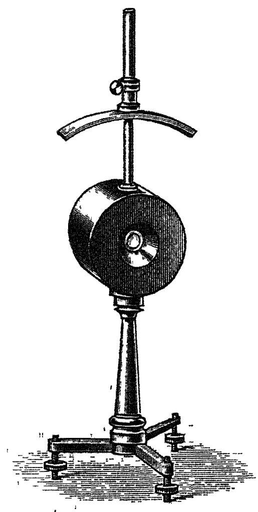


FIG. 341—A mirror galvanometer.

magnetised watch-spring. A controlling magnet is supported above the instrument by a vertical upright, on which the position of the magnet can be adjusted as required

Fig. 342 shows how a lamp and scale is used in conjunction with a galvanometer. The beam of light is equivalent to a pointer of length equal to twice the distance between the mirror

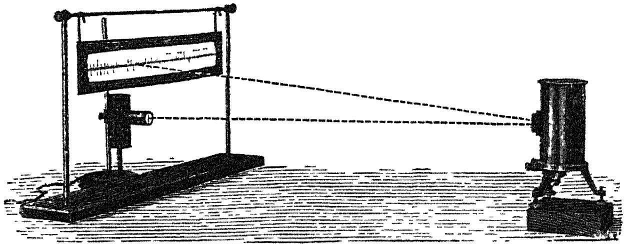


FIG. 342.—Galvanometer, with lamp and scale.

and the scale. In Fig 342 the beam of light is derived from an electric incandescent filament enclosed in a metal case fitted with a horizontal adjustable tube, in which a lens is fitted. The beam of light is directed upon the mirror of the galvanometer and reflected back to the divided scale, and deflections are observed by focussing the image of a fine vertical line scratched on the surface of the lens. A concave mirror is generally used, and the cross-line can then be focussed without using an auxiliary lens.

Suspended coil galvanometers.—The action of a magnet upon a conductor conveying a current is the basis of a type of galvanometer which possesses considerable advantages. The **d'Arsonval** (Fig. 343) is the most familiar pattern. The magnetic field is derived from a cylindrical magnet M built up of magnetised rings of hard steel. The rectangular coil C is hung on a stretched strip of phosphor-bronze, which serves to convey the current to the coil. The current is conveyed away through a very fine spiral spring, the lower end of which is

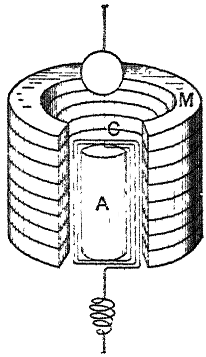


FIG. 343.—A suspended coil galvanometer.

connected to one of the terminal screws on the base-board of the instrument.

The instrument is adjusted so that when no current is passing the plane of the coil is parallel to the magnetic lines of force. When a current passes, the vertical sides of each turn of wire in the coil is acted upon by a force; and since the two sets of forces will be acting in opposite directions, they constitute a couple, which tends to rotate the coil into a position with its plane perpendicular to the lines of force. The rotation is opposed by the torsion set up in the suspension; and the restoring force due to this torsion is proportional to the angle through which the lower end of the suspension is twisted.

Hence, if the magnetic field is uniform within the range of movement of the coil, and also radial towards the vertical axis of the coil, the moment of the couple due to the magnetic forces, and therefore the current, will also be proportional to the angle through which the coil is deflected. Uniformity and radial direction of the magnetic field are obtained by means of the soft-iron cylinder A which is fixed in position between the pole faces of the magnet; and these pole faces are usually curved, concentric with the axis of A, and made wider than represented in Fig 343. The following are the chief advantages of this type of galvanometer:

(i) The deflections are scarcely affected by external magnetic fields.

(ii) The instrument may face in any direction, since the zero position of the coil is independent of the direction of the magnetic field in which it is suspended.

The tangent galvanometer.—In order that a galvanometer may obey the tangent law, it is necessary that the controlling force should be due to a uniform magnetic field (such as that of the earth), and that the field created by the current in the coil should be uniform within the region in which the needle is capable of moving. When the coil is circular and of considerable diameter, the field at its centre due to a current passing round it will be fairly uniform. Hence if a very short magnetised needle is suspended at the centre of a circular coil, which is placed with its plane in the magnetic meridian, then all the conditions for obeying

the tangent law will be fulfilled. Such an instrument is called a **tangent galvanometer**

Fig. 344 represents a suitable form of tangent galvanometer for simple experiments. Three separate coils may advantageously be wound on the circular wooden frame (about 20 cm. diameter), and connected to separate binding-screws fixed to the base-board of the instrument. One coil may conveniently consist of three or four turns of thick copper wire, for use with fairly strong currents, the other coils may consist of fifty and one hundred turns (respectively) of thin copper wire, for use with weaker currents. A horizontal circular scale is fixed at the centre of the coil, and a magnetised needle (2 cm. long) is suspended by means of a single fibre of unspun silk just above the centre of the scale. A long pointer is attached to the centre of the needle and at right angles to its axis; a suitable pointer consists of a strip of thin aluminium sheet, bent on each side of the centre, as shown in Fig 344 The use of a silk fibre introduces torsion when the needle is deflected, but its controlling force is small when compared with that due to the earth's magnetic field (unless the magnet is but feebly magnetised), this error to which the instrument is liable, may be more completely avoided by pivoting the needle on a vertical metal point fixed through the centre of the scale. The needle is protected from air currents by placing a glass shade over the instrument.

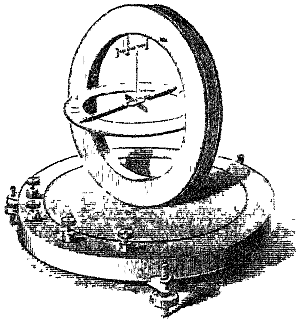


FIG. 344.—A tangent galvanometer.

The deflecting force (f) near the centre of the coil varies directly as the strength C of the current, the length l of the wire, and the strength m of the magnet's pole, and inversely as the square of the distance r between the wire and the pole. If the radius of the coil be r, and consists only of one turn, then the length of wire in the coil $= 2r \times \pi$.

Hence $$f = \frac{C \times 2\pi r \times m}{r^2} = \frac{C \times 2\pi \times m}{r}.$$

But, deflecting force (f) = intensity of field (F) × pole-strength (m), hence $$F = f/m = 2\pi C/r.$$

If the coil consists of n turns of wire, then

$$F = 2\pi n C/r.$$

Comparison of currents by means of the tangent galvanometer.—It has been shown (p. 483) that

$$F = H \tan \theta,$$

where F is the intensity of the magnetic field at the centre of a circular coil conveying a current, H is the horizontal intensity of the earth's magnetic field, and θ is the angle of deflection.

But $$F = 2\pi n C/r;$$

hence $$2\pi n C/r = H \tan \theta,$$

or $$C = \frac{rH}{2\pi n} \cdot \tan \theta.$$

It is evident therefore that **the current strength is proportional to the tangent of the angle of deflection.** Hence two or more currents may be compared by passing them individually through the same tangent galvanometer, and observing the angles of deflection. The numerical values of the tangents of these angles may be obtained from mathematical tables.

When using the instrument, error of observation may arise from three causes.

(i) **Parallax.** The eye should be vertically over the pointer when reading the deflection. This is ensured by mounting the circular paper scale, of which the central portion is removed, on a plane mirror. The eye is moved until its image appears to be immediately below the end of the pointer which is being observed.

(ii) **The suspending fibre may not coincide with the centre of the circular scale.** Errors due to this are eliminated by taking the mean of the readings of both ends of the pointer.

(iii) **The axis of the magnet and the plane of the coil may not coincide with the magnetic meridian.** The former possible error is probably due to torsion in the suspending fibre. The errors are eliminated by observing the deflections when the current is reversed, and taking the mean of the four readings obtained.

The absolute unit of current.—So far the strength of a current has been indicated only by the symbol C, to which a numerical value cannot be given until a *unit* of current strength has been decided upon. The absolute unit which is universally recognised is based upon the expression $2\pi C/r$ for the intensity of the magnetic field at the centre of a circular coil of wire consisting of

one turn only. If the current be supposed to traverse a portion of the circuit equal to $1/2\pi$ part of the whole circumference (*i.e* a portion equal in length to the radius of the coil), the intensity of the field will be C/r, and if the radius be 1 cm. the intensity will be equal to C units. Hence C will be equal to unity when the intensity of the field is equal to unity. Therefore, unit current may be defined thus: **A current has unit strength when 1 cm. length of its circuit, bent into the form of an arc of 1 cm. radius, exerts a force of one dyne on a unit magnet-pole placed at the centre of the arc.**

The 'practical' unit of current, called the **ampere**, is equal to $\frac{1}{10}$ part of the absolute unit.

The unit of quantity.—The unit of quantity is defined as that which is conveyed in one second by a steady current of one unit. The 'practical' unit of quantity, called the **coulomb**, is that which is conveyed by a current of one ampere in one second.

The reduction-factor of a tangent galvanometer—It has been shown previously (p. 488) that, in the case of a tangent galvanometer,

$$C = \frac{rH}{2\pi n} \cdot \tan\theta,$$

where C is the current in absolute units. If the current be expressed in amperes, then, since the ampere is one-tenth part of the absolute unit,

$$C = \frac{10rH}{2\pi n} \cdot \tan\theta \text{ amps.}$$

If the values of H, r, and n are known, the quantity $10rH/2\pi n$ may be regarded as a constant quantity for the instrument; and the current passing through the instrument may be calculated by multiplying $\tan\theta$ by this quantity. The expression $10rH/2\pi n$ is termed the **reduction-factor** of the galvanometer, and is usually denoted by the symbol k. Hence

$$C = k \tan\theta.$$

The value of k may be determined either by direct measurement of H, r, and n; or it may be determined indirectly by electro-chemical methods (p. 520).

EXERCISES ON CHAPTER XXXVII.

1. Describe the construction and use of a tangent galvanometer.

2. Explain fully why the deflection of the needle of a tangent galvanometer is independent of the pole strength of the needle.

3. What is meant by the *sensibility* of a galvanometer? Describe some simple method of increasing the sensibility of (i) an astatic galvanometer, (ii) a mirror galvanometer

4. How does the *controlling force* in an astatic galvanometer differ from that in a tangent galvanometer? And explain why the former instrument does not obey the tangent law.

5. Describe and explain the action of some form of sensitive galvanometer with which you are acquainted, pointing out in detail the factors that contribute to the sensitiveness

If you found that the deflection produced was off the scale, how would you reduce the sensitivness of the galvanometer?

6. Show that a galvanometer with a single needle may be made more sensitive by placing a magnet in a suitable position in its neighbourhood. Give a sketch showing how you would mount the magnet so that by moving the magnet the sensitiveness may easily be altered; indicate the position of the poles of the magnet when the sensitiveness of the instrument is as great as possible.

7. The coil of a given tangent galvanometer can be rotated about a vertical axis while the scale upon which the deflection of the needle is read remains fixed Describe and explain in detail how the deflection of the needle will alter (the current through the coil remaining constant) when the coil is turned continuously through 360° from its original position in the meridian.

8. Define the absolute unit of current, and explain how the definition is derived from the fundamental principle of the tangent galvanometer.

9. The coil of a tangent galvanometer consists of 30 turns of wire, of mean radius 8 cm. If the horizontal intensity of the earth's field is 0 36 unit, find the *reduction-factor* of the instrument.

10. A current of 0 1 ampere produces a deflection of 20° in a tangent galvanometer in a position where the horizontal intensity is 0 36 unit. What current will be required to produce the same deflection in a position where the horizonal intensity is 0·32 unit.

11. The same current is sent through two tangent galvanometers connected in series, and causes deflection of 30° and 60° respectively. Find the ratio of the reduction factors of the two instruments.

12. What is the intensity of the magnetic field at the centre of the coil of a tangent galvanometer of 20 turns of wire and 25 cm. mean radius when traversed by a current of 0·2 ampere?

13. Calculate the strength of the current in C.G.S. units and also in amperes from the following data Radius of coil, 12 cm. Number of turns in coil, 10. Deflection of needle, 45°. Value of earth's horizontal force, 0 36

14. Discuss the several forces or moments which act on the needle of a tangent galvanometer when deflected by the action of a current

passing through the coil of a galvanometer, and deduce the law of action of the instrument.

15. A coil of six turns, each of which is 1 metre in diameter, deflects a compass needle at its centre through 45° Find the strength of the current in amperes, having given that H = 0·36 C G S units.

16. A current flows through two tangent galvanometers in series, each of which consists of a single ring of copper, the radius of one ring being three times that of the other. In which of the galvanometers will the deflection of the needle be greater? If the greater deflection be 60°, what will the smaller be?

17. Of how many turns of wire must the coil of a tangent galvanometer consist, if the radius of the coil is 15 cm, and if a current of 0·01 ampere is to produce a deflection of 30°? (H = 0·36.)

CHAPTER XXXVIII.

E.M.F. AND RESISTANCE. OHM'S LAW.

Electromotive force.—Electricity tends to move from regions, or conductors, having higher electric potential to those having lower electric potential; and the transference is due to what is termed a potential-difference (p. 456) between the regions or conductors. The grouping of metals and liquids in any type of voltaic cell results in one of the metals having a higher electric potential than the other; consequently, when the plates are connected by means of a conducting material, such as a metal wire, a current of electricity flows along the conductor from the plate at higher potential to that at lower potential. **So long as the current continues, work is being done by the electric forces; and, in the case of a simple circuit, such as that under consideration, this work re-appears as heat generated in the wire.**

Just as the mechanical work which a falling body is capable of doing is equal to the product of the mass of the body and the vertical distance through which it falls (*i.e.* the 'difference of gravitational potential'); so in the electrical case, the work done in the conductor is equal to the product of the quantity of electricity which passes and the P.D. between the ends of the conductor. From this statement a definition of the unit P.D. may be derived by the following reasoning: The quantity of heat generated in the conductor must be expressed in terms of its equivalent quantity of work, *i.e.* in **ergs**; the number of heat units, therefore, must be multiplied by **Joule's equivalent** ($4{\cdot}2 \times 10^7$ ergs). If, then, the heat generated is equivalent to W ergs, and if the quantity of electricity conveyed is expressed by the product *current* × *time*, then

$$W = (C \times t) \times V$$

$$\text{or} \quad V = W/Ct,$$

where V is the P.D. between the ends of the conductor. Hence, **unit P.D. is that between the ends of a conductor wherein 1 absolute unit of current is dissipating 1 erg of work per second.**

This unit, called the *absolute* (or C.G.S.) unit, is far too small for practical purposes. The practical unit of P.D., with other practical units, was agreed to by the Paris Congress of 1881; it was fixed as being equal to 10^5 absolute units, this multiple being selected perhaps because it was approximately equal to the E.M.F. of a Daniell cell, which was then regarded as the most trustworthy standard of E.M.F. This practical unit is called the **volt**[1]

Current depends upon E.M.F.—We should anticipate that the strength of the current flowing along a wire would depend upon the difference of potential between the two ends (or, in other words, upon the E.M.F. of the battery). When two similar cells in-series are used instead of one cell, we double the E.M.F. in the circuit, but the resistance (p. 494) is also slightly increased since the additional cell has resistance; hence the current is not quite twice as great. If cells having no resistance could be used, the current from two similar cells would be exactly twice as great as that from one. We may say that **the current in a wire is directly proportional to the potential difference between its two ends.**

This is known as **Ohm's Law**, which can be more rigorously demonstrated by the method described in the following section.

EXPT. 309.—**Strength of current.** Connect up a large Bunsen cell (or an accumulator), a 4-metre length of No. 22 German-silver wire, and the low resistance coil of a tangent galvanometer. Note the angle of deflection. Substitute two similar cells connected together in series instead of the single cell, and note the angle of deflection. Obtain the numerical values of the tangents of these angles, and observe that the value is nearly twice as great in the second case.

Ohm's law.—G. S. Ohm,[2] in 1826, conducted original experiments which resulted in the statement of the following simple relationship: **In any wire at uniform temperature, the current is**

[1] Alessandro Volta (1745-1827), Professor of Physics in Pavia University.

[2] G. S. Ohm (1789-1854) was the son of a German locksmith, and was appointed professor of physics in Munich in 1849

directly proportional to the potential difference between its ends; or, E/C is a constant ratio (where E and C represent the potential difference and the current respectively).

The numerical magnitude of the ratio E/C is a measure of the *Resistance* of the conductor. The constancy of the ratio E/C may be proved by applying the following principle: If any two points (A and C, Fig. 345) on a long thin wire AB, conveying a current, are touched by the ends of a thin wire AR_2C, a weak current will be generated and will traverse the thin wire from A to C (R_2 is a *high* resistance, *e.g.* a pencil line drawn on matt-glass). This weak current may be detected by including in its path a sensitive galvanometer (MG). We can also include in the same circuit

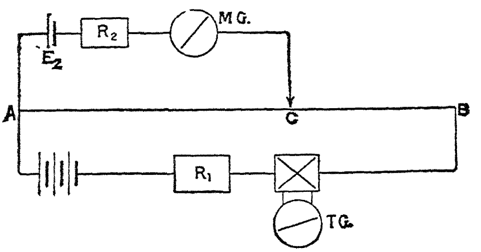


FIG 345.—Illustration of Ohm's law.

another source of electro-motive force E_2 (say, a standard cell), placed so as to tend to send a current in the opposite direction. If this *opposing* electro-motive force is equal to that due to the potential difference between A and C, then no current will traverse the wire, and no deflection will be produced in MG. This point C may be determined by trial, and the strength of the current between A and C may be observed by inserting a tangent galvanometer (TG) in the circuit. If *two* standard cells are used instead of E_2, and if C remains fixed, it will be found necessary to *double* the current strength between A and C in order to obtain no deflection in MG. If *three* standard cells are used, the current in AB must be made *three* times as great.

EXPT. 310.—**Illustration of Ohm's law.** Connect up the apparatus as shown in Fig. 345, and adjust R_1 so that a deflection of about 15° is obtained in TG. Find a point C such that no current traverses MG when one standard cell is used. Read the deflection in TG. Insert *two* standard cells in place of E_2. Make contact at C, reduce R_1 until

there is no deflection in MG, and read the deflection in TG. Repeat with three standard cells. Enter the observations thus.

Standard Cells (n)	Deflections in TG		Mean Deflection	tan α.	$\frac{n}{\tan \alpha}$
	East End	West End			
1	10° , 12°·1 } 11°·05	12°·4 , 10° } 11°·2	11°·1	0·196	5·102
2.	20°·5 , 22°·2 } 21°·3	22°·8 , 20° } 21°·4	21°·35	0·391	5·115

The absolute (or C.G.S.) unit of resistance.—The close relationship between Current, E.M.F. and Resistance, as expressed in Ohm's law enables us to define the unit of resistance in terms of the other units. The absolute unit of resistance is defined as follows:—
A conductor has unit resistance when unit potential difference between its ends causes a current of unit strength to flow through it.

The ohm and the ampere.—Since the absolute unit of resistance is far too small for practical purposes, it was decided, by the Paris Congress of 1881, to take as the practical unit a resistance equal to 10^9 absolute units, because this was approximately the resistance of the standard unit previously in use. This practical unit is termed the **ohm.**

In order that all the practical units might comply with Ohm's law, it was necessary to regard the practical unit of current, the **ampere,** as equal to $10^8 \div 10^9 = 10^{-1}$ absolute unit of current.

Graphic representation of resistance, E.M.F., and current.—In Fig. 346 let AB represent a copper wire along which a current is flowing from A to B. If the wire be of uniform material and cross-section, the resistance of each cm length of the wire will be the same; hence two cm. length would have twice the resistance of one cm. length. In other words, the resistance will be proportional to the length, and if AB represents the length of the wire, it may also be regarded as a graphic representation of its resistance.

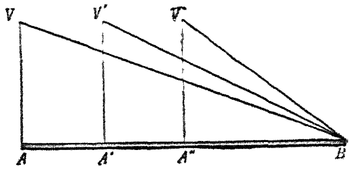


FIG. 346.

Let the potential at A be represented by AV, and that at B zero. The fall of potential along the wire will be uniform and represented by the line VB.

When the wire is shortened to A′B its resistance will be less than before, and V′B will represent the fall of potential. If the wire is still further shortened to A″B, then V″B will represent the fall of potential.

A simple experiment demonstrates that when the wire is shortened the current traversing the wire is increased: can this increase be suggested in the diagram? An increased current in the experiment is accompanied by an increase in the angle VBA in the diagram can we regard the latter as a representation of the current?

This is evidently possible if we consider not the angle itself but rather the *tangent* of the angle, for then the tangent of VBA = VA/AB, or expressed in words—

$$\text{The strength of the current} = \frac{\text{difference of potential}}{\text{resistance}}.$$

Fig. 347 is a similar diagram, which shows how the potential difference between *any* two points of a uniform wire may be indicated. The potentials at A and B are AV_1 and AV_2, and the potential difference is represented by the length $V_1 a$.

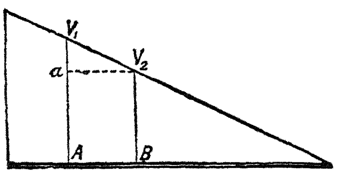


FIG. 347.

The resistance of a wire to the passage of electricity along it depends upon (i) the metal of which the wire is made, (ii) the length of the wire, (iii) the cross-section. This may be illustrated by simple qualitative experiments, but fuller consideration of the relation between resistance and current will be obtained by experiments with the metre bridge (p. 502).

EXPT. 311.—**Variation of resistance.** (1) Connect one end of a piece of German-silver[1] wire (No 22,[2] 2 metres long) to one pole of a large Bunsen cell (or an accumulator), and connect the other end to a terminal of the short thick coil of a tangent galvanometer. Connect the other terminals of the cell and the galvanometer together by means of a short copper wire. The current traversing the German-silver wire

[1] German silver is an alloy of the following metals —copper 60 parts, zinc 26 parts, and nickel 14 parts.

[2] These numbers refer to the *Standard Wire Gauge* (S.W.G.).

also traverses the galvanometer, and the tangent of the angle of deflection is a measure of the magnitude of the current Note the deflection

(ii) Break the circuit between the cell and the German-silver wire and insert into the circuit another piece of the same wire (1 metre long). The current is now opposed by the resistance of a total length of 3 metres of German-silver wire, and the current is *less* than before Evidently the resistance of a wire depends upon its length

(iii) Remove the metre length of German-silver wire, and substitute for it a metre length of copper wire (No 22, S W G). Note the deflection The current is greater than in (ii), but less than in (i), showing that the resistance of copper is less than that of German silver.

(iv) Remove the copper wire, and substitute for it a metre length of iron wire (No 22, S W.G.) The deflection observed indicates that iron is a better conductor than German silver, but not so good as copper.

(v) Remove the iron wire, and substitute for it a metre length of No 26 copper wire The deflection indicates that a thin copper wire offers more resistance than a thick one.

Resistance of liquid conductors, and therefore of voltaic cells — The fact that cells offer resistance to currents was anticipated in the previous section by recommending the use of a *large* Bunsen cell or an accumulator; the cell itself has resistance, and therefore the *total* resistance in the circuit is only partly due to the connecting wires

Just as the resistance of a wire depends upon the metal, its length, and its cross-section, so also does the resistance of a voltaic cell depend upon the materials of which it is made, and upon the length and cross-section of the liquid which the current has to traverse between the two poles of the cell. This can be readily shown by means of a modified form of the simple voltaic cell.

EXPT 312 —**Internal resistance** Pierce the axis of an ordinary cork with the supporting wire of the copper plate of a simple voltaic cell, and mount the zinc plate on a cork in a similar manner Fix the corks in the screw clamps of two separate retort-holders, so that the metal plates are vertical and just above the level of the table. In this manner the plates may be rigidly supported within a shallow glass dish, *e.g* a crystallising dish, and their distance apart and their depth of immersion within the dish varied. Fill the dish with very dilute sulphuric acid, and connect the plates to the thick coil of a tangent galvanometer by means of copper wires. Place the plates close together and observe the deflection. Separate them gradually and observe how the deflection

diminishes, showing that the resistance of the cell is increased when the length of the liquid column between the two plates is increased.

Now raise the plates slightly, or remove some of the acid by means of a pipette, so as to reduce the cross-section of the liquid column. Notice how the deflection diminishes as the cross-section of the liquid column becomes less.

The foregoing experiment indicates the advantage of using a large cell instead of a small one. The E.M.F. of the cell simply depends upon the materials used, and is quite independent of the *size*; but the resistance depends very largely upon the size, and only becomes negligible when a cell with large plates close together (such as an accumulator) is used.

FIG. 348. Expt. 313.

EXPT. 313.—**Resistance of liquids.** Fit a cork into each end of a glass tube, 40 cm. long and about 2 cm. diameter (Fig. 348). Boil the corks in paraffin wax. Cut two circular discs of sheet copper, sufficiently large to pass readily into the tube, and solder a long piece of thick copper wire to the centre of each disc. Through the centre of the corks pierce a hole sufficiently large for the copper wire to pass through while fitting tightly. Connect up a voltaic cell with a galvanometer of low resistance, and insert into the circuit sufficient thin wire to give a deflection of about 45°. Fill the long glass tube with dilute sulphuric acid (about $2\frac{1}{2}$ %), and separate the copper discs as much as possible. Break the circuit and include the column of dilute acid. Notice the diminished deflection. Push the discs nearer together, and notice how the deflection increases.

The dilute acid evidently offers resistance to the passage of the current, and the resistance depends upon the length of the column through which the current has to pass.

Application of Ohm's law.—By giving to the symbols C, E, and R their numerical values in practical units, we are able to regard the expression $C = E/R$ as a correct mathematical equation, and to use it in the solution of problems in which the numerical values of only two of the symbols are known. Thus, if the difference of potential between the ends of a wire is E volts, and if the resistance of the wire is R ohms, then the ratio E/R will be the numerical value of the current in amperes.

EXAMPLE.—The resistance of a mile of ordinary iron telegraph wire is 9 ohms, and the potential difference between its two ends is 1·25 volts. What is the magnitude of the current flowing through the wire?

$$E=1{\cdot}25,\ R=9$$

Hence $$C=\frac{E}{R}=\frac{1{\cdot}25}{9}=0{\cdot}14 \text{ ampere.}$$

As a general rule the equation $C=E/R$ is applied to the entire circuit traversed by the current, including the battery as well as the external wires, both of which offer a resistance to the passage of the current. Hence the symbol R includes both the resistance of the wire (usually termed the *external resistance*) and also that of the battery (usually termed the *internal resistance*). It is better to represent these component resistances by separate symbols, and to write the equation thus—

$$C=\frac{E}{R+r},$$

where R = the external resistance, and r = the internal resistance. Since the battery has resistance, a portion of its E.M.F. will be used in driving the current through the battery, and only the remainder of the total E.M.F. will be available for driving the current through the wire. This is rendered more evident by writing the above equation thus—

E	=	CR	+	Cr
(Total E.M.F.)		(E.M.F. used in external circuit.)		(E.M.F. used in internal circuit.)

This is represented diagrammatically in Fig. 349, where AB represents the internal resistance and BC the external resistance. AE is the total E.M.F., and Ee is the portion used up in overcoming the resistance of the cell, while BE′ represents the difference of potential between the ends of the wire. The current is represented by the tangent of the angle ECA, hence

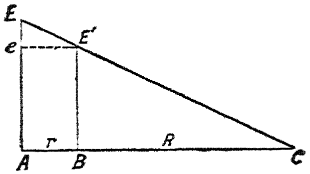


FIG. 349.

$$C=\frac{Ee}{r}, \text{ and also } C=\frac{BE'}{R};$$

or, $$Ee=Cr, \text{ and } BE'=CR.$$

Therefore $$AE=Ee+BE'=Cr+CR.$$

EXAMPLE 1.—A Grove cell has an internal resistance of 0·5 ohm, and its total E.M.F. is 1·9 volt. The poles are joined by a wire, the resistance of which is 1·5 ohm. Find the current produced and the potential difference between the terminals of the cells.

$$R=1{\cdot}5,\quad r=0{\cdot}5,\quad E=1{\cdot}9.$$

$$C=\frac{E}{R+r}=\frac{1{\cdot}9}{1{\cdot}5+0{\cdot}5}=\frac{1{\cdot}9}{2}=0{\cdot}95 \text{ ampere.}$$

Potential difference between the ends of wire $=CR=0{\cdot}95\times1{\cdot}5$

$$=1{\cdot}425 \text{ volt.}$$

EXAMPLE 2.—The total E.M.F. of a battery is 10 volts. When the poles of the battery are connected by a wire a current of 2 amperes is obtained, and the potential difference of the battery poles drops to 7·5 volts. Find the resistance of the battery and of the wire.

$$C=\frac{E}{R+r},\quad\text{or}\quad R+r=\frac{E}{C}=\frac{10}{2}=5 \text{ ohms.}$$

Potential difference between the ends of wire $=CR$;

or $$7{\cdot}5=2\times R.$$

Hence $$R=3{\cdot}75 \text{ ohms.}$$

But $R+r=5$ ohms, therefore $r=5-3{\cdot}75=1{\cdot}25$ ohms.

Divided external circuits.—When a number of conductors are arranged with their ends in contact so that a current entering at one end has several paths open to it, they are said to be arranged **in parallel.** Fig. 350 represents a voltaic cell AB, with its poles connected together by two wires in parallel, the resistances of which are r_1 and r_2. The potential difference between the ends of the wires is the same in both cases; let it be denoted by E.

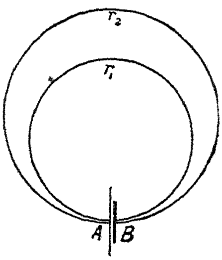


FIG. 350.

If c_1 is the current traversing the wire r_1, then

$$c_1=E/r_1.$$

If c_2 is the current traversing the wire r_2, then

$$c_2=E/r_2.$$

The total current C traversing the circuit is equal to the sum of c_1 and c_2;

or, $$C=c_1+c_2=\frac{E}{r_1}+\frac{E}{r_2}=E\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=E\left(\frac{r_2+r_1}{r_1r_2}\right)=\frac{E}{\dfrac{r_1r_2}{r_1+r_2}}$$

Hence **the combined resistance of two wires in parallel is equal to their product divided by their sum.**

EXAMPLE.—The poles of an accumulator (E.M.F. = 2 volts) are connected by two wires in parallel, the resistances of which are 5 ohms and 6 ohms respectively. If the internal resistance of the accumulator is 0·1 ohm, find the total current traversing the circuit.

The external resistance $=\frac{r_1 r_2}{r_1+r_2}=\frac{5\times 6}{11}=\frac{30}{11}=2{\cdot}73$ ohms approximately.

The total resistance $=2{\cdot}73+0{\cdot}1=2{\cdot}83$ ohms.

The total current $=\frac{E}{R}=\frac{2}{2{\cdot}83}=0{\cdot}707$ ampere.

Special case of conductors in parallel.—If the component resistances in a divided external circuit are equal to one another, then the above formula may be considerably simplified. Thus, if $r_2=r_1$, then

$$\frac{r_1 r_2}{r_1+r_2}=\frac{r_1^2}{2r_1}=\frac{r_1}{2}.$$

We may imagine the two wires to be merged into one wire of twice the cross-section of either; the resistance of this thicker wire would be half that of the thinner wire, showing that the resistance of a wire varies inversely as its cross-section.

The Wheatstone net.—Let the points A and C (Fig. 351) be connected by two conductors ABC and ADC in parallel. The current entering at A divides into two portions, c_1 and c_2, which rejoin at C. The potential falls gradually along each branch; and a point D, in ADC, may be found which has the same potential as the point B. The position of D may be determined by connecting the two points through a galvanometer G, and adjusting the point of contact at D until there is no deflection. Under this condition, since no current passes through G, the currents c_1 and c_2 are individually uniform along the conductors ABC and ADC.

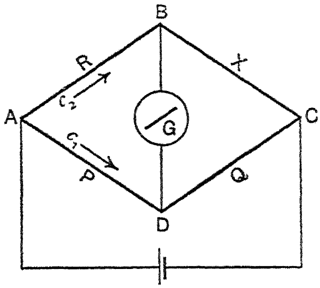


FIG. 351.—The Wheatstone net.

If V_A, V_B, V_C, and V_D are the potentials of the points A, B, C,

and D respectively; and if P, Q, R, and X are the resistances of AD, DC, AB, and BC respectively, then, by Ohm's law,

$$V_A - V_B = c_2 R,$$

$$V_A - V_D = c_1 P.$$

But $$V_A - V_B = V_A - V_D;$$

therefore $$c_2 C = c_1 P; \quad \text{or} \quad c_1/c_2 = R/P.$$

Similarly $$c_2 X = c_1 Q; \quad \text{or} \quad c_1/c_2 = X/Q.$$

Hence $$X/Q = R/P,$$

or $$\frac{X}{R} = \frac{Q}{P}.$$

This result explains how it is possible, in an experiment with four separate resistances, to determine any one of them if the other three are known; or to obtain the ratio of any two of them if the ratio of the remaining two is known.

The metre bridge—The metre bridge (Fig. 352) is the simplest application of the previous paragraph to the measure-

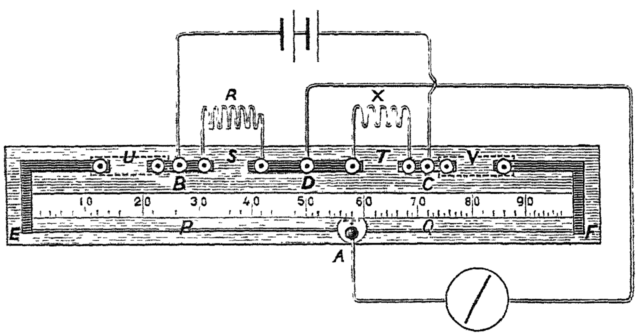


FIG 352—The metre bridge.

ment of an unknown resistance. It consists of a uniform German-silver (or iridio-platinum) wire, one metre long, stretched alongside a metre scale, and with its ends soldered to stout copper strips E and F. There are four gaps between copper strips fixed along the other edge of the board, but in simple measurements the gaps U and V are closed by copper strips held in position by binding screws. The resistances R and X which are to be compared are

attached to binding screws as shown. The galvanometer circuit is closed by depressing the knob A, which makes contact between a knife-edge and the bridge wire. Having found by trial the position of A which gives no deflection, then, since the wire is uniform and the resistance of any portion is proportional to its length, the ratio R/X will be equal to the ratio of the lengths P and Q.

EXPERIMENTS WITH THE METRE BRIDGE.

EXPT. 314.—**The Construction of a 1-ohm coil.** Connect up the apparatus as shown in Fig. 352. Measure out 1 metre of manganin wire (No. 22), and remove the silk covering from the ends. Insert the wire in the gap T, and a standard 1-ohm coil in the gap S. Find the point of contact on the bridge wire which gives no deflection in the galvanometer. Calculate the resistance of the wire, and note that the resistance is rather greater than one ohm. Shorten the wire slightly, and again determine its resistance. Repeat this until no deflection is obtained when the point of contact is exactly at the middle of the bridge wire.

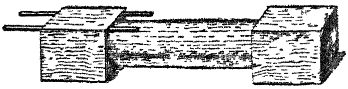


FIG. 353.

Before removing the wire from the bridge, bend the ends of the wire to a right angle just where the wire leaves the binding screws. Solder to the end of each wire a length (10 cm.) of thick copper wire, adjusting the wire so that the soldering terminates at the points where the wire is bent. Carefully wash the soldered joints in water. Insert the copper wires through the holes bored in the ends of the wooden cylinder provided. Double the wire together at its middle point, then wrap it round the cylinder, and tie it in position with cotton thread. Re-determine the resistance as accurately as possible, and write this in pencil on the cylinder.

EXPT. 315.—**The resistance of a wire varies directly as its length.** Cut two pieces of different lengths of No. 28 German-silver wire. Bare the ends of the wires, and bend the bared ends to a right angle. Measure the length between the bends of each wire. Measure the resistance of each wire, taking care that the wire leaves the binding screws of the bridge just where the bend is situated. Prove that the ratio of the lengths is equal to the ratio of the resistances.

EXPT. 316.—**The resistance of a wire varies inversely as the cross-section.**—Measure the resistance (R_1) of 1 metre of German-silver wire (No. 28 S.W.G.), and measure its diameter (d_1) by means of a micrometer

screw gauge Measure the resistance (R_2) of 1 metre of German-silver wire (No. 22 S.W G), and also its diameter (d_2).

If s_1 and s_2 are the cross-sections of the wires, and r_1 and r_2 their *radii*, then $\frac{s_1}{s_2}=\frac{\pi\times r_1^2}{\pi\times r_2^2}=\frac{r_1^2}{r_2^2}$.

Prove that $\frac{R_1}{R_2}=\frac{s_2}{s_1}=\frac{r_2^2}{r_1^2}$.

EXPT 317.—**The resistance of two wires in parallel.** Connect in parallel the two German-silver wires used in the previous experiment, and measure the resistance (R) between the extreme ends. Prove that

$$R=R_1R_2/(R_1+R_2)$$

EXPT 318.—**The resistance of a wire depends upon its temperature**—Fig. 354 represents a spiral of iron wire (No 28), about 2 metres long, with its ends soldered to short pieces of thick copper wire which pass through a cork fitted in a glass boiling tube. The apparatus is fitted with a thermometer and stirrer, and the tube is nearly filled with paraffin oil

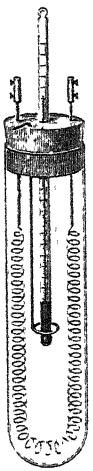


FIG 354 —To illustrate Expt. 318

Place a deep beaker full of water on a tripod, and fix the tube containing the wire spiral in the water. Connect up the ends of the spiral by means of thick copper wires to the binding screws of the bridge. After the tube has been in the water for about five minutes, stir the paraffin oil, and note the temperature Measure the resistance of the spiral Slowly warm the water, and frequently stir the oil When the temperature has risen about 10° C., remove the flame, stir the oil, and repeat the observations of temperature and resistance. Repeat these readings at higher temperatures. From the first and the last observations determine how much a wire of 100-ohms resistance would increase in resistance due to a change in temperature of 1° C., thus,

$$\frac{R_2-R_1}{R_1}\times\frac{100}{(T_2-T_1)}.$$

Enter your observations thus:

Temperature.	Resistance.	Per cent increase due to rise of 1° C

Specific Resistance.—The specific resistance of any metal is the resistance of a cube of the metal, each edge of the cube being 1 cm. long. Such a cube may be regarded as a wire 1 cm. long and 1 sq. cm. cross-section, if the dimensions of the wire are altered to l cm. in length and s sq. cm. in cross-section, then

$$\text{Resistance (R)} = \text{Specific resistance } (k) \times \frac{l}{s},$$

or,
$$R = \frac{kl}{\pi r^2}$$

In order to determine k for any metal, it is necessary to measure the length, cross-section, and resistance of a piece of that metal in the form of a wire.

EXPT. 319.—**The specific resistance of a metal.**—Measure the length, cross-section, and resistance of a piece of manganin wire by the method described in Expt. 314. Enter your results thus.

Metal	Length (l)	Cross-section (πr^2)	Resistance (R)	$\frac{R \times \pi r^2}{l}$
Manganin.				

COMPARISON OF THE E.M.F OF VOLTAIC CELLS

The potentiometer.—The principle of the potentiometer has been described previously (p. 494) in the demonstration of Ohm's Law, and a more complete statement may be given here with advantage.

In Fig. 355 AB represents a long uniform wire joined in series with a battery E of constant E.M.F., a tangent galvanometer G_1, and an adjustable resistance R. If the positive terminal of another cell E_2 is joined to A, the positive pole of E_2 will have the same potential as A; and the P.D. between A and the free end k of a

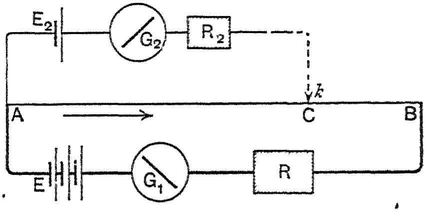


FIG. 355.—The potentiometer.

wire attached to the negative pole of E_2 will be equal to the P.D. between the terminals of E_2. If the E.M.F. of the battery E is sufficiently great, a point C may be found on AB which has the same potential as *k*; and if *k* is brought into contact with this point, the tendency of E_2 to send a current through a sensitive galvanometer G_2 in one direction is just balanced by the tendency of the P.D. between A and C to send a current through C_2 in the opposite direction, and no current therefore traverses this circuit. Since the wire AB is uniform, the length AC is a measure of the E.M.F. of E_2. The point C is determined by adjusting the point of contact so that there is no deflection in the galvanometer G_2. A high resistance R_2 is inserted to protect the galvanometer G_2, and also to prevent any momentary current of sufficient strength to cause polarisation in the cell E_2.

EXPT. 320.—**Variation of the E.M.F. of a cell, due to polarisation.** Fit up a potentiometer as shown in Fig. 355. Measure the length AC required to balance the E.M.F. of a Leclanché cell placed at E_2. Disconnect the Leclanché cell, and *short-circuit* its terminals through a low resistance, *e.g* 2-5 ohms. Allow this to remain for five minutes, then disconnect it and at once measure again its E.M.F. by means of the potentiometer. Again short-circuit it through the low resistance, and, after five minutes, repeat the measurement of its E.M.F. Now allow the cell to remain on *open circuit*, and measure its E.M.F. every few minutes. Tabulate the observations, and describe whether they afford information as to loss of E.M.F. due to polarisation, and as to the subsequent recovery of the cell.

EXPT. 321.—**Comparison of the E.M.F. of voltaic cells (potentiometer method).** Connect up the apparatus as in Fig. 355. Frequently observe the deflection in the tangent galvanometer, and keep it constant by adjusting R (if necessary). Note the length of the wire AC required to balance the E.M.F. of each cell when placed successively in the position E_2. Enter your observations in the following manner:

Type of Cell.	Deflection in Galvanometer	Length of Wire (AC)
1 Daniell		$l_1 =$
2. Leclanché		$l_2 =$
3.		
4		

Express the E.M.F. of each cell in terms of that of the Daniell thus—

$$\frac{\text{Leclanché}}{\text{Daniell}}=\frac{l_2}{l_1}=$$

If two cells (E.M.F. denoted by E_1 and E_2) are connected in series with a tangent galvanometer, then $\tan\alpha$ is proportional to (E_1+E_2) If E_2 is then reversed, *tan α* will be less than before, and will be proportional to E_1-E_2 (assuming that E_1 is greater than E_2). Hence

$$\frac{\tan\alpha_1}{\tan\alpha_2}=\frac{E_1+E_2}{E_1-E_2},$$

or,

$$\frac{E_1}{E_2}=\frac{\tan\alpha_1+\tan\alpha_2}{\tan\alpha_1-\tan\alpha_2}.$$

EXPT. 322.—**Comparison of the E.M.F. of voltaic cells (method of sum and difference).**[1] (1) Connect up a Daniell cell (E.M.F. $=E_2$) with another cell (E.M.F. $=E_1$) in series, with a commutator, resistance, and tangent galvanometer. Note the deflection of both ends of the needle, and also when the current is reversed. Reverse the cell having the lower E.M.F., and repeat the observations Enter your results thus:—

Comparison of Leclanché (E_1) *and Daniell* (E_2).

Cells.	Deflections		Mean Deflection (α)	$\tan\alpha$
	East End.	West End		
In conjunction (E_1+E_2)	(i.)		$\alpha_1=$	$\tan\alpha_1=$
	(ii.)			
In opposition (E_1-E_2)	(i.)		$\alpha_2=$	$\tan\alpha_2=$
	(ii.)			

Calculate the value of the ratio E_1/E_2 by means of the above equation.

The current generated in a simple circuit (consisting of a high resistance and a mirror galvanometer) is, by Ohm's law, proportional to the E.M.F. of the cell. The E.M.F. of different types of cell may be compared by inserting them successively in such a

[1] This method is suitable only when the E.M.F of the cells differs by at least 20 %.

circuit, and comparing the current generated in each case. If the circuit includes a high resistance the internal resistance of the cells may be neglected; and if a sensitive mirror galvanometer be used, the deflections observed may be regarded as proportional to the current passing through the instrument.

EXPT. 323.—**Comparison of the E.M.F. of volatic cells (direct deflection method).** Connect in series the galvanometer, high resistance, commutator, and one of the cells supplied. Adjust the high resistance until a moderate deflection is observed. Note the deflection, reverse the current, and again note the deflection. Enter the observations thus:

Type of Cell.	Deflection		Mean Deflection
	Right	Left	

Calculate the E.M.F. of the cells in terms of that of the Daniell.

GROUPING OF CELLS.

The various methods of grouping cells together so as to form a battery have been described already on p. 462.

Cells in series.—If n cells are connected together in series, and if E and r are the E.M.F. and the internal resistance of each cell, then

$$\text{The total E.M.F.} = nE.$$

$$\text{,, internal resistance} = nr.$$

$$\text{Then, by Ohm's Law, } C = \frac{nE}{R+nr} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Fig. 356 represents a battery of two cells in series. The continuous and the thick dotted lines are the potential diagrams when the circuit is open and closed respectively. The lengths AB and BC represent the internal resistances of the cells, and CD represents the external resistance. AV (or CV′) is the total E.M.F. The current is represented by the ratio AV/AD (*i.e.* by $\tan\theta$). Before the circuit is closed the potential difference between the terminals is CV′, but as soon as the circuit is closed the potential difference at the terminals falls to Cv. The

remainder of the total E.M.F. (viz. V′v) is used up in overcoming the internal resistance of the two cells and the resistance of the connecting wire BB. (The latter is usually very small, and may be disregarded.) In this case equation (1) becomes $C = \frac{2E}{R + 2r}$.

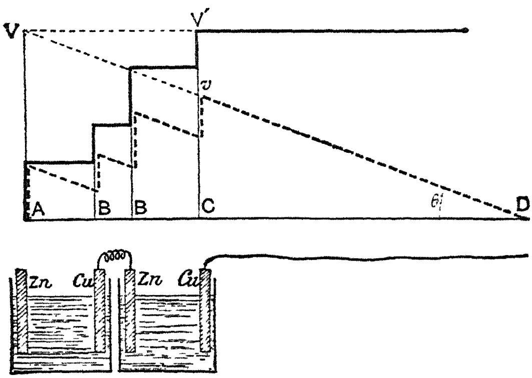


FIG. 356 – Potential diagram of a simple circuit containing two cells in series.

Special Case.—Suppose that one of the n cells is accidentally reversed, so that it tends to send a current in the opposite direction. What will be the final result? There are $n-1$ cells tending to send a current in one direction, and with an E.M.F. $= (n-1)E$, while there is one cell tending to reverse the current with an E.M.F. $= E$. The resultant E.M.F. $= (n-1)E - E = (n-2)E$.

Hence, by Ohm's Law, $C = \frac{(n-2)E}{R + nr}$.

Cells in parallel.—If m cells are connected together in parallel, the E.M.F. will be the same as that of one cell. The arrangement will be equivalent to one large cell, the plates of which are m times as large as those of a single cell, hence the internal resistance will $= \frac{r}{m}$ (where $r =$ the resistance of a single cell).

By Ohm's Law,
$$C = \frac{E}{R + \frac{r}{m}} \qquad \text{. . . . . (2)}$$

If the cells are arranged in m rows, each row containing n cells in series, then the resistance of each row is nr. The effect of having m rows side by side will be equivalent to enlarging the

plates of each cell m times, and the total internal resistance will be $\frac{nr}{m}$. The total E.M.F. will be nE (*i.e.* the same as n single cells in series).

By Ohm's Law,
$$C=\frac{nE}{R+\frac{nr}{m}} \quad \ldots\ldots \qquad (3)$$

Arrangement of cells for maximum current—It is clear, from equation (1), when r is small compared with R, that the current obtained is approximately proportional to the number of cells used. But if R is small compared with r, then the current is scarcely increased by an increase in the number of cells, since the total resistance $(R+nr)$ will be increased almost in the same proportion as the E.M.F., in this case it is advantageous to connect the cells in parallel so as to reduce the internal resistance. It can be proved mathematically that a maximum current is obtained when the cells are so arranged that the internal resistance is equal to the external resistance.

EXAMPLE 1.—The poles of a battery of three cells (each having an internal resistance = 2 ohms, and an E.M.F. = 1 volt) are joined by a wire having a resistance of 0·5 ohm. Find the magnitude of the current (i) with three cells in series, (ii) with two cells in series, and (iii) with three cells in parallel.

$$\text{(i)}\quad C=\frac{nE}{R+nr}=\frac{3}{0{\cdot}5+6}=\frac{3}{6{\cdot}5}=0{\cdot}46 \text{ ampere.}$$

$$\text{(ii)}\quad C=\frac{nE}{R+nr}=\frac{2}{0{\cdot}5+4}=\frac{2}{4{\cdot}5}=0{\cdot}44 \text{ ampere.}$$

$$\text{(iii)}\quad C=\frac{E}{R+\frac{r}{n}}=\frac{1}{0{\cdot}5+\frac{2}{3}}=\frac{1}{1{\cdot}16}=0{\cdot}86 \text{ ampere.}$$

Evidently two cells will give approximately the same current as three cells, and a much greater current is obtained if the three cells are in parallel.

EXAMPLE 2.—Repeat the calculations in Example 1, but making the external resistance much greater. Let R = 20 ohms.

$$\text{(i)}\quad C=\frac{3}{20+6}=\frac{3}{26}=0{\cdot}115 \text{ ampere.}$$

(ii) $C=\frac{2}{20+4}=\frac{2}{24}=0{\cdot}083$ ampere.

(iii) $C=\frac{1}{20+\frac{2}{3}}=\frac{1}{20{\cdot}16}=0{\cdot}049$ ampere.

(iv) With a single cell, $C=\frac{1}{20+2}=\frac{1}{22}=0{\cdot}045$ ampere

In this case it is evidently an advantage to increase the number of cells in series, and a single cell gives almost the same current as several cells connected in parallel.

EXPT. 324.—**Arrangement of cells for high and low resistances.**—Connect up in series one Leclanché cell, a resistance of 100 ohms, the commutator, and a tangent galvanometer. Note the deflection. Repeat with two cells, in series, with three cells in series, and with three cells in parallel.

Repeat these observations, but use a 5-ohm resistance instead of the 100-ohm resistance. Enter your observations in the following manner

Leclanché Cells	Resistance of Coil.	Deflection		Mean Deflection (a)	tan a.
		East End.	West End		
1 cell	5 ohms				
2 cells in series	"				
3 cells in series	"				
3 cells in parallel	"				

Note which arrangement of cells gives a maximum current through the high resistance, and through the low resistance.

EXERCISES ON CHAPTER XXXVIII.

1. How would you show that a copper wire is a better conductor of electricity than a similar iron wire? Make a sketch of the apparatus you would use.

2. Describe a voltaic battery, and state how it can be shown that it is not a constant battery.

3. Describe the construction of a Daniell's cell. If a Daniell and a Grove were each made to send a current through a long thin coil, which would give the stronger current, and why?

4. Describe, as fully as you can, some one method of comparing the strength of two different batteries.

5. State Ohm's Law, and explain the terms used.

An incandescent electric lamp takes a current of 0·5 ampere when connected to a circuit of 100 volts. What is the resistance of the lamp?

6. A single cell is connected to a galvanometer by long fine wires, giving a deflection of 10°. If a second similar cell is connected in parallel with the first, the deflection becomes 11°, but if connected in series, the deflection is increased to 19°. Explain this.

7. How is polarisation prevented in the Daniell cell? How does a large Daniell cell differ from a small one in respect of (1) electromotive force, (2) resistance?

8. It is intended to set up 100 Grove cells in series, but by mistake 10 cells are arranged in opposition to the rest. What is the relation of potential difference of the terminals on open circuit to that which would have been obtained if the mistake had not been made?

9. What is meant by the Electromotive Force of a voltaic cell? If you were given two cells, how would you test which had the greater electromotive force?

10. You are given two voltaic cells which are identically alike. The current traversing a simple circuit, including the two cells in series, is not quite twice as great as that obtained when only one of the cells is used (the circuit being otherwise the same). Why is this?

11. The resistance of two wires in series is 15 ohms, and in parallel $3\frac{3}{5}$ ohms. What is the resistance of each wire?

12. A wire has a resistance of 20·5 ohms. What must be the resistance of a wire joined in parallel with it so that the combined resistance is 20 ohms?

13. A tangent galvanometer, resistance 6·5 ohms, is connected in parallel with a resistance of 3·25 ohms. The reduction factor of the galvanometer is 0·30. If the deflection is 30°, find the total current traversing the circuit.

14. The +ve poles of two Daniell's cells, one of them twice as large as the other, are connected together by a short wire, and the circuit is completed by connecting the −ve poles together by means of a long thin wire. Will any current traverse the circuit? Give reason for your answer.

15. A cell having an E.M.F. of 2 volts and a resistance of 0·5 ohm is connected up with three lengths of wire having resistances of 1, 2, and 3 ohms respectively, the wires being in series. Find the difference in potential between the ends of the middle wire.

16. Being given 4 voltaic cells, each of E.M.F. 2 volts and resistance 0·2 ohm, find the currents they would produce in external resistances of 0·1 ohm and 1 ohm respectively when the cells are connected

up (1) in parallel and (2) in series Find also the differences of potential between the ends of each external resistance for each arrangement of the cells.

17. A storage cell with a single pair of plates connected by a wire of 0·8 ohm resistance gives the same current as a similar cell, with plates twice as broad, twice as deep, and twice as far apart, which are connected by a resistance of 0·9 ohm Find the resistance of each cell.

Why are the plates of a storage cell usually of large surface?

18. The E.M.F. between the poles of a battery is 12 volts when the external circuit is "open," and 10 volts when it is closed by a resistance such that a current of 6 amperes is passing Find the resistance of the battery.

19. The zinc pole of a Daniell cell being joined to the platinum pole of a Grove cell, the other poles are connected up with a tangent galvanometer, and produce a current of 0·5665 ampere, the zinc pole of one is next joined to the zinc pole of the other, and the +ve poles are connected with the galvanometer by the same wires as before, whereby a current is produced whose value is 0·0875 ampere. Deduce the ratio of the E.M.F. of the two cells

20. A battery of which the E.M.F. is 1 volt and the internal resistance 1 ohm, is connected to a galvanometer of which the resistance is 2 ohms. What is the current in the circuit? How is the current through the galvanometer affected by joining its terminals by a wire of 2 ohms resistance?

21. Upon what factors does the internal resistance of a battery depend?

A current is sent through a wire of 0·5 ohm resistance by attaching its ends to the terminals of a Daniell cell of internal resistance 0·5 ohm. What is the resistance of a second Daniell if when it is connected in series with the first the current is unaltered?

In what proportion would the current through the wire alter if the cells were joined to it in parallel? Explain how you obtain your results.

22. Four cells, each of 2 volts E.M.F., and 0·1 ohm internal resistance, are used to send a current through a wire of resistance 0·1 ohm. Compare the currents in the wire when the cells are (i) in series, (ii) in two parallel rows, each with two in series, (iii) all parallel

23. A galvanometer connected (*a*) in series, (*b*) in parallel, with a resistance of 3 ohms and a battery of constant E.M.F and negligible resistance, indicates currents which are in the ratio of 3 to 4 Find the resistance of the galvanometer.

24. A wire is formed into a circle, 1 foot in diameter, and two points, A and B, a quarter of the circumference apart, are connected to the poles of a battery of E.M.F. 2 volts, and resistance 5 ohms. If 1 foot of the wire have a resistance of 6 ohms, find the current in the battery and in the two parts of the wire.

25. Four wires, AB, BC, CD, and DA, are arranged so as to form a rectangle, and their resistances are 1, 2, 3, and 4 ohms respectively. The opposite corners A and C are then connected to a voltaic cell of E.M.F. 2 volts. If, as the result, the difference of potential between A and C is 1·4 volts, determine the difference of potential between B and D.

Show that if B and D were connected by a thick copper wire of no appreciable resistance, the current in AB would be four times that in AD.

26. A telegraph line, including instruments, has a total resistance of 2000 ohms and is to be worked by Daniell cells. If the internal resistance and E.M.F. of each cell is 8 ohms and 1·07 volts respectively, how many cells will be required in order to transmit a current of ·025 ampere?

27. A reflecting galvanometer of 250 ohms resistance is shunted with 25 ohms. A cell of negligible resistance and an E.M.F. of 1·5 volts is connected in series with the galvanometer through a resistance of 10,000 ohms. A deflection of 200 scale divisions is observed. What is the sensibility of the galvanometer (*i.e.* the current required to give a deflection of one scale division)?

28. The resistance of 100 metres of copper wire (No. 24 S.W.G.; diameter$=0·0559$ cm.) is 6·63 ohms at 0° C. What is the specific resistance of the copper?

29. A column of mercury 106·3 cm. long and 1 sq. millimetre cross-section has a resistance of 1 ohm at 0° C. What is its specific resistance?

30. Thirty accumulators, each having an E.M.F. of 2·1 volts and a resistance of 0·002 ohm, are employed to feed incandescent lamps (joined in parallel). If the lamps require 45 volts and 1·25 amperes each, what is the maximum number of lamps that can be employed?

31. Edison glow-lamps requiring 108 volts potential difference, and 0·72 ampere each, are required to be fed by accumulators each having 2·1 volts E.M.F. and 0·0017 ohm resistance. What is the least number of such accumulators arranged in series that must be employed to feed 200 of these lamps in parallel?

CHAPTER XXXIX.

CHEMICAL EFFECTS OF AN ELECTRIC CURRENT.

Electrolysis.—All conductors of electricity may be divided into two groups, namely, (i) metals (solid or molten), mercury, and liquids which are not decomposed when a current passes through them, and (ii) those compounds, whether fused or in solution, which undergo decomposition by the current.

The latter are termed **electrolytes**: and when traversed by an electric current, they are said to undergo **electrolysis**. Dilute sulphuric acid, hydrochloric acid, and chemical salts (*e.g.* copper sulphate, sodium chloride, sodium sulphate, etc.) are typical electrolytes. Perfectly pure liquids, *e.g.* water, sulphuric acid, and alcohol, are not capable of electrolytic decomposition.

A current is conveyed to and from an electrolyte by immersing in it rods or plates of a metal or of carbon; these are termed the **electrodes**. That by which the current enters is termed the **anode**, and that by which it leaves is termed the **kathode**. The elements (or groups of elements) liberated are termed **ions**; the ion liberated at the anode is termed the **anion**, and that liberated at the kathode is termed the **kation**.

EXPT. 325.—**Electrolytic decomposition of solutions.** (i) Connect two short lengths of platinum wire to copper wires attached to the poles of a battery. Dip the wires into a beaker of dilute sulphuric acid. Notice how bubbles of gas are liberated from the wires.

(i) Dip the platinum wires into a solution of copper sulphate. Allow the current to pass for a few moments, and observe how the kathode becomes coated with a layer of copper. Observe what takes place at the anode. Repeat the experiment, using copper electrodes, and note whether the effects observed are the same as before.

Fig. 357 represents a simple form of apparatus (called a **water voltameter**) for the electrolysis of water. The vessel supported in the stand is made from a glass funnel, and the lower end is closed with a cork and a layer of paraffin wax. The wires through the cork terminate in strips of platinum foil. The vessel and the test tubes are filled with dilute sulphuric acid; and, as the current passes, hydrogen and oxygen are slowly liberated from the kathode and anode respectively.

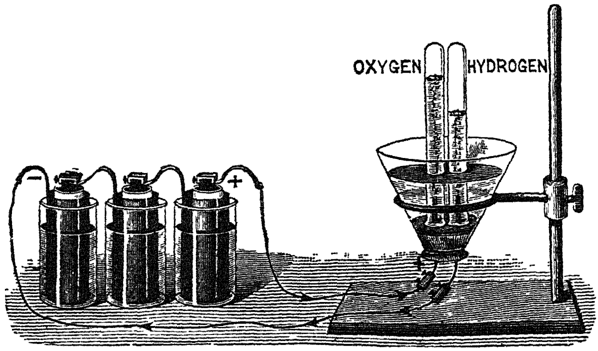


FIG 357 —The electrolysis of water

EXPT. 326 —**Electrolysis of water.** Nearly fill the funnel of a water-voltameter with dilute sulphuric acid; fill the test-tubes with a similar acid, and invert them over the platinum strips. Connect the copper wires to the terminals of a Bunsen battery (of at least two cells). Notice how the kation accumulates twice as rapidly as the anion. Break the circuit, and remove the test-tubes (carefully closing the end with the thumb before removing it from the acid). Verify that the kation is hydrogen, and that the anion is oxygen.

Theory of electrolysis.—The electrolysis of water may be represented thus:

H_2O		H_2	+	O.
Water	is decomposed into	2 vols. *Hydrogen*	and	1 vol. *Oxygen.*

The theoretical explanation of the electrolysis of water is, however, not so simple as would appear from the above equation, since this does not take into consideration the presence of the sulphuric

acid which is essential to the experiment. The potential difference set up between the electrodes causes a breaking-up of the H_2SO_4 molecules, thus

$$H_2SO_4 = SO_4 + H_2.$$

The hydrogen is drawn towards the kathode and is there liberated, while the *sulphion* (SO_4) is drawn towards the anode where it acts upon a molecule of water, thus

$$SO_4 + H_2O = H_2SO_4 + O,$$

and in this manner re-forms sulphuric acid and liberates oxygen.

The student will remember that the accumulation of hydrogen on the copper plate of a simple voltaic cell causes *polarisation* (p. 458), and an opposing E.M.F. (or *back* E.M.F.) is thereby set up, since the hydrogen is a readily-oxidizable element, and behaves in a similar manner to the zinc plate of a simple voltaic cell.

In the water voltameter this back E.M.F. is set up. If E = total E.M.F. of battery, and E' = back E.M.F. in the voltameter, the resultant E.M.F. for the complete circuit is $E - E'$, and the magnitude of the current obtained depends directly upon the magnitude of this resultant E.M.F. If $E' = E$ then no current will be obtained. In the case of a water voltameter $E' = 1{\cdot}47$ volts, so that the E.M.F. of the battery must be greater than this in order to electrolyse water. This explains why a single Bunsen cell (E.M.F. = 1·9 volts) will electrolyse water, and why it is necessary to use at least two Daniell cells (E.M.F. = 1·07).

If the dilute acid is replaced by copper sulphate, copper is liberated at the kathode instead of hydrogen, but the changes at the anode are the same as in the water voltameter. Since copper is not so readily oxidized as hydrogen, it follows that the back E.M.F. in this case is less than that of the water voltameter. This holds good if platinum electrodes are used, but the conditions are altered if copper electrodes are used; oxygen is not necessarily set free, since it may combine with the copper anode and form copper oxide.

$$Cu + O = CuO.$$

In the presence of sulphuric acid, the CuO will dissolve to form copper sulphate.

$$CuO + H_2SO_4 = CuSO_4 + H_2O.$$

The extent of this reaction depends upon the amount of acid present, but it is assured if the acid is added to the original electrolyte. Moreover, this reaction is the more important since the

back E.M F. will no longer exist, for the chemical energy absorbed in removing the copper from the solution at the kathode is restored by the solution of an equal weight of copper at the anode. It will also be observed that the re-formation of $CuSO_4$ at the anode ensures the strength of the solution being maintained

Faraday's laws of electrolysis—Faraday, in 1833, fully investigated the phenomena of electrolysis, and deduced the following laws:

(i) **The mass of an ion set free by a current is proportional to the quantity of electricity which has passed.**

Thus, the amount of chemical action due to a current depends directly upon both the strength of the current and also upon the time; a weak current flowing for a given time is equivalent to a strong current flowing for a relatively shorter time.

(ii) **If several different electrolytes are included in the same circuit, the relative masses of the liberated ions are proportional to their chemical equivalents.**

The chemical equivalent of an element is the weight of it which will combine with, or replace, 1 part by weight of hydrogen It is numerically equal to the atomic weight of the element, compared to that of hydrogen, divided by the valency, the valency being the number of hydrogen atoms which will combine with, or are replaced by, one atom of the element.

Electro-chemical equivalents.—The electro-chemical equivalent (E.C.E.) of an element is the weight in grams deposited by the unit quantity of electricity (1 coulomb). The accurate determination of the E.C E. of at least one element is important, since by the second law of Faraday this determination may be used for the purpose of calculating the E.C.E. of all other elements. Lord Rayleigh has found that one coulomb of electricity deposits 0·001118 gm. of silver. A current of one ampere flowing for one second will deposit this weight of silver: this constitutes a very useful definition of the ampere.

Since the chemical equivalent of silver is 107·07, the E C.E. of hydrogen is $0{\cdot}001118/107{\cdot}07 = 0{\cdot}00001044$. In a similar manner the E.C.E. of any other element may be calculated.

ELECTRO-CHEMICAL EQUIVALENTS.

Element	Chemical Equivalent	E C E (grams per coulomb)
Silver	107 07	0 001118
Hydrogen	1 00	0 00001044
Oxygen	7 935	0 00008285
Copper	31 54	0 0003293
Nickel	29 12	0 0003040
Gold	65·21	0 0006808

EXAMPLE —If the E C.E. of nickel be 0·000304, calculate how much electricity is required to give a coating of nickel 0·1 mm. thick to a surface of 1000 sq. cms. (density of nickel = 8·8 gm per c.c.).

$$\text{Volume of nickel} = 1000 \times 0{\cdot}01 = 10 \text{ c c.}$$
$$\text{Mass} \quad ,, \quad ,, \quad = 10 \times 8{\cdot}8 = 88 \text{ grams.}$$

$$\text{Quantity of electricity required} = 88 / 0{\cdot}000304 = 289{,}600 \text{ coulombs.}$$

Voltameters —The accuracy with which the E.C.E of several elements is known enables the process of electrolysis to be used for the measurement of current. the method is particularly useful in the case of very weak currents Any appliance devised for this purpose is termed a voltameter.

(1) **The copper voltameter** —Fig. 358 represents a convenient form of this voltameter: the two outer plates of copper form the anode, and the central plate is the kathode, which should be much smaller than the anode plates. The plates hang from copper wires C which are supported by two vulcanite rods, V and V′. The solution used is a 15% solution of copper sulphate, to each litre of which 5 c.c. of concentrated sulphuric acid have been added The kathode should be sufficiently large to allow 50 sq. cm. of surface for each ampere of current.

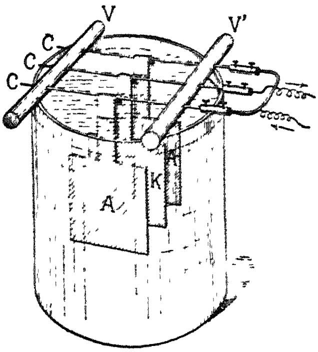


FIG 358 —A voltameter

(ii) **The water voltameter** (Fig. 359)—Two platinum wires are fixed through a rubber stopper and terminated in platinum strips immersed in dilute sulphuric acid (25 %). The observed volume of gases must be corrected for temperature and pressure, and also for the vapour tension of water. As the gases are appreciably soluble in water, the current should be passed through the voltameter for some time before the evolved gases are collected for measurement.

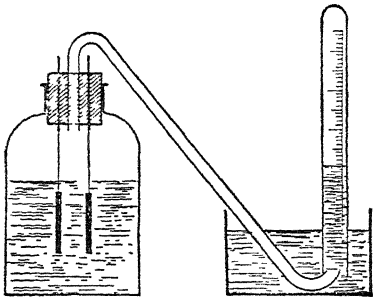


FIG 359—A water voltameter.

The mass of hydrogen liberated by one ampere in one second is $1{\cdot}044 \times 10^{-5}$ gm.; since the density of hydrogen at 0° C. and 76 cm pressure is 0·0000896 gm. per c.c., this mass of gas will occupy 0·1165 c.c. The volume of oxygen liberated under the same conditions is 0·0582 c.c. Hence the total volume of mixed gases set free by one ampere in one second is 0·1747 c.c.

EXPERIMENTS WITH A COPPER VOLTAMETER.

EXPT. 327—**Determination of the reduction factor of a tangent galvanometer.** Thoroughly clean the copper plates with sand-paper. Connect up the battery, voltameter (V), adjustable resistance (R), commutator (C), and tangent galvanometer (T.G.), as shown in Fig. 360. Adjust R until a convenient deflection is obtained. Break the circuit, remove the kathode, wash it in distilled water, then in alcohol, and dry it quickly over a spirit flame. Weigh it accurately, and replace it in position. Note the time by your watch at the instant when the circuit is completed by means of the commutator. Note the deflection, reading both ends of the pointer; keep it constant by adjusting R if necessary. After about 15 minutes, quickly reverse the commutator, and again read the deflection. After another interval of about 15 minutes, note the time and instantly break the circuit

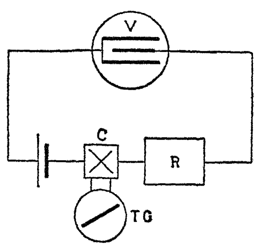


FIG. 360.

at C. Remove the kathode, wash and dry it as before, and weigh it accurately.

If t = duration of experiment, in seconds,

W = wgt. of copper deposited,

α = average deflection,

C = current, in amperes ;

then $C = W/(0{\cdot}0003293 \times t)$

and $K = W/(0{\cdot}0003293 \times t \times \tan \alpha)$.

EXPT. 328.—**Verification of Faraday's first law of electrolysis.** Connect up a battery, copper voltameter, adjustable resistance, commutator and tangent galvanometer as in the previous experiment. Adjust the resistance to give a deflection of about 30°. After weighing the kathode allow a steady current to pass for an accurately measured period of time, *e.g.* 30 minutes, taking the same precautions as before. Determine (i) the weight of copper deposited, and (ii) the product $K \tan \alpha \times$ time. Repeat the experiment, but reduce the resistance so that a deflection of about 50° is obtained, and allow the current to pass for a shorter period of time than before. Determine the weight of copper deposited and the product $K \tan \alpha \times$ time. From the results, find whether the weight of copper deposited is proportional to the quantity of electricity which has passed.

Secondary cells, or accumulators.—When dilute sulphuric acid is electrolysed between lead plates the anode becomes coated with lead peroxide, PbO_2, while the kathode remains unaltered. On breaking the circuit, and connecting the terminals by a wire, a **polarisation current** is obtained which passes through the cell in the reverse direction to that of the charging current. An arrangement of this kind is termed a **Secondary Cell.** The original type of this cell, devised by M. Planté, consisted of two sheets of lead rolled up together and separated by felt or similar material. Frequent charging and discharging results in the formation of porous or spongy lead on the surface of the plates, thus increasing the amount of available surface. With a view to accelerate this process of *forming*, the plates now used consist of lead *grids* (Fig. 361), into the spaces of which is firmly pressed a paste

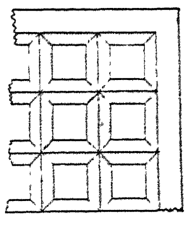


FIG. 361.—Grids of an accumulator

made from oxides of lead and sulphuric acid; in both cases lead sulphate is formed

The reactions which take place during the *forming* of the plates is as follows.

$$\text{At anode: } PbSO_4 + O + H_2O = PbO_2 + 2H_2SO_4.$$
$$\text{At kathode: } PbSO_4 + H_2 = Pb + H_2SO_4.$$

During the discharge of the cell the following reactions take place:

$$\text{At the positive plate: } \left.\begin{array}{l} PbO_2 + H_2 = PbO + H_2O, \\ PbO + H_2SO_4 = PbSO_4 + H_2O. \end{array}\right\}$$

$$\text{At the negative plate: } \left.\begin{array}{l} Pb + O = PbO, \\ PbO + H_2SO_4 = PbSO_4 + H_2O. \end{array}\right\}$$

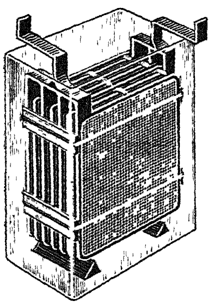


FIG. 362.—An accumulator

Evidently, during the passage of the current through the cell from the negative to the positive plate, sulphuric acid is electrolysed. The hydrogen travels with the current and is liberated at the positive plate.

The voltage of an accumulator is about 2·2 volts when fully charged.

Cells are made usually with several positive and negative plates arranged alternately and close together, the two outer plates always being negative. Fig. 362 represents a typical modern accumulator.

INDUSTRIAL APPLICATIONS OF ELECTROLYSIS.

The student will have observed that, in the electrolysis of sulphuric acid (or, as it may be termed, *hydrogen* sulphate) and of copper sulphate, the hydrogen and the copper traverse the electrolytic cell in the same direction. This statement holds good in the electrolysis of all metallic salts, and it is an invariable rule that **the metallic ion always travels with the current.**

Electro-plating is a term which denotes the deposition of a thin layer of a metal on any object by means of an electric current. The metals which are most frequently deposited in this manner are nickel, silver, gold, and copper. The objects to be

plated are thoroughly cleaned, and then suspended from copper wires in a liquid bath, containing in solution a salt of the metal which is to be deposited. The copper wires form the kathode, and a plate of the metal to be deposited forms the anode. The solution used in copper-plating contains copper sulphate, and is rendered slightly acid by the addition of sulphuric acid; that used in nickel-plating contains nickel-ammonium sulphate and ammonium sulphate. Silver is deposited from a solution of the double cyanide of silver and potassium; and gold is deposited from the double cyanide of gold and potassium.

Electro-typing is the process by which the surface of any object is coated with a layer of copper sufficiently thick to allow it to be subsequently removed and used as a copy of the original object. The surface of the object is coated with black-lead in order to make the surface a conductor. Coins and medals may be reproduced by taking a plaster cast of the coin; the face of the cast is then coated with black-lead, and copper is deposited on the conducting surface. Printer's type and wood engravings may be reproduced by taking a cast in wax or in papier-mâché, and obtaining a copper reproduction which is afterwards strengthened by being backed with a thick layer of type-metal.

Electro-metallurgy includes the manufacture on a large scale of aluminium, which is obtained by the electrolysis of fused oxide of aluminium, to which cryolite (a double fluoride of aluminium and sodium) is added to serve as a flux. The oxide is contained in a large iron vessel, which forms the kathode in the circuit; the anode consists of several stout carbon rods. Oxygen is liberated at the anode and combines with the carbon to form carbon monoxide; the aluminium gradually accumulates in the bottom of the iron vessel.

Previous to 1808 the caustic alkalis were supposed to be chemical elements, but in that year Sir Humphry Davy succeeded in decomposing caustic soda and potash by means of electrolysis. In his first experiment he placed a piece of slightly moistened caustic soda on a platinum plate connected to the positive pole of a battery, and touched the upper surface of the caustic with a platinum wire joined to the negative pole. Oxygen was liberated from the plate, and on the wire small globules of metal

appeared which rapidly tarnished in the air, and detonated when the wire was immersed in water. At the present time sodium and potassium are made chiefly by the electrolysis of fused caustic soda, iron electrodes being used. Caustic soda is now made largely by the electrolytic decomposition of common salt.

EXERCISES ON CHAPTER XXXIX.

1. State and explain Faraday's laws of electrolysis. Describe in detail how you would find experimentally the ratio of the electro-chemical equivalents of hydrogen and copper.

2. Describe carefully what takes place when an electric current is passed through a solution of copper sulphate (1) with platinum electrodes, and (2) with copper electrodes.

3. How would you proceed and what data would you require in order to measure a given current in amperes by means of (1) a tangent galvanometer; (2) electrolysis of copper sulphate?

What do you consider to be the relative advantages and disadvantages of the two methods?

4. Plates of copper and platinum are dipped into a solution of copper sulphate, and a current is passed through the cell from the copper to the platinum. Describe the effects produced; also what happens when the current is reversed.

5. Describe the construction of a water voltameter, and explain the chemical action in the electrolyte. A current is sent through dilute solutions of sulphuric acid and copper sulphate placed in series; how much copper would be liberated in the one for every gramme of hydrogen in the other?

6. Explain how you would decompose water by means of the electric current.

A current is passed through acidulated water contained in a vessel, and the gases liberated are collected in two test tubes A and B, hydrogen in A, and oxygen in B. After a time the pole wires are exchanged, so that the current goes through the water in the other direction and the oxygen is now collected in A and hydrogen in B. At the end of the experiment it is found that the total volume of the gases collected in A is three-fourths of the total volume of the gases in B. Show that the volume of hydrogen in A is two-fifths of the volume of hydrogen in B

7. An electric current (which is the same in all parts of the trough) flows horizontally in a trough filled with copper sulphate. A rod of copper is then supported horizontally in the trough, with its length parallel to the direction in which the current is flowing. How will the rod be affected by the current?

8. The E.M.F. of a storage cell is almost exactly double the E.M.F. of a Daniell. How would you test this statement without the aid of a galvanometer? How do you explain the difference of E.M.F.? Describe carefully the chemical changes that would occur in a Daniell if it were connected, in opposition, to a storage cell.

9. Describe some form of secondary battery. State (1) how you would charge it; (2) which would be the positive pole.
What are the advantages and disadvantages of such a battery as compared with a Leclanché cell?

10. Describe a storage cell or accumulator, and explain the advantage of using large plates.
Explain the experiments you would perform in order to distinguish between three cells—one a storage cell, another a Daniell, and the third a Leclanché—of which the terminals only were accessible in each case.

11. Explain the term *electro-chemical equivalent*. If 3 amperes deposit 4 grams of silver in 20 minutes, what is the electro-chemical equivalent of silver?

12. How much silver will be deposited by a current of 5 amperes in 1 minute?
How long will it take a current of 5 amperes to deposit 5 grams of copper?

13. A piece of metal weighing 200 grams is to be plated with $2\frac{1}{2}$% of its weight in gold. If the current strength is 1 ampere, how long will it take to deposit the required weight of gold?

14. A tangent galvanometer and a copper voltameter were connected in series and included in the same circuit. A constant current was sent through the circuit for 30 minutes, and the weight of copper deposited was found to be 0·272 gram. If the deflection of the galvanometer needle is 30°, calculate the *reduction factor* of the galvanometer.

15. A metal plate, having a surface of 200 sq. cm. is to be silver-plated. If a current of 0·5 ampere is used for a period of 1 hour, what thickness of silver will be deposited on the plate (density of silver = 10·6 gm. per c.c.)?

16. A tangent galvanometer has a current passed through it which deflects it 45°. The same current passes through a copper voltameter, where it deposits 0·3 gram of copper in 30 minutes. If the electro-chemical equivalent of copper is 0·00033 gram ampere-second, find the value of the current, and show how to determine the current for any other reading of the galvanometer.

17. How long must a constant current of 500 amperes pass through a bath for the electrolytic deposition of copper in order to deposit sufficient copper to make 1 kilometre of No. 16 S.W.G. wire (diam. = 0·163 cm.)? The density of copper is 8·95 gm. per c.c.

18. A current is measured by means of a water voltameter The density of the dilute acid was 1 2 gm. per c c. In 5 minutes 25 c c. of mixed gases were collected Assuming that the gas was saturated with moisture, calculate the current strength, having given that

height of column of dilute acid	= 10 cm.,
density of mercury	= 13 56 gm per c.c
reading (corrected) of barometer	= 75 62 cm.
temperature of room	= 20° C,
vapour tension of water at 20° C.	= 17·4 mm

CHAPTER XL.

THERMAL EFFECTS OF AN ELECTRIC CURRENT. THERMO-ELECTRIC CURRENTS.

Conversion of electrical energy into heat.—Unit potential difference has already been defined as that which requires the expenditure of unit work in order to convey unit quantity between two points, the potentials of which differ by unity. If the unit quantity proceeds from the point at higher potential to the point at lower potential, then unit work will be done *by* the electric forces. In a simple electrical circuit this work reappears in the form of *heat*.

If Q coulombs of electricity traverse a wire, between the ends of which there is a potential difference of E volts, then the measure of the work done in the wire is $(Q \times E)$ practical units. (This unit of work is called the *Joule*.[1])

If this is expressed in absolute units, since 1 coulomb $= \frac{1}{10}$ absolute unit of Quantity, and 1 volt $= 10^8$ absolute units of P.D., the work done

$$= QE\ (\tfrac{1}{10} \times 10^8)\ \text{ergs} = QE \times 10^7\ \text{ergs}.$$

Hence 1 Joule $= 10^7$ ergs

Since $Q = Ct$, the work done $= ECt$ Joules

But, by Ohm's Law, $E = CR$

Hence ECt Joules $= C^2Rt$ Joules $= (C^2Rt \times 10^7)$ ergs.

This is an expression for the quantity of work which reappears as *heat* in a simple circuit.

EXPT. 329.—**Generation of heat in a simple circuit.** Connect two accumulators, or two large Bunsen cells, in series. Connect the poles, by means of thick copper wires, to the ends of a short piece of platinum wire (No. 32 S.W.G.). Observe how the wire is heated, and

[1] James Prescott Joule (1818-1889), born at Salford.

perhaps even glows. If the wire be too long it will not glow, since the total resistance is too great to allow sufficient current for the experiment; the resistance may be reduced either by shortening the wire, or by reducing the resistance of a portion by immersing it in a vessel of cold water, when the remaining portion will glow brightly.

The relationship between the heat generated and the resistance may be well shown by passing a fairly strong current through a chain, the alternate links of which are made of thin platinum and silver wire (both of the same diameter). Platinum has a much higher specific resistance than silver, and more heat will therefore be generated in the platinum than in the silver, with the result that the former will glow brightly while the latter will remain comparatively cool.

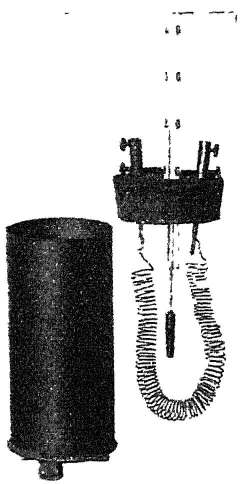


FIG. 363—Apparatus for measuring heat developed in a wire.

Joule's Law.—**The heat generated in a simple circuit is proportional (i) to the square of the current, (ii) to the resistance, (iii) to the time during which the current continues.** The general principle of the apparatus which Joule used in the experimental proof of this law may be understood by reference to Fig. 363. The ends of an open coil of thin German-silver wire are connected to thick copper wires passing through a wide cork fitting into the top of a thin metal vessel (made of sheet brass or copper and called a **calorimeter**) containing water. A thermometer is fixed through the centre of the cork so that its bulb is immersed in the water. In order to prevent the current from leaking through the water instead of traversing the wire, it is advisable to coat the surface of the wire with a thin insulating layer of shellac by dipping the wire into shellac varnish and heating it in an air-bath to 140° C.

EXPT. 330.—**Demonstration of Joule's Law.** (i) Pour a measured quantity of water into the calorimeter sufficient to cover the German-

silver wire Read the thermometer. Complete the circuit, which includes a tangent galvanometer and one accumulator. Note the time. Read the deflection, and allow the current to pass until the temperature has risen, say 3° C Occasionally shake the calorimeter slightly so as to allow the water to become uniformly heated. Note the time at the instant of breaking the circuit Repeat the experiment, using two cells, and allowing the current to pass for the same period of time as before.

$$\text{Prove that } \frac{\text{Rise of temp}_1}{\text{Rise of temp}_2} = \frac{(\tan\theta_1)^2}{(\tan\theta_2)^2}.$$

(ii) Replace the water by the same volume of fresh cold water. Use one cell only, and repeat Expt. (i), but allow the current to pass for twice the length of time. Observe that the rise in temperature is twice as great as before, or

$$\frac{\text{Rise of temp.}_1}{\text{Rise of temp}_3} = \frac{\text{time}_1}{\text{time}_3}.$$

(iii) Connect together two calorimeters of the same size in series, but let the spiral in one of them be *twice as long* as that in the other. Pour equal volumes of cold water into the calorimeters, and observe the rise in temperature of the water in the two vessels after a current has passed for a short time. Notice that the rise in temperature due to the longer spiral is twice as great as that due to the shorter spiral. Hence the heat generated is proportional to the resistance.

The amount of heat generated by a current is measured in terms of the **calorie.** If W gms. = weight of water used, and T° C. = rise in temperature, heat generated = (W × T) calories

By elaborate experiments Joule determined that the energy equivalent to one calorie, expressed in units of work, is $(4{\cdot}2 \times 10^7)$ ergs.

But the work done in a simple electric circuit = $(C^2Rt \times 10^7)$ ergs.

Hence the number of heat units generated in a simple circuit

$$= \left(\frac{C^2Rt \times 10^7}{4{\cdot}2 \times 10^7}\right) = \frac{C^2Rt}{4{\cdot}2} \text{ calories.}$$

This result indicates that the measurement of the heat developed in a wire of known resistance affords a means of measuring the strength of the current traversing the wire. The amount of heat generated in the calorimeter, (W × T), is equated to $\frac{C^2Rt}{4{\cdot}2}$, thus

$$W \times T = \frac{C^2Rt}{4{\cdot}2},$$

or,

$$C = \sqrt{\frac{W \times T \times 4{\cdot}2}{R \times t}}.$$

The essential data are (i) the resistance of the wire, (ii) the weight of water in calorimeter, (iii) the rise in temperature, and (iv) the time.

The incandescent electric lamp.—The incandescent lamp is an application of the principle that a conductor is heated when traversed by an electric current, and that it is rendered incandescent when the material of the conductor has a high melting point and a high resistance. The first lamp, introduced by Edison in 1878, consisted of a fine platinum wire; but the liability of the wire to fuse prevented its commercial success A filament of carbon was found to be far more satisfactory. Since carbon readily burns in air, the filament must be enclosed in an exhausted glass vessel; the current passes into the filament by means of platinum wires fused into the ends of the glass vessel.

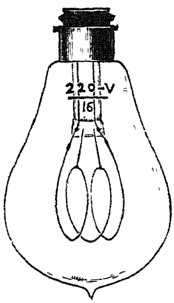


FIG. 364 —An incandescent electric lamp

The filaments were made formerly from thin strips of bamboo, which were wrapped round a carbon *former*, to give the desired shape to the filament, and surrounded by powdered carbon contained in a crucible, which was then heated to a high temperature in a furnace. The filaments are now prepared artificially from a soluble cellulose, which is prepared by dissolving cotton wool in zinc chloride. The thick liquid is forced through a die, thus giving a uniform thread which, when dry, resembles catgut; this is cut into suitable lengths and 'carbonised.'

The electric power consumed by a lamp is expressed, in **watts**, by the product of the voltage at the terminals and the current. The energy expended in the lamp appears partly as heat and partly as light. So far as the true function of the lamp is concerned, the energy which appears as heat is wasted; and, when a lamp is working under normal conditions, this constitutes fully 95 % of the total energy. This loss may be diminished if the filament be made brighter by increasing the voltage; but this is accompanied by the slow volatilisation of the carbon, which is deposited on the surface of the glass,

thus diminishing the candle-power and, consequently, the life of the lamp.

As a general rule the consumption of energy in an incandescent lamp is slightly less than *four watts for each candle-power*. Thus a 16 c.p. lamp on a 220-volt circuit will require a current of about 0.28 ampere. Incandescent lamps cannot be worked at less than 2.5 watts per candle-power but even then the life of the lamp is very short.

If a number of lamps are worked from one source of current, they are usually connected together in parallel (p. 500).

Lamps with metallic filaments.—The chief disadvantages of the carbon filament are (i) that it begins to disintegrate at 1600 C., and (ii) that its resistance diminishes with rise of temperature, and therefore it is very sensitive to fluctuations in the voltage. In 1905, Dr. von Bolton produced pure tantalum from the mineral tantalite. This metal has a very high melting-point, about 2300° C.; it is very suitable therefore for making lamp filaments. When consuming 1·5 watts per candle-power its temperature is only 1850° C. Another great advantage is that it is less sensitive to fluctuations of the voltage, since its resistance increases with rise of temperature. As the conductivity is fairly high, the filament must be very long and narrow: the standard type of lamp has a filament 65 cm. long and 0·05 mm. diameter.

The "Osram" lamp, which has a fine filament of tungsten, is probably the most successful of metallic filament incandescent lamps. The difficulty of producing very fine, yet strong, filaments of tungsten has been overcome; and it is now possible to obtain lamps of low candle-power suitable for use with currents of usual town voltages. A 16 candle-power, 220-volt, 28-watt Osram lamp, with a filament only about 0·015 mm. in diameter, is now an article of commerce.

"Tungsten, being more refractory than carbon, can be kept continuously at a higher temperature without volatilising, and a tungsten filament would give three times the light of a carbon filament for the same consumption of power and the same life. But as the specific resistance of tungsten is lower than that of carbon it is necessary to produce a finer filament than the carbon one, and to find a means of supporting a greater length

of this fine filament in the lamp. These difficulties were at first only partly overcome. The filament of a 25 candle-power 220-volt carbon lamp has a diameter of about 0·16 mm., and a length of about 350 mm., while the filament of a tungsten lamp of the same candle-power and voltage is about 0·02 mm. in diameter and 850 mm. long." (*Nature*, Oct. 19, 1911.)

The electric arc.—Another convenient method of producing a powerful light is by forming an electric arc between carbon poles. On allowing two sticks of compressed carbon, connected to the terminals of a battery or dynamo (giving a potential difference of at least 30 volts) to touch, and then separating them, the current continues to pass as an arc between the carbon points. The maintenance of the arc is due to the fact that carbon volatilises at a very high temperature; and the vapour thus formed gives the necessary conductivity to the arc. Since the arc has comparatively high resistance, much heat is generated locally and the temperature of the carbon points is maintained. After use the end of the positive carbon becomes hollow or crater-shaped, and that of the negative carbon pointed (Fig. 365). The positive carbon gives much the brighter light. The efficiency of an arc lamp is greater than that of an incandescent lamp: thus, about 1 watt is required for each candle-power, and fully 10 % of the total energy is converted into light.

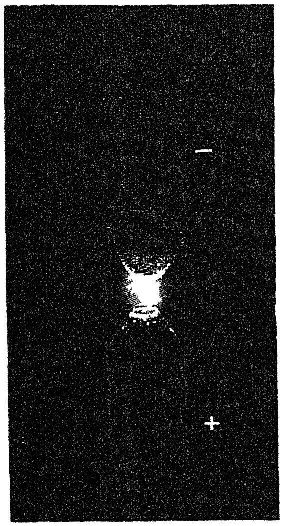


FIG. 365.—The electric arc

The carbon rods gradually wear away, partly owing to the transfer of carbon from the positive to the negative rod, and partly to the oxidation of the carbon. It is necessary, therefore, to have some method of regulating and adjusting their distance apart. Space does not permit a description here of the many ingenious devices which have been designed for this purpose. In most cases the device works automatically, and is controlled

by the current passing through the lamp itself; in simple cases, such as the electric lantern, a regulator worked by hand may be found sufficient.

Safety fuses and other applications of heating effects.—The heat generated in a wire conveying a current is applied as a device for protecting circuits against abnormal currents which might prove dangerous. Such devices are termed **fuses.** These consist usually of a short length of wire, of a metal or alloy of comparatively high specific resistance and low melting-point: and the diameter is selected so that a current about 50% stronger than that required in the circuit will heat the wire up to its melting-point and so break the circuit. The relationship between the diameter of the wire and the maximum current may be expressed by the equation $d=(C/a)^{\frac{2}{3}}$, where a is a constant depending upon the metal or alloy. If d be expressed in millimetres, the value of a is 80 for copper, 12·8 for tin and 10·8 for lead.

Blasting fuses consist of a short piece of thin platinum wire, inserted in the base of the detonating charge, the ends of which are connected by long insulated wires to a distant battery. The charge is fired by passing a strong current through the platinum wire.

In surgery, a short piece of platinum wire heated to redness by means of a current is frequently used for the purpose of cauterising animal tissue **(electric cautery).**

Two bars of metal may be joined by placing them in contact end to end, and passing a strong current across the junction **(electric welding).** The point of contact, offering a comparatively high resistance, is locally heated to a sufficiently high temperature to weld the surfaces together.

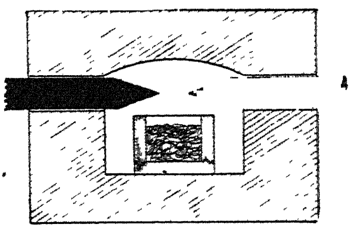


FIG. 366.—Moissan's electric furnace.

The electric furnace.—Much of our present knowledge of this application is due to Prof. Moissan, whose initial type of furnace is represented in Fig. 366; it consists of two superposed blocks of lime or of limestone. Thick carbon electrodes are

inserted through holes in the opposite walls. In order to avoid the formation of calcium carbide by the action of carbon on the limestone, the cavity is lined with alternate layers of magnesia and carbon. The heat derived from the arc is reflected downwards from the cover, and subjects the crucible to a 'toasting' action. This type of furnace is too costly for commercial purposes, and in such cases the resistance type is used. Here the carbon electrodes are buried in the substance to be fused, the ends being connected initially by fragments of badly conducted material, such as gas-carbon, on passing a current the gas-carbon is fused, and the electrodes are thus joined by a semi-fluid mass of relatively high resistance. In this type of furnace, therefore, an arc is never formed.

In this manner **calcium carbide** is now manufactured from a mixture of pure lime and coke dust. As the carbide is formed it fuses and sinks to the bottom of the vessel. Similarly **carborundum**, which is a silicide of carbon and used as an abrasive, is made from a mixture of coke and sand.

THERMO-ELECTRIC CURRENTS.

Production of electricity by heat.—When two arches, one of iron and one of German-silver, are joined as in Fig 367, a difference of potential exists at each of the junctions. At A, it tends to send a current in the direction AD. The direction at C is again towards the iron, *i.e.* along CD. But though these forces urging the flow of electricity exist, and there is a complete circuit in which a current could flow, no current is produced. The fact is that the electric forces at A and C balance each other. When the force at either junction is strengthened, by any means, this equilibrium no longer holds good. A current will flow in the direction determined by the stronger force. On heating the junction A, a current flows round the circuit in the direction shown by the large arrow in Fig. 368. The enhanced force at A overpowers that at C, and

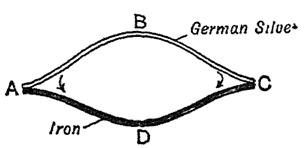


FIG. 367.—Electric force at junctions of two metals

while the difference of temperature of the two junctions lasts, the current continues. The energy necessary for the maintenance of the current is supplied by the heat of the flame. The arrangement is, in fact, an electric battery using heat, instead of the energy of chemical action, to drive the current.

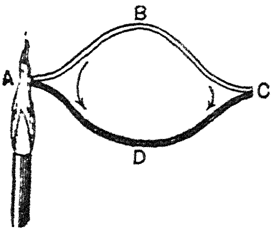


FIG. 368.—Production of a thermo-electric current.

When an iron and German-silver junction is immersed in hot water, while the other ends are connected to a galvanometer, a current through the instrument is indicated by a deflection of the needle. On cooling the water, the flow of electricity is lessened. On removing the junction from the warm bath, the current ceases. So far as this experiment goes, the current is stronger the greater the difference of temperature between the junctions. When an iron-copper junction is heated gradually in a Bunsen flame and connected as before to a galvanometer, it is observed that as the temperature rises the current increases, becomes constant, then decreases in intensity, and finally becomes reversed just before the zinc begins to melt. Instead of the current always increasing with the difference of temperature it may fall to nothing or become reversed in direction.

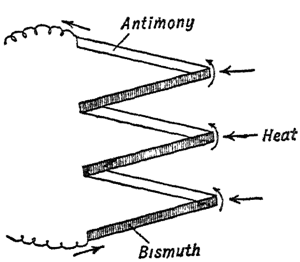


FIG 369.—Current from compound metallic strip.

This thermo-electric inversion takes place with most pairs of metals at particular temperatures.

The thermopile.—The production of electric current by heat is used less as a source of electricity than as a detector of radiation. When a compound strip is made as shown in Fig. 369, and the right hand junctions are heated, their current-producing effects are added. A small rise of temperature will produce a current sufficiently strong to be detected by a galvanometer. In the actual instrument, which consists of alternate rods of antimony and bismuth, the metals are compactly bunched together

as shown in Fig. 370 Contact is prevented except at the junctions, by inserting sheets of mica as indicated by the thickened lines of Fig. 370 (ii). The figure shows also the mounted instrument with a collecting and protecting cone on the face exposed towards the source of heat. The fine wires lead to the galvanometer by which the currents are detected The thermopile, as the instrument is named, is very sensitive to radiation.

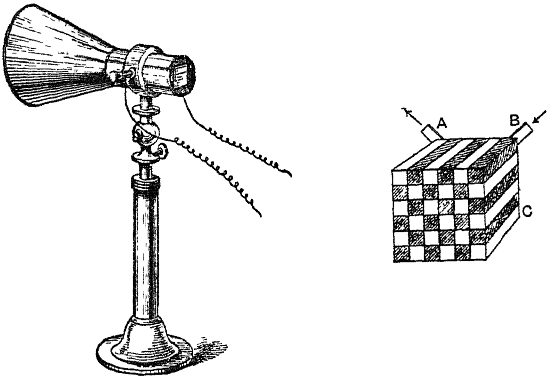


FIG 370.—A thermopile. The separate diagram shows the arrangement of the compound rods.

For observing high temperatures such as those of molten metals, a thermo-couple, as these metal junctions are called, can be used. The junction, usually of platinum joined to a platinum-rhodium alloy, is protected by a fire-clay cylinder and immersed in the liquid metal. The deflection of the galvanometer gives information as to the temperature. The thermo-couple here acts as a high temperature thermometer or **pyrometer.**

EXPT. 331.—**Thermo-electric currents.** (i) Solder an iron and a German-silver wire together at one end. Solder the other ends to long copper wires connected to a mirror galvanometer (of *low* resistance). Heat the iron and German-silver junction by immersing it in hot water. Observe the deflection of the galvanometer. Cool the water. Notice the diminished deflection. Take the junction out of the warm water. Note that the deflection falls to zero.

(ii) Connect together a number of strips of iron and copper in

the manner shown in Fig. 369 Place the alternate junctions over a Bunsen flame. Connect up with a galvanometer. Observe the large deflection of the needle

(iii) Join together the ends of an iron and a copper wire. Solder copper wires to the free ends of the wires. Connect the wires to the galvanometer. Place the iron-copper junction in a Bunsen flame. Observe that the deflection of the galvanometer increases, remains stationary, and then diminishes, and becomes reversed as the temperature of the junction increases.

EXERCISES ON CHAPTER XL.

1. The current from a voltaic battery is passed at the same time through a thin wire and through dilute sulphuric acid, connected in series. What will happen to the wire and to the dilute acid, and what change (if any) will be produced in each case by reversing the battery connections, so as to alter the direction of the current through the wire and liquid?

2. An electric current is passed through a platinum wire and a copper wire of the same size, arranged in series. If the strength of the current is sufficiently increased, the platinum becomes red-hot while the copper remains dark. Explain this.

3. A current flows through a copper wire, which is thicker at one end than at the other. If there is any difference either (1) in the strength of the current at, or (2) in the temperature of, the two ends of the wire, state how they differ from each other, and why.

4. Assuming that the rate of production of heat by a current in a wire varies as the product of the resistance and the square of the current, compare the amount of the heat developed by a current of 2 amperes in 3 minutes in a wire 3 feet long, with that produced by a current of 3 amperes in 2 minutes in 2 feet of the same wire.

5. Two wires of the same size and length, one of copper and the other of iron, are joined in series and connected to the poles of a battery. In this case the iron wire becomes hotter than the copper. The two wires are then connected in parallel to the same battery, and the copper is observed to become hotter than the iron. Explain these observations.

6. A current is sent through a piece of fine wire by a voltaic cell, the resistance of which is very small compared with that of the wire. How will the heat produced be altered if the length of the wire is halved?

7. Two wires are connected in series with a voltaic cell, the resistance of the cell being very small compared to that of either wire, and it is found that the heat developed in one wire is twice that developed in the other. Compare the quantities of heat developed per second when the wires are in turn connected to the same cell.

8. The plates of a cell, the resistance of which is inappreciable, are connected by a platinum wire. How would the rate of development of heat in the wire, and the rate of consumption of zinc in the cell, be affected if the wire were drawn out uniformly to double its length?

9. A coil of bare German silver wire of known length is attached to the terminals of a storage cell, and is observed to become warm. Why is this, and what are the factors that determine the rate of generation of heat within the wire?

What length of German silver wire, of half the cross-sectional area of the above one, must be joined in series with it so that the rate of generation of heat within the first wire may be reduced by three-fourths?

10. The terminals of a voltaic battery of resistance 1 ohm are connected by two wires *in parallel*, their resistances being 6 and 8 ohms respectively. The difference of potential between the terminals is 2 volts. Find the currents, and compare the rates at which energy is expended in the wires. Find also the electro-motive force of the battery.

11. A current is passed through a voltameter and through a coil of wire in series with it. If the current is altered in such a way that the heat produced in the coil is doubled, show what change will be produced in the rate at which chemical action takes place in the voltameter.

12. Find how many grams of water would be heated 1° C. by immersing in it a wire coil whose resistance is 7 ohms, and passing a current of 0.3 ampere for 10 minutes, supposing all the heat to be communicated to the water.

13. A wire of 5.23 ohms resistance was placed in a calorimeter containing 1000 grammes of water, and a current of 5 amperes was sent through the coil for 10 minutes. If the inital temperature of the water is 10° C., what will be its final temperature?

14. A current is passed through a wire of 5 ohms resistance placed in a calorimeter. A steady stream of water is kept flowing through the calorimeter at the rate of 15 c.c. per minute, and the heating effect was such that the water was 4° C. warmer on leaving the calorimeter than it was on entering. Find the strength of the current.

15. A current was sent through the low-resistance coil of a tangent galvanometer and through a 1 ohm resistance immersed in 100 grams of water. In 40 minutes the rise of temperature was 15·8 degrees Centigrade, and the mean deflection was 32°. Calculate the strength of the current, and the reduction factor of the galvanometer.

[tan 32° = 0·625.]

16. Write a short essay on the incandescent electric lamp, dealing particularly with any improvements which have been made within the last few years.

CHAPTER XLI.

ELECTRO-MAGNETIC INDUCTION. THE RHUMKORFF COIL. THE TELEPHONE. RÖNTGEN RAYS.

Faraday's experiments.—We have seen already that a magnetic field is set up in the space surrounding a wire conveying a current. If the current and magnetic field are connected indissolubly we might anticipate that the creation of a magnetic field round a closed circuit will cause a current to traverse the circuit. Faraday, in 1831, showed that whenever a circuit is moved in a magnetic field in such a manner as to alter the number of lines of force passing through the circuit, then an E.M.F. is set up, proportional in magnitude to the rate of change in the number of lines of force passing through it, and continuing so long as the change continues. This relationship between the rate of change in the number of lines of force and the magnitude of the E.M.F. set up, is known as **Faraday's Law**.

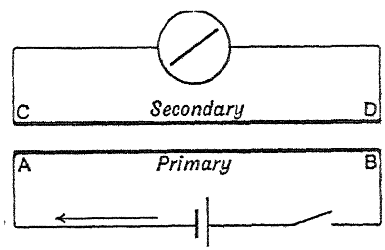


FIG. 371.—Faraday's initial experiment on induced currents.

Fig. 371 represents Faraday's initial experiment. AB and CD are two parallel wires, the former joined to a battery and key, the latter to a galvanometer. AB is termed the *primary*, and CD the *secondary* circuit. On closing the primary circuit a momentary current is observed in CD, and its direction is *opposite* to that in AB. On breaking the primary circuit a

momentary current, in the *same* direction as that in AB, is observed in CD.

Faraday applied the term **Induced Currents** to currents generated in this manner. He also observed that (i) **an inverse induced current is obtained when the primary current begins, or when it increases, or when it approaches the secondary circuit**; and (ii) **a direct induced current is obtained when the primary current ceases, or diminishes, or is removed to a greater distance.**

Much longer wires can be manipulated if these are wound into coils, as shown in Fig. 372. Also, the effects are more marked when a soft iron core is inserted in the primary coil, since this increases the number of magnetic lines of force associated with the coil.

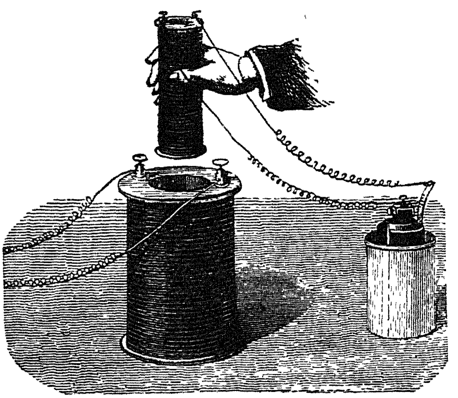


FIG. 372.—Production of induced currents.

EXPT. 332.—**Inverse and direct induced currents.** (i) Observe which terminal of a galvanometer must be positive in order to give a deflection to the right or left. Place the primary coil inside the secondary, and at some distance from the galvanometer. Close the primary circuit and observe the direction of deflection. Note how the needle comes back to zero; and how it is deflected in the opposite direction when the primary circuit is broken. Verify that the induced currents are *inverse* and *direct* respectively

Repeat the observations with an iron core inserted in the primary.

(ii) Remove the primary coil to a distance, and close its circuit. Move it quickly towards the secondary and, when the needle is again at rest, remove it to a distance. Verify the inverse and direct current.

(iii) Insert the primary coil into the secondary, and include an adjustable resistance in the primary circuit. Verify that an increase or diminution of the primary current induces an inverse or direct current in the secondary.

(iv) Repeat Expt. (ii), using a bar-magnet instead of the primary coil Verify that when a *north-seeking* pole is brought towards the secondary coil the induced current is such that the near end of the coil acquires *north-seeking* polarity; also that when the same pole is withdrawn the polarity of the same end of the coil is *south-seeking*.

Lenz's law.—Energy is required in order to generate the induced currents observed in the preceding experiments. Since there is no source of energy in the secondary coil the energy represented by the induced currents must be due to some external agency, in fact, in Expts. 332, (ii) and (iv), it originates from the **mechanical work** done in overcoming the mutual forces of attraction or repulsion which exist between the currents in the two circuits. Thus, when the primary circuit (or a magnet pole) is approaching, the direction of the induced current is such as to exert a repulsion; also, when receding, the induced current will exert an attraction. Lenz's law may be stated thus: **The induced current is in such a direction that its reaction tends to stop the motion or change to which the induced current is due.**

This can be understood more clearly by considering the case of a coil of wire (cotton-covered) approaching, or receding from, the pole of a magnet. Suitable coils may be made about 10 cm. diameter, and of 50 turns of wire. By referring to Fig. 373 (i) it

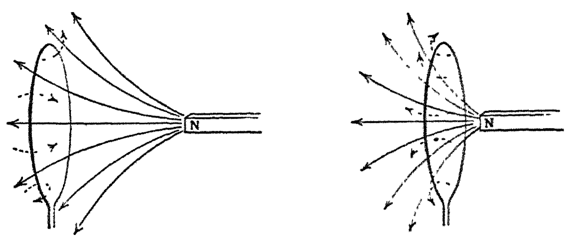


FIG. 373.—Directions of induced currents in a coil of wire

is evident that the approach of the coil results in an increase in the number of magnetic lines threaded through the coil; and, by Lenz's law, the induced current will be in such a direction as to give N.-seeking polarity to the near face of the coil, thus

creating a flux of magnetic lines through the coil in the *opposite* direction to those due to the magnetic pole. Similarly, when the coil is receding, as in Fig. 373 (ii), the near face of the coil acquires S.-seeking polarity; and the lines of force due to the current now pass through the coil in the *same* direction as those due to the magnet. All such cases comply with the following rule: **When the number of lines of force passing through a circuit is increased or diminished, the induced current will be in that direction which tends to keep the number of lines constant.**

Induced E.M.F.—Since a current cannot be set up in a closed circuit unless there is established in the circuit an E.M.F., it is evident that the fundamental effect of changing the number of magnetic lines of force enclosed within the circuit is to create an E.M.F. in the circuit. The magnitude of this E.M.F. is dependent solely upon the *rate* at which the number of lines of force through the circuit is changing, whereas the resulting current depends also upon the resistance of the circuit. Also, the E.M.F. is established whether the circuit is open or closed, but necessarily a current is generated only when the circuit is closed.

If the circuit consists of two turns of wire in series, the same E.M.F. is induced in each turn, and the total E.M.F. between the extreme ends is twice as great as that generated in a single turn; if the coil consists of n turns, the total induced E.M.F. is n times as great.

It can be proved that **the induced E.M.F., expressed in absolute units, is equal to the rate of change in the number of lines of force enclosed within the circuit.** Thus, if the number of lines of force through the circuit changes from N_1 to N_2 in an interval of time t, then

$$E = (N_1 - N_2)/t; \qquad \text{(1)}$$

and, if the circuit be closed, since $C = E/R$,

$$C = (N_1 - N_2)/Rt. \qquad \text{(2)}$$

Equation (1) may be used as the basis of a definition of the absolute unit of E.M.F., from which is derived the practical unit—the volt (p. 492). The absolute unit of E.M.F. is then defined as that which is set up in a single circuit when the number of lines of force through the circuit changes by unity per second.

The Dynamo.—The dynamo is an extremely important application of the principles explained in the foregoing paragraphs. Fig. 374 represents a rectangular coil which is supposed to rotate round a horizontal axis and within the magnetic field due to the

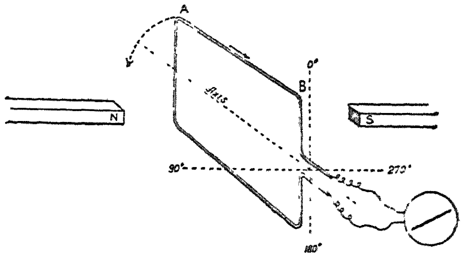


FIG 374 —Principle of the dynamo.

opposite poles of two magnets placed on opposite sides of the coil. If the direction of rotation be anti-clockwise, as indicated by the arrow, it is evident that by Lenz's law, the direction of the induced E.M.F. in the side AB will be from A to B, and in the reverse direction in the opposite side of the coil, until the coil has rotated through 180°: and if the rotation is continued through a further 180°, the direction of the induced E.M.F. is reversed. When the ends of the coil are connected to rings (Fig. 375) which rotate with the coil and which are in contact with metal or carbon *brushes* B_1 and B_2, the current produced

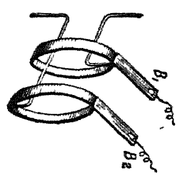


FIG. 375 —Collecting brushes of an alternator

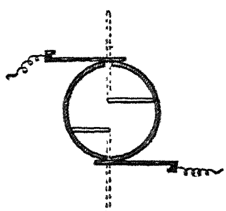


FIG. 376.—Commutator of a continuous current dynamo.

may be collected. The current so obtained alternates in direction every time the coil passes the vertical position, and the machine may be termed an **alternating-current dynamo** or an **alternator.**

When a split ring (Fig. 376) is used, the end of the coil

connected to each brush is changed every half revolution; and by this device the direction of the current is kept constant. The machine is then termed a **continuous current** (or **direct current**) **dynamo.**

A consideration of Fig. 374 will suggest that, when the speed of rotation is uniform, the *rate* at which lines of force are cut by the horizontal sides of the rectangle is zero at the instants when the coil is vertical, and the rate gradually increases to a maximum when the coil is horizontal. Hence the E.M.F. is continually fluctuating, and an unsteady current results. In practice a steady current is obtained by using a large number of coils wound uniformly on a cylindrical spindle of soft iron, and connected together either in series or in parallel, thus, when the induced E.M.F. in some of the coils is a minimum, that in others is at the same instant a maximum, and a steady current is obtained. This arrangement of spindle and coils is termed the *armature.* In machines designed to generate large currents the armature is rotated, by a steam or a gas engine, or by water power, between the poles of a powerful electro-magnet.

The Rhumkorff coil.—The Rhumkorff coil is a practical application of Expt 332 (iii), in which an induced E.M.F. is set up between the ends of a secondary coil by making and breaking the circuit of an electro-magnet which is placed inside the secondary circuit. Fig. 377 is a diagrammatic representation of the essential parts of

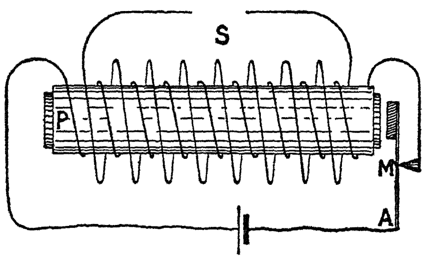


FIG 377 —Diagram of a Rhumkorff coil.

the instrument. P is the primary coil with its core consisting of a bundle of soft-iron wires. The primary circuit is rapidly *made* and *broken* by the contrivance shown at M; a flexible

spring is fixed vertically at A, and a piece of soft iron is attached to the upper end of the spring and near to the soft-iron core. When the primary circuit is made the current traverses the coil of wire and passes from the point (at M) into the spring, returning to the battery through A. The iron core becomes strongly magnetised and attracts the soft iron at the end of the spring, thus *breaking* the circuit at M. The iron core immediately ceases to be a magnet and the spring returns to its original position, thus *making* the circuit once more. This series of operations proceeds with great rapidity, and the spring vibrates rapidly to and fro, the circuit being made and broken in every complete vibration of the spring. The secondary coil, S, is wound round the primary circuit, and an induced E.M.F. is set up in each turn of the secondary. The total P.D. between the ends of S is equal to the sum of the potential differences set up in each individual turn of the secondary coil. In practice, P consists of several hundred turns of fairly thick silk-covered copper wire and S consists of several thousand turns of fine silk-copper wire. It is possible to obtain between the ends of the secondary coil a potential difference sufficient to yield sparks 15-20 inches long.

The telephone.—In 1876, Graham Bell invented the magneto-telephone, which is still used as the 'receiver' in telephonic systems. It consists simply of a very thin iron diaphragm (A, Fig. 378) fixed round the edge and with its centre near to the end face of a soft iron cylinder (B) fixed on to the end of a cylindrical permanent magnet. When air-waves, such as are associated with sound, fall upon the diaphragm, vibrations are set up in the iron, and the disturbance so caused in the distribution of the magnetic lines of force generates induced currents in a coil (D) of thin wire wound round the soft iron cylinder. The ends of this coil are joined to the *line wires*, which are connected at the distant end to an exactly similar instrument. The induced currents traverse the coil of wire inside

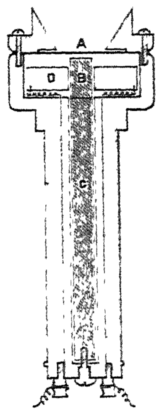


FIG. 378.—Section of a telephone.

this latter instrument, causing rapid variations in the pole-strength of the magnet which operates on the iron disc fixed near its end. Thus vibrations are set up in this disc which correspond exactly with those of the first instrument, and the original sound-waves are exactly imitated. The two instruments are termed the **transmitter** and the **receiver** respectively. With this arrangement no battery is required.

EXPT. 333.—**Principle of telephone.** Wind at least 50 turns of thin insulated copper wire round one pole of a large horse-shoe magnet. Join the ends of this coil to the terminals of a low resistance mirror galvanometer Quickly bring near to the poles a strip of soft iron, and notice the momentary current caused by the magnetic disturbance. Quickly withdraw the iron to a distance, and notice the momentary current in the reverse direction.

At the present time another type of transmitter is used. This is based upon the discovery made by Hughes, in 1878, that if a *loose contact* is included in a simple battery circuit, sound-waves

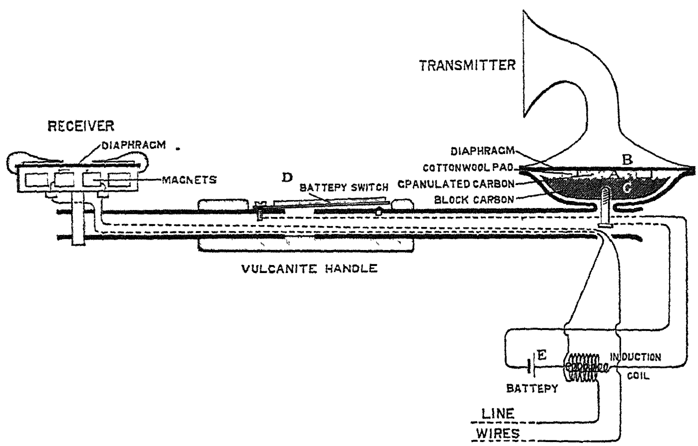


FIG 379 —A telephone transmitter and receiver.

falling on the points of loose contact cause variations in the resistance, and therefore in the current. If this varying current be sent through the coil of a Bell receiver, the original sound-

waves are reproduced The variations in resistance which can be obtained thus are but slight, and if these are to produce sufficiently great variations in the current it is essential that the total resistance in the circuit be small This is impossible if the line wires connecting the receiver are very long. The difficulty is overcome by inserting a small induction coil in the circuit, near the transmitter, the variable current passing through the primary circuit of the coil. The ends of the line wires are joined to the secondary circuit of the coil, and the variations of electro-motive force there induced are sufficiently great to send through the line wires currents which vary sufficiently to operate the receiver at the distant end.

Fig. 379 represents the principle of a modern type of transmitter and receiver. The loose contact in the transmitter is obtained by packing a shallow layer of granulated carbon (A) between the diaphragm (B) and a block of carbon (C). The circuit of the battery (E) includes this loose contact, a battery switch (D) which is closed only when the instrument is held in the hand, and the primary (P) of the induction coil. The line wires are joined through the secondary (S) of the induction coil to the receiver, which, though different in construction, is identical in principle with that represented in Fig. 378.

Kathode rays.—The discharge of electricity through a rarefied gas is observed by means of metal electrodes, fixed inside the ends of a long glass tube from which the air has been removed more or less completely. When the electrodes are connected

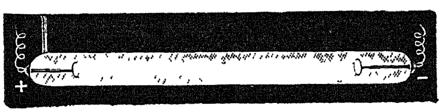


FIG. 380.—Electric discharge in rarefied gas

to the terminals of a Rhumkorff coil the tube is traversed by a feebly luminous column which may extend along the whole length of the tube (Fig. 380) This shows that a rarefied gas is a good conductor of electricity. When the rarefaction is increased,

the luminous column disappears, and the tube is filled with a phosphorescent light distributed over the surface of the glass. The colour of the phosphorescence depends upon the nature of the glass; it is bright green with soda glass, and blue with lead glass.

A remarkable series of experiments on these phenomena carried out in 1879 by Sir W. Crookes led him to believe that in the discharge through a highly rarefied gas particles of matter charged negatively are repelled with great velocity from the surface of the kathode, and that these particles travel in straight lines until they strike the walls of the tube or the surface of any object placed in their path. The stream-lines of the particles are known as **kathode rays.**

It was suggested by Crookes that the particles constituting the kathode rays are not solid, liquid, or even gaseous, but that they consist of *ultra-atomic corpuscles*, much smaller than the atom. More recent experiments have shown that the particles have a mass of only about 1/1000 of that of a hydrogen atom, and that whatever gas may be used their mass is the same.

FIG 381.—Röntgen-ray photograph of a human hand.

Röntgen rays.—Prof. Rontgen, when investigating the phenomena of kathode rays in 1895, observed that a covered photographic plate, which was lying by chance near the apparatus, was affected just as though exposed to ordinary light. He concluded that the effect must be due to some unknown form of radiation; and the uncertainty of their character led him to apply the term 'X-rays.' The rays differ from kathode rays in that they pass through many solid substances with comparatively little absorption. Metals and compounds of heavy metals (*e g.*, lead glass) are opaque to the rays, but non-metallic substances are comparatively transparent.

Rontgen observed that the flesh of the hand is transparent, and more so than the bones. Consequently, if the hand be laid on a photographic plate (protected, if necessary, from ordinary light by being enclosed in an opaque paper envelope) and held in the path of the rays, a 'negative' or 'radiograph' is obtained which shows clearly the details of the bones (Fig. 381).

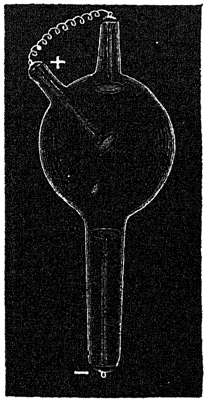


FIG. 382—A Rontgen-ray tube.

Fig. 382 represents a type of tube now used frequently for producing X-rays. The kathode is a concave aluminium disc, and the anode is a round piece of platinum foil placed at the centre of curvature of the kathode and inclined at 45° to its axis. On passing a discharge through the tube, the kathode rays are concentrated upon a point on the surface of the anode, and the X-rays originate from the point where this bombardment occurs.

EXERCISES ON CHAPTER XLI.

1. State the simple laws of electro-magnetic induction, and describe simple experiments illustrating them

2. One pole of a strong bar-magnet is put through a copper ring and quickly taken out again. This is done repeatedly and quickly. Although the magnet and ring are not allowed to rub against each other, the ring becomes slightly heated. Why is this?

3. How could you temporarily stop or weaken the current in a wire, without disconnecting it from the battery, by means of the motion of another wire through which a current is passing?

4. A bar-magnet is allowed to drop vertically through a hank of insulated copper wire placed horizontally. What electrical effects, if any, are produced in the wire?

5. Describe experiments to prove that the current produced by moving a magnet tends to stop the motion, and that the motion produced by the magnetic action of a current tends to stop the current.

6. Supposing that a magnetic north pole is just below the middle of the sheet of paper on which you are writing, and that the south

pole is distant, and supposing that a copper ring is laid flat on the paper and drawn across the middle of the sheet from left to right, draw figures at various points of the course, showing the directions of the induced currents, and give an explanation of your figures.

7. A coil of wire is wound upon the north pole of a magnet, and the pole is then presented to a piece of iron which it draws towards it. Determine the direction of the current induced in the coil.

In what form has energy disappeared to appear as heat developed by the current?

8. Two coils of insulated wire lie, one inside the other, on a table. The outer is in series with a galvanometer; the inner can be connected in series with a battery of negligible resistance. Explain the indications of the galvanometer when a current is made, kept on for some time, and then interrupted in the inner coil.

Would the indications be different on repeating after (1) reducing the number of turns of wire in the inner coil, (2) placing the inner coil upright? Give reasons

9. A few turns of fine insulated wire are wrapped round the centre of an iron nail, and the ends of the wire connected to a galvanometer. State and explain the effect on the galvanometer (1) when the nail is put across the poles of a horse-shoe magnet, slowly or quickly, (2) when it is removed slowly or quickly.

10. State Lenz's law, and describe a simple experiment by which it could be demonstrated.

11. Describe, and illustrate by a diagram, the essential parts of a Rhumkorff coil.

12. Describe a method of obtaining a large E.M.F. by means of the current supplied by a small E.M.F.

PHYSICAL TABLES.*

(1) Equivalents of Metric Weights and Measures in terms of Imperial Units.

METRIC TO IMPERIAL.

Linear Measure:

1 millimetre (mm.) ($\frac{1}{1000}$ m.) - - -	= 0·03937 inch.
1 centimetre ($\frac{1}{100}$ m.) - - - -	= 0·3937 „
1 decimetre ($\frac{1}{10}$ m.) - - - -	= 3·937 inches.
1 metre (m.) - - - - - -	= **39 37 inches.** **3 28 feet.** **1 09 yards.**
1 dekametre (10 m.) - - - -	= 10 936 yards.
1 hectometre (100 m.) - - - -	= 109 36 „
1 kilometre (1000 m.) - - - -	= 0·62 mile.

Square Measure:

1 square centimetre - - - -	= 0·155 square inch.
1 square decimetre (100 square centimetres) - - - - - -	= 15 50 square inches.
1 square metre (100 square decimetres) -	= 10 76 square feet. 1·19 square yards.

Cubic Measure:

1 cubic centimetre - - - - -	= 0·06 cubic inch.
1 cubic decimetre (c.d.) (1000 cubic centimetres) - - - - - -	= 61·02 cubic inches.
1 cubic metre (1000 cubic decimetres) -	= 35·31 cubic feet. 1·31 cubic yards.

Measures of Capacity:

1 centilitre ($\frac{1}{100}$ litre) - - - -	= 0·070 gill.
1 decilitre ($\frac{1}{10}$ litre) - - - -	= 0 176 pint.
1 litre - - - - - - -	= **1·76 pints.**

* Chiefly compiled from "Physical Tables," published by the Smithsonian Institution, Washington, U.S.A.

Mass.		Avoirdupois.
1 milligram ($\frac{1}{1000}$ gm.) - - - -	=	0·015 grain.
1 centigram ($\frac{1}{100}$ gm.) - - - -	=	0·154 ,,
1 decigram ($\frac{1}{10}$ gm.) - - - -	=	1·543 grains.
1 gram (1 gm.) - - - - -	=	15·432 ,,
1 dekagram (10 gm) - - - -	=	5 644 drams.
1 hectogram (100 gm.) - - - -	=	3 527 oz.
1 kilogram (1000 gms.) - - - -	=	**2·20 lb. or 15432·356 grains.**

(2) Equivalents of Imperial Weights and Measures in terms of Metric Units.

IMPERIAL TO METRIC.

Linear Measure:		
1 inch - - - - - -	=	25·4 millimetres.
1 foot (12 inches) - - - -	=	0·30 metre.
1 yard (3 feet) - - - - -	=	**0·914 metre.**
Square Measure·		
1 square inch - - - - -	=	6·45 sq. centimetres.
1 square foot (144 square inches) -	=	9·29 sq. decimetres
1 square yard (9 square feet) - -	=	0·836 square metre.
Cubic Measure.		
1 cubic inch - - - - -	=	16 387 cub. centimetres.
1 cubic foot (1728 cubic inches) - -	=	0·028 cub. metre.
1 cubic yard (27 cubic feet) - -	=	0·764 cub. metre.
Measures of Capacity:		
1 gill - - - - - - - -	=	1·42 decilitres.
1 pint (4 gills) - - - - -	=	0 568 litre.
1 quart (2 pints) - - - -	=	1·136 litres.
1 gallon (4 quarts) - - - -	=	**4 546 litres.**
Apothecaries Measure:		
1 minim - - - - - - -	=	0 059 millilitre.
1 fluid drachm (60 minims) -	=	3·552 millilitres.
1 fluid ounce (8 drachms) - - -	=	2·841 centilitres.
1 pint (20 fluid ounces) - - -	=	0 568 litre.
1 gallon (8 pints or 160 fluid ounces) -	=	**4·546 litres.**
Avoirdupois Weight:		
1 grain - - - - - - -	=	0·065 gram.
1 dram - - - - - - -	=	1·77 grams.
1 ounce (16 drams or 437·5 grains) -	=	28·35 ,,
1 pound (16 ounces or 7000 grains) -	=	**0·4536 kilogram.**

(3) Mensuration.

$\pi = 3{\cdot}14159$.	$\frac{1}{\pi} = 0{\cdot}3183$.
$2\pi = 6{\cdot}28318$.	$\sqrt{\pi} = 1{\cdot}7724$.
$\pi^2 = 9{\cdot}8696$	

Lengths.

Circumference of circle of radius r - - - - -	$=2\pi r$.
" " diameter d - - -	$=\pi d$
" ellipse with semi-axes a and b -	$=2\pi\sqrt{\frac{a^2+b^2}{2}}$.

Areas

Triangle, base b, perpendicular h - - - - -	$=\frac{bh}{2}$.
Rectangle, sides b, h - - - - - - - - -	$=bh$.
Parallelogram, base b, perpendicular h - - -	$=bh$.
Circle, radius r - - - - - - - - -	$=\pi r^2$.
Circle, diameter d - - - - - - - -	$=\frac{\pi d^2}{4}$.
Ellipse, semi-axes a, b - - - - - - -	$=\pi ab$
Surface of cube, edge a - - - - - - -	$=6a^2$.
Surface of sphere, radius r - - - - - -	$=4\pi r^2$.
Curved surface of right cylinder, radius r, height h -	$=2\pi rh$.
Total surface of right cylinder, " " -	$=2\pi r(r+h)$.
Curved surface of right cone, radius r, altitude h, slant height s	$=\pi rs$. $=\pi r\sqrt{r^2+h^2}$.
Total surface of right cone - - - - - -	$=\pi r(s+r)$.

Volumes.

Cube, edge a - - - - - - - - -	$=a^3$.
Rectangular parallelopiped, edges a, b, c - - - -	$=abc$.
Pyramid, area of base a, altitude h - - - - -	$=\frac{ah}{3}$.
Cone, with circular base, radius r, altitude h - -	$=\frac{\pi r^2 h}{3}$.
Cylinder or prism, area of base a, altitude h - -	$=ah$.
Sphere, radius r - - - - - - - -	$=\frac{4}{3}\pi r^3$.

(4) Density, or Mass per Unit Volume.

Common Solids.

Substance	Mass of 1 Cubic Centimetre in Grams	Mass of 1 Cubic Foot in Lbs.
Anthracite	1·4–1·8	87–112
Beeswax	0·96–0·97	60–61
Brass	8·2–8·7	511–542
Brick	2·0–2·2	125–137
Bronze	8·74–8·89	545–555
Butter	0·86–0·87	53–54
Caoutchouc	0·92–0·99	57–62
Chalk	1·9–2·8	118–175
Ebonite	1·15	72
Felspar	2·53–2·58	158–161
Flint	2·63	164
German Silver	8·30–8·77	518–547
Glass (Common)	2·4–2·8	150–175
" (Flint)	2·9–4·5	180–280
Glue	1·27	80
Granite	2·5–3·0	156–187
Graphite	1·9–2·3	120–140
Ice	0·88–0·91	53–57
Ivory	1·83–1·92	114–120
Leather (Dry)	0·86	54
Limestone	2·46–2·86	154–178
Magnetite	4·9–5·2	306–324
Marble	2·5–2·8	157–177
Paper	0·7–1·15	44–72
Paraffin	0·87–0·91	54–57
Pitch	1·07	67
Porcelain	2·3–2·5	143–156
Pumice Stone	0·37–0·9	23–56
Quartz	2·65	165
Resin	1·07	67
Rock Salt	2·28–2·41	142–150
Sandstone	2·2–2·5	137–156
Slag (Furnace)	2·5–3·0	156–187
Slate	2·6–2·7	162–168
Starch	1·53	95
Sugar	1·61	100
Tallow	0·91–0·97	570–605

Metals

Substance.	Mass of 1 Cubic Centimetre in Grams	Mass of 1 Cubic Foot in Lbs
Aluminium	2·56–2·80	160–175
Copper	8·80–8·95	549–558
Gold	19·26–19·34	1202–1207
Iron	7·03–7·90	439–493
Lead	11·00–11·36	686–709
Magnesium	1·69–1·75	105–109
Mercury	13·596	848
Nickel	8·30–8·90	517–555
Platinum	21·20–21·70	1322–1354
Silver	10·40–10·57	649–659
Tin	6·97–7·30	435–455
Zinc	7·04–7·19	439–449

Liquids.

Substance.	Mass of 1 Cubic Centimetre in Grams.	Mass of 1 Cubic Foot in Lbs
Alcohol (Ethyl)	0·791	49·4
„ (Methyl)	0·810	50·5
„ (Proof Spirit)	0·916	57·2
Benzine	0·899	56·1
Carbolic Acid (Crude)	0·950–0·965	59·2–60·2
Carbon Bisulphide	1·293	80·6
Ether	0·736	45·9
Glycerin	1·260	78·6
Naphtha (Wood)	0·848–0·810	52·9–50·5
Oil, Camphor	0·910	56·8
„ Castor	0·969	60·5
„ Linseed	0·942	58·8
„ Mineral	0·900–0·925	56·2–57·7
„ Olive	0·918	57·3
„ Turpentine	0·873	54·2
Petroleum	0·878	54·8
Sea Water	1·025	64·0
Water	1·000	62·4

Gases.

Substance	Specific Gravity	Mass of 1 Litre in Grams	Mass of 1 Cubic Foot in Lbs.
Air - - -	1·000	1·293	0·0807
Carbon Dioxide -	1·529	1·974	0·1232
Hydrogen - -	0·0696	0·089	0·0056
Nitrogen - -	0·972	1·250	0·0780
Oxygen - - -	1·105	1·430	0·0893
Steam at 100° C -	0·469	0·581	0·0363

Woods.

Substance.	Mass of 1 Cubic Centimetre in Grams.	Mass of 1 Cubic Foot in Lbs.
Ash - - - -	0·65–0·85	40–53
Beech - - -	0·70–0·90	43–56
Birch - - - -	0·51–0·77	32–48
Box - - - -	0·95–1·16	59–72
Cedar - - -	0·49–0·57	30–35
Cherry - - -	0·70–0·90	43–56
Cork - - - -	0·22–0·26	14–16
Ebony - - -	1·11–1·33	69–83
Elm - - - -	0·54–0·60	34–37
Lignum Vitae - -	1·17–1·33	73–83
Lime - - - -	0·32–0·59	20–37
Mahogany (Spanish) -	0·85	53
Maple - - -	0·62–0·75	39–47
Oak - - - -	0·60–0·90	37–56
Poplar - - -	0·35–0·5	22–31
Sycamore - - -	0·40–0·60	24–37
Teak - - - -	0·66–0·98	41–61
Walnut - - -	0·64–0·70	40–43
Willow - - -	0·40–0·60	24–37

(5) Acceleration due to Gravity.

Latitude.	Increase of Velocity per Second due to the Earth's Attraction		
30°	979·3 cm.	385·5 in.	32·1 ft.
40	980·1	385·9	32·1
50	981·0	386·2	32·2
60	981·9	386·6	32·2

(6) Length of Seconds Pendulum.

Latitude.	Centimetres.	Inches.
30°	99 2	39 1
40	99 3	39 1
50	99·4	39·1
60	99 5	39 2

(7) Pressure.

A standard atmosphere is the pressure of a vertical column of pure mercury, having a height of 760 mm. and temperature 0° C. under standard gravity at latitude 45° and at sea level.

1 standard atmosphere = 1033 grams per sq. cm.
= 14·7 lbs. per sq. in.
= 2116 lbs per sq ft.

Pressure of mercurial column 1 inch high
= 34·5 grams per sq. cm.
= 0 491 lbs. per sq. in.

A column of water 2·3 ft. high corresponds to a pressure of 1 lb. per square inch.

(8) Imperial Standard Wire Gauge.

Descriptive Number.	Diameter.		Area of Cross Section	
	Inches	Centimetres.	Square Inches	Square Centimetres.
14	0·080	0 203	0·0050	0 0324
16	0·064	0·162	0 0032	0·0207
18	0·048	0·121	0 0018	0·0116
20	0·036	0 091	0 0010	0·0065
22	0 028	0·071	0 0006	0·0039
24	0·022	0 055	0·0004	0·0024
26	0·018	0·045	0 00025	0·0016
27	0·016	0·041	0·00021	0·00114
28	0 015	0·037	0 00017	0 00111
30	0·012	0·031	0 00012	0 00078
32	0·011	0·027	0·00009	0·00059
34	0 009	0·023	0 00007	0·00043
36	0·008	0·019	0 00004	0 00029
38	0 006	0·015	0·00003	0 00018
40	0·005	0 012	0·00002	0·00012

(9) Melting Points and Latent Heats of Fusion.

	Melting Point.	Latent Heat.
Beeswax - - - - - -	61·8° C.	42·3
Butter - - - - - -	28–33	—
Ice - - - - - -	0	79·3
Lard - - - - - -	36–40	—
Naphthalene - - - - -	79·8	35·6
Paraffin Wax (ordinary white solid)	50–55	35·1
Spermaceti - - - - -	43·9	36·9
Sulphur - - - - -	115	—

(10) Boiling Points and Latent Heats of Vaporization.

	Boiling Point.	Latent Heat.
Alcohol (Ethyl) - - - -	78° C.	205
Benzene (or Benzol) - - -	80–83	92·9
Carbon Bisulphide - - -	47	84
Ether - - - - - -	35	90·4
Hydrochloric Acid - - -	110	—
Nitric Acid - - - - -	86	—
Steam - - - - - -	100	536
Sulphuric Acid - - - -	338	—
Turpentine - - - - -	159	74

(11) Coefficients of Linear Expansion of Solids.

	Expansion per Degree C.
Aluminium - - - - - -	0·0000231
Brass - - - - - -	0·0000193
Copper - - - - - -	0·0000168
Glass (Tube) - - - - -	0·0000083
Iron - - - - - - -	0·0000109
Lead - - - - - -	0·0000292
Platinum - - - - - -	0·0000089
Silver - - - - - -	0·0000192
Zinc - - - - - - -	0·0000292

(12) Coefficients of Cubical Expansion of Liquids.

Alcohol (Ethyl), (0° to 80°)	0 00104
Benzene	0 00138
Carbon Bisulphide	0 00147
Ether (−15° to +38°)	0 00215
Glycerin	0 00053
Mercury	0 00018
Olive Oil	0 00074
Petroleum	0 00099
Sulphuric Acid	0 00049
Turpentine	0 00105
Water (10° − 100° C)	0·00043

(13) Volume and Density of Water at different Temperatures.

(Metric Units.)

$t°$.	Volume of Unit Mass	Density.	$t°$.	Volume of Unit Mass.	Density.
0°	1·0001	0 9999	55°	1·0144	0 9858
4°	1·0000	1 0000	60°	1·0170	0·9834
10°	1 0003	0 9997	65°	1·0197	0 9807
15°	1 0009	0 9991	70°	1·0226	0·9780
20°	1·0018	0·9982	75°	1·0257	0·9750
25°	1 0029	0 9971	80°	1 0289	0·9720
30°	1 0044	0·9957	85°	1·0322	0·9688
35°	1·0059	0·9941	90°	1·0357	0·9656
40°	1·0077	0·9924	95°	1·0394	0 9621
45°	1·0097	0·9903	100°	1·0433	0 9587
50°	1 0120	0 9882			

(14) Coefficients of Expansion of Gases.

	Increase of Pressure at Constant Volume.	Increase of Volume at Constant Pressure
Hydrogen	0 003669	0·003661
Air	0·003665	0 003671
Carbon Dioxide	0·003706	0·003710

(20) Coefficients of Thermal Conductivities.

Substance.	Coefficient (k)	Substance.	Coefficient (k)
Air	0·000568	Marble	0·005
Aluminium 0°	0·343	Mercury 0°	0·0148
100°	0·362	50°	0·0189
Asbestos	0·00043	Olive Oil	0·000395
Beeswax	0·00009	Paraffin 0°	0·00023
Brass 0°	0·2041	100°	0·00168
100°	0·2540	Plaster of Paris	0·0013
Carbon Dioxide	0·000307	Sawdust	0·00012
Copper 0°	0·7189	Silver 0°	0·960
100°	0·7226	Slate	0·0034
Flannel	0·000035	Turpentine	0·00325
Glass	0·0023	Vulcanite	0·00087
Granite	0·0053	Water 0°	0·0012
Iron 0°	0·1665	30°	0·0016
100°	0·1627	Zinc	0·303
Lead 0°	0·0836		
100°	0·0764		

(21) Indices of Refraction Relative to Air.

Solids.			
Crown Glass	1·53	Fluor Spar	1·43
Diamond	2·42	Ice	1·31
Emerald	1·58	Rock Salt	1·54
Flint Glass	1·62	Ruby	1·77
Liquids.			
Alcohol	1·36	Olive Oil	1·47
Benzene	1·49	Sulphuric Acid	1·42
Carbon Bisulphide	1·63	Turpentine	1·46
Glycerin	1·47	Water	1·33

(22) Velocity of Sound.

Substance.	Metres per Second.	Feet per Second.
Aluminium	5104	16740
Brass	3500	11480
Copper	3560	11670
Iron	5130	16820
Platinum	2690	8815
Silver	2610	8553
Marble	3810	12500
Slate	4510	14800
Glass	5000-6000	16410-19690
Ivory	3013	9886
Ash, along the fibre	4760	15310
„ across the rings	1390	4570
„ along the rings	1260	4140
Oak	3850	12620
Pine	3320	10900
Poplar	4280	14050
Alcohol	1264	4148
Turpentine	1212	3977
Water	1437	4714
Temp. 0° C.		
Air	332	1090
Carbon Dioxide	262	858
Ammonia	415	1361
Hydrogen	1286	4221
Illuminating Gas	490	1609
Oxygen	317	1041

(23) Electro-Chemical Equivalents.

Element.	Atomic Weight (O=16).	Chemical Equivalent.	Electro-Chemical Equivalent (in Grams per Coulomb)
Aluminium	27·1	8·96	0·0000935
Copper	63·6	31·54	0·0003293
Gold	197·2	65·21	0·0006809
Hydrogen	1·008	(1)	0·0000104
Oxygen	16·00	7·935	0·0000828
Nickel	58·7	29·12	0·0003040
Silver	107·93	107·73	0·0011180
Zinc	65·4	32·45	0·0003387

(24) Mean Values during 1908 of the Magnetic Elements at various Observatories.

From Kew Summaries of Results of Terrestrial Magnetism.

Place.	Latitude	Longitude.	Declination	Inclination	Force in C G S Units	
					Hori-zontal	Vertical.
	° ′	° ′	° ′	° ′		
N Magnetic Pole	70 5 N	96 33 W	—	90 N	—	—
Sitka (Alaska)	57 3 N	135 20 W	30 10 7 E	74 36 5 N	0 1557	0 5653
Eskdalemuir (Dumfries)	55 19 N	3 12 W	18 33 3 W	69 37 3 N	0 1683	0 4531
Stonyhurst - -	53 51 N	2 28 W	17 35 6 W	68 44 2 N	0 1743	0 4480
Wilhelmshaven -	53 32 N	8 9 E	11 54 1 W	67 31 N	0 1817	0 4390
Potsdam - -	52 23 N	13 4 E	9 18 0 W	66 19 5 N	0 1885	0 4299
De Bilt (Utrecht) -	52 5 N	5 11 E	13 12 8 W	66 47 2 N	0 1855	0 433
Valencia (Ireland)	51 56 N	10 15 W	20 55 7 W	68 16 3 N	0 1787	0 4484
Kew - - -	51 28 N	0 19 W	16 16 9 W	67 0 9 N	0 1851	0 4365
Greenwich - -	51 28 N	0	15 53 5 W	66 56 3 N	0 1853	0 4352
Uccle (Brussels) -	50 48 N	4 21 E	13 36 7 W	66 1 6 N	0 1906	0 4287
Falmouth - -	50 0 N	5 5 W	17 54 7 W	66 31 4 N	0 1880	0 4328
Val Joyeux (near Paris)	48 49 N	2 1 E	14 39 6 W	64 44 6 N	0 1973	0 4183
Odessa - -	46 26 N	30 46 E	3 53 5 W	62 22 1 N	0 2176	0 4156
Pola - - -	44 52 N	13 51 E	8 43 2 W	60 6 8 N	0 2221	0 3864
Agincourt (Toronto)	43 47 N	79 16 W	5 54 0 W	74 36 9 N	0 1634	0 5939
Coimbra - -	40 12 N	8 25 W	16 46 2 W	58 57 3 N	0 2295	0 3812
Baldwin (Kansas)	38 47 N	95 10 W	8 33 0 E	68 47 8 N	0 2171	0 5597
Cheltenham (Maryland)	38 44 N	76 50 W	5 31 1 W	70 30 5 N	0 1994	0 5634
Athens - -	37 58 N	23 43 E	4 52 9 W	52 11·7 N	0 2620	0 3361
San Fernando -	36 28 N	6 12 W	15 25 6 W	54 48 4 N	0 2483	0 3521
Tokio - - -	35 41 N	139 45 E	4 53 2 W	48 56 9 N	0 2999	0 3444
Zi-ka-wei (China)	31 12 N	121 26 E	2 35 4 W	45 35·4 N	0 3308	0 3377
Dehra Dun - -	30 19 N	78 3 E	2 36 7 E	43 42 2 N	0 3329	0 3182
Helwan - -	29 52 N	31 20 E	2 55 7 W	40 39 4 N	0 3003	0 2579
Barrackpore (Calcutta)	22 46 N	88 22 E	1 5 6 E	30 34 6 N	0·3730	0 2204
Hongkong - -	22 18 N	114 10 E	0 3 8 E	31 2·4 N	0 3706	0 2230
Honolulu - -	21 19 N	158 4 W	9 25 7 E	39 55 3 N	0 2919	0 2442
Toungoo - -	18 56 N	96 27 E	0 34 4 E	23 1 9 N	0 3876	0 1648
Alibág (Bombay)	18 38 N	72 52 E	1 2 2 E	23 21 8 N	0 3686	0·1592
Vieques (Porto Rico)	18 9 N	65 26 W	2 2 5 W	49 36·3 N	0 2905	0 3414
Kodaikánal - -	10 14 N	77 28 E	0 45 4 W	3 33 2 N	0·3743	0 0232
St. Paul de Loanda	8 48 S	13 13 E	16 20 4 W	35 21 7 S	0 2018	0 1432
Apia (Samoa) -	13 48 S	171 46 W	9 41 9 E	29 21 7 S	0 3561	0 2004
Mauritius - -	20 6 S	57 33 E	9 14·3 W	53 44 9 S	0 2341	0 3193
S. Magnetic Pole	72 25 S	154 0 E	—	90 S	—	—

(25) Electromotive Force of Voltaic Cells.

Name of Cell.	Solution for Negative Pole	Solution for Positive Pole	E M F. in Volts
Bunsen (i) -	1 part H_2SO_4 to 12 parts water	Fuming HNO_3	1·94
" (ii) -	"	$HNO_3 (\Delta = 1{\cdot}38)$	1·86
Bichromate -	12 parts $K_2Cr_2O_7$ to 25 parts H_2SO_4 and 100 parts H_2O	1 part	2·00
Daniell (i) -	1 part H_2SO_4 to 4 parts H_2O	Saturated solution of $CuSO_4 5H_2O$	1·06
(ii) -	1 part H_2SO_4 to 12 parts H_2O	"	1·09
Grove - -	1 part H_2SO_4 to 12 parts H_2O	Fuming HNO_3	1·93
Leclanché - -	Sal-ammoniac	—	1·46

(26) Specific Resistance of Metals and Alloys.

Substance.	Specific Resistance (in Microhms or Millionths of an Ohm) at 0° C.	Temperature Coefficient.
Elements—		
Silver (annealed) - -	1·5–1·7	0·0038
Copper (annealed) - -	1·58–2·20	0·0043
Tungsten - - - -	5·0	0·0051
Iron (soft wire) - - -	9·7–12·0	0·0058
Platinum - - - -	9·0–15·0	0·0034
Mercury - - - -	94·1	0·0008
Alloys—		
German Silver {Cu 50 %, Zn 30, Ni 20} -	20–26	0·0004
Phosphor-bronze - -	8·48	0·0007
Platinoid - - - -	43·60	0·00025
Manganin {Cu 84 %, Mn 12, Ni 4} -	42·28	0·00002
Constantan or Eureka -	48·0	±0·00001
Non-metal—		
Carbon (lamp filament) -	$4{\cdot}0 \times 10^6$	−0·0003

(27) Trigonometrical Ratios.

Angle Deg	Sine.	Tangent	Cotangent.	Cosine	
0°	0	0	∞	1	90°
1	0.0175	0.0175	57.2900	0.9998	89
2	0.0349	0.0349	28.6363	0.9994	88
3	0.0523	0.0524	19.0811	0.9986	87
4	0.0698	0.0699	14.3006	0.9976	86
5	0.0872	0.0875	11.4301	0.9962	85
6	0.1045	0.1051	9.5144	0.9945	84
7	0.1219	0.1228	8.1443	0.9925	83
8	0.1392	0.1405	7.1154	0.9903	82
9	0.1564	0.1584	6.3138	0.9877	81
10	0.1736	0.1763	5.6713	0.9848	80
11	0.1908	0.1944	5.1446	0.9816	79
12	0.2079	0.2126	4.7046	0.9781	78
13	0.2250	0.2309	4.3315	0.9744	77
14	0.2419	0.2493	4.0108	0.9703	76
15	0.2588	0.2679	3.7321	0.9659	75
16	0.2756	0.2867	3.4874	0.9613	74
17	0.2924	0.3057	3.2709	0.9563	73
18	0.3090	0.3249	3.0777	0.9511	72
19	0.3256	0.3443	2.9042	0.9455	71
20	0.3420	0.3640	2.7475	0.9397	70
21	0.3584	0.3839	2.6051	0.9336	69
22	0.3746	0.4040	2.4751	0.9272	68
23	0.3907	0.4245	2.3559	0.9205	67
24	0.4067	0.4452	2.2460	0.9135	66
25	0.4226	0.4663	2.1445	0.9063	65
26	0.4384	0.4877	2.0503	0.8988	64
27	0.4540	0.5095	1.9626	0.8910	63
28	0.4695	0.5317	1.8807	0.8829	62
29	0.4848	0.5543	1.8040	0.8746	61
30	0.5000	0.5774	1.7321	0.8660	60
31	0.5150	0.6009	1.6643	0.8572	59
32	0.5299	0.6249	1.6003	0.8480	58
33	0.5446	0.6494	1.5399	0.8387	57
34	0.5592	0.6745	1.4826	0.8290	56
35	0.5736	0.7002	1.4281	0.8192	55
36	0.5878	0.7265	1.3764	0.8090	54
37	0.6018	0.7536	1.3270	0.7986	53
38	0.6157	0.7813	1.2799	0.7880	52
39	0.6293	0.8098	1.2349	0.7771	51
40	0.6428	0.8391	1.1918	0.7660	50
41	0.6561	0.8693	1.1504	0.7547	49
42	0.6691	0.9004	1.1106	0.7431	48
43	0.6820	0.9325	1.0724	0.7314	47
44	0.6947	0.9657	1.0355	0.7193	46
45	0.7071	1.0000	1.0000	0.7071	45
	Cosine.	Cotangent.	Tangent.	Sine.	Angle. Deg

TYPICAL EXAMINATION PAPERS.

MADRAS SECONDARY SCHOOL LEAVING CERTIFICATE.

Physics.

Eight questions only to be attempted.

1. How would you proceed to obtain data to enable you to plot a curve showing the relation between the stretching force and consequent elongation of a piece of rubber?
If the curve assumed the form of a straight line, what would you infer?

2 How would you determine exactly the thickness of a piece of platinum foil?
What other experiment would you perform to verify your result?

3 A body floats in water; how would you experimentally determine the exact relation between the weight of the body and the weight of the water displaced by the body?
A salt solution has a density of 1·120 gm. per c.c.; how many c c. of water must be added to one litre to reduce the density to 1·115 gm. per c.c.?

4. Describe exactly the construction and use of a vernier graduated to measure one-tenth of a millimetre.

5. Describe in detail any method for the determination of the specific heat of alcohol.

6. What is meant by "Capacity for heat"?
Describe any experiment you have performed to compare the capacities for heat of any two substances.
State the nature of your results.

7. On pouring cold water into a beaker a liquid is sometimes deposited on the outside surface of the beaker. What is this liquid? Why does it make its appearance, and how is this phenomenon utilized in determining the humidity of the atmosphere?

8. Without using an air-pump, how would you determine the weight of one cubic centimetre of air?

9. Describe in detail the construction and use of any thermometer which is used for a special purpose.

10. What results relating to the formation of an image by a single plane mirror have you obtained experimentally?
How did you arrive at your results?

11. What is meant by the pole of a magnet?
How would you determine experimentally the positions of the poles of a mass of loadstone? What magnetic laws enable you to make the discovery?

12 Describe any one experiment, other than the electrolysis of water, you have performed with reference to any of the effects produced by an electric current.
What did you infer from your experiment?

MADRAS UNIVERSITY MATRICULATION.

Elementary Physics.

1. You are given a small piece of lead, and are required to find out by means of it what is the weight of a cubic inch of lead. What apparatus would you need, and how would you use it?

2 How would you determine experimentally whether the surface of a mirror is level or not?

3. Describe how a string of a musical instrument moves when it emits a note. How is the sound transmitted to the ear?

4. How would you compare the heat given out by equal masses of copper, iron, and lead in cooling from 100° C. to 0° C?

5. Explain the difference in the effect of heating a vessel of water at the top and at the bottom.

6 State the laws of reflection of light. Describe what you see when you look at the image of a pencil which you hold upright in front of a convex mirror and which you move slowly away from the mirror.

7. Describe the Leyden jar, and explain the use of each part

8. What changes occur in a Grove's battery when it is sending a current? Why is it called a constant battery? To what is the constancy due?

9. Explain what is meant by energy. How would you show that when a glass rod is electrified it possesses energy?

Elementary Science.

PART I.

1. What is the unit of time employed in science? How is the pendulum used to measure time in terms of this unit?

2. What is the principle of Archimedes? Describe an experiment by which it is proved. Explain how it is applied in finding the specific gravity of a substance.

3. Draw a diagram showing how a real image of an object is formed by a convex lens. State the law regarding the distances of the image and the object from the lens, and describe how it is verified.

4. Describe any instrument for measuring the strength of an electric current, and explain its action.

Elementary Science.

PART I.

1. Describe any two methods of determining the area of a plane figure with an irregular boundary.

2 Describe a method of determining the specific gravity of a solid body which floats in water. How would you determine the specific gravity of sugar?

3. A vertical slit is made in a window, and the upper half of it is covered with deep red glass and the lower half with deep blue glass. Show by a diagram how the slit will appear if you look at it through a prism with its edge vertical and turned towards your right. Account for the appearances seen.

4 A Grove's battery is enclosed in a box, its poles alone being outside. How would you determine the nature of the poles without opening the box?

PUNJAB UNIVERSITY MATRICULATION.

Physics.

1 State fully the points of difference between the *mass* and the *weight* of a body. How would you show that the weight of a body varies as its mass?

2. Explain as fully as you can the terms *force*, *work* and *energy*. Give examples of a correct use of these terms. Show with reference to any simple machine that you can at best get as much work out of a machine as you put into it.

3. Explain the principle of the screw as a mechanical power, and show its relation to the inclined plane.

4. Describe the best kind of barometer you have seen. What will be the effect of making a *small* hole in the barometer tube, (1) above the level of mercury in the tube, (2) somewhere between the level of mercury in the tube and that in the cistern? How will you show that there is no air in the upper part of a filled barometer?

5. How can you show that, as a rule, black substances are good absorbents of heat? How are the radiation, reflection and absorption of heat related to one another?

6. Show by means of diagrams how with a double convex lens an image of a lighted candle may be seen, (1) inverted and magnified, (2) inverted and minified, (3) erect and magnified.

7. What is an echo? Mention the *essential* conditions for the production of a single and a multiple echo

Compare the intensities of sound at two places 700 and 1,200 feet respectively from the origin of sound.

8. A positively charged body is held *near* a wall. Will the wall be charged? If so, how will you proceed to show that it is charged?

9. Describe, as fully as you can, some one method of comparing the strength of two different batteries.

10. As far as our present knowledge extends, what similarity and what dissimilarity appears to you to exist between current electricity and frictional electricity?

Physics.

1. Draw diagrams showing two arrangements of pulleys by which a power of 3 lbs. may be just able to support a weight of 24 lbs.

2. Explain the terms Kinetic energy and Potential energy.

A body weighing 2 tons is moving at the rate of 25 miles an hour. Find its Kinetic energy in absolute units and in foot-pounds.

3. What is meant by Latent heat of fusion?

A ball of iron weighing 5 lbs and heated to 250° C. is placed in melting ice. Find how much of the ice will be converted into water. The specific heat of iron = ·114.

4. Describe any two experiments to show that liquids generally are bad conductors of heat.

5. (*a*) State the conditions that affect the intensity of sound.

(*b*) How would you experimentally verify the sound law of the transverse vibrations of strings?

6. What is meant by the law of inverse squares? Show how the law is made use of in comparing two sources of light.

7. Describe and sketch the astronomical telescope

8. State the experiments and the reasoning from which it is inferred that the earth is a magnet.

9. Describe any experiments to prove that equal quantities of + electricity and − electricity are always produced together.

10. (*a*) Explain the cause of Polarisation, and describe the chief methods of preventing it.

(*b*) Write out Ohm's law.

Physics.

1. Why does a rope dancer use a long pole? Why does a man carrying a heavy body in his right hand instinctively throw out his left arm?

2. State the principle of the syphon. Describe very briefly a few applications (omitting the fountain) of the principle of the syphon.

3. Give a sketch of the ordinary Tate's air pump. Describe its action as clearly as you can. Particularly indicate the positions of the valves, and explain their action.

4. What is meant by radiant heat? Describe any simple experiment showing that a black piece of cloth absorbs more radiant heat than a white piece.

5. Describe the construction, action and use of a camera obscura What is the simplest arrangement of lenses forming a telescope?

6. What is meant by a sound wave? Define pitch, intensity and timbre of a note, and state upon what physical elements each of them depends.

7. Describe a method of charging a Leyden jar negatively by means of the prime conductor of an ordinary glass plate machine. Can we also make use of the negative electricity of the glass plate? How can you show that negative electricity is also produced when a plate machine is worked?

8. Describe an ordinary voltaic cell. Explain clearly why such a cell gives a very much weaker current than, say, a Bunsen's cell. Is it absolutely necessary to use a zinc plate in all kinds of cells?

9. Describe the wedge as a mechanical power. Some writers do not include the wedge in the list of simple machines. Why?

ALLAHABAD UNIVERSITY MATRICULATION.

Physics.

N.B.—Nine questions only to be attempted.

1. How would you measure the area of an irregular figure drawn on a sheet of paper?

2. How would you measure the diameter of a very thin wire?

3. How would you test a coin to see if it is pure gold?

4. What is meant by the specific heat of a substance?

5. How would you measure the specific heat of copper?

6. Given an ungraduated thermometer, how would you graduate it?

7. State the laws of reflection.

8. Draw the image of an object placed in front of a plane mirror.

9. Draw the image (of a pin) formed by a double convex lens.

10. How would you show that equal and opposite charges are produced by electrostatic induction?

11. What is a compass? What is it used for?

CALCUTTA UNIVERSITY INTERMEDIATE EXAMINATION.

Physics.

First Paper.

Only Six questions are to be answered, which must include at least Two out of Group B.

Group A.

1. State the principle of conservation of energy, and give an illustration.

A railway train is moving with uniform speed (*a*) on a level country, (*b*) uphill. Explain how the energy supplied by burning coal in the engine is being expended in the two cases.

2. Describe experiments to show that water exerts pressure in all directions.

A plate of 10 metres square is placed horizontally 1 metre below the surface of water, when the height of the mercury barometer is 760 millimetres. What will be the total pressure on the plate? (The density of mercury = 13·6 gm. per c.c.)

3. If you were given a piece of wood cut in the form of a cube, how would you very roughly determine its specific gravity without using a balance?

A Nicholson's hydrometer weighs 200 grammes and requires 50 grammes in the upper pan to sink it to the fixed mark. What weight must be added to or subtracted from the weights in the upper pan to bring it to the fixed mark, when it is placed in a liquid of specific gravity 1·2?

4. State Boyle's law and describe experiments made to verify it.

A faulty barometer contains some air which occupies 10 c.c. If it stands at 740 mm, when a true barometer indicates a pressure of 750 mm., find the volume the air will occupy at the standard pressure 760 mm.

5. The mean coefficient of expansion of mercury between 0° C. and t° C. is α, and the following table gives corresponding values of a and t:

t	α
0	0·00018179
100	0 00018216
150	0 00018261
200	0 00018323
250	0·00018403
300	0·00018500

Plot a curve to illustrate the relation between a and t, and find from your curve the value of a at 220° C.

6 Indicate what goes on in the body emitting a musical note, and the medium which transmits it

You are given a tall jar, the requisite quantity of water, and a tuning-fork. Describe how you will find the vibration frequency of the tuning-fork.

GROUP B.

7. What do you mean by the specific heat of a substance?

A lump of platinum weighing one hundred grammes is heated in a flame until its temperature has reached that of the flame. It is then removed and dropped quickly into a calorimeter which has a water equivalent of 5 grammes and contains 495 grammes of water If the temperature of the water rises from 22° C. to 30° C, find the temperature of the flame. (The specific heat of platinum is ·0365)

8. What do you mean by the expression *vapour tension*?

Three barometer tubes are filled with mercury and their open ends plunged into a vessel of mercury in the usual way. Into the vacuum of one a little air is introduced, and into that of another a few drops of water. What will be the effect in each case on the height of the mercury of plunging the (three) tubes further into the cistern? Give reasons for your answer

9. What is the cause of the cooling effect produced in a room when a grass (khus khus) screen moistened with water is placed in front of the door?

Steam at 100° C. is allowed to pass into a vessel containing 10 grammes of ice and 100 grammes of water at 0° C, until all the ice is melted and the temperature is raised to 5° C. Neglecting the water equivalent of the vessel and the loss due to radiation, etc, calculate how much steam is condensed. (The latent heat of steam = 536, and the latent heat of water = 80.)

SECOND PAPER.

Only Seven questions are to be answered, which must include either question 1 or question 2 of Group A, and question 1 of Group B.

GROUP A.

1 Describe with full experimental details a method of determining the focal length of a convex lens

Solve the following problem by drawing a diagram to scale, with the help of the squared paper provided.

An object 6 centimetres high is placed at a distance of 40 centimetres from a thin convex lens, and an image is formed on the other side of the lens, the height of the image being 4 centimetres. Find the focal length of the lens approximately

2. Explain the formation of images by a concave mirror.

An object, of height 5 centimetres, is placed at a distance of 40 centimetres from the surface of a concave mirror (measured along the

axis of the mirror) whose radius of curvature is 20 centimetres. Find the position and the size of the image, without calculation, as in the preceding problem

3. Why do opaque objects appear coloured? Why does a mixture of ordinary blue and yellow pigments appear green? How would you make a stick of red sealing-wax appear black?

4. You are given a slit, a convex lens, a screen, and a prism. Show how you would arrange these to obtain a pure spectrum. Explain how it is that the spectrum is not pure when the lens is not used.

Group B

1. A metal globe (insulated) is charged with positive electricity. (*a*) Another insulated metal globe without any charge is placed near it. (*b*) The latter globe is momentarily connected with the earth. (*c*) Both are then enclosed in an insulated metal case (*d*) The case is connected to earth

Explain what will happen in each case, and how you will proceed to test your conclusions

2. Describe any two arrangements for maintaining a steady current of electricity in a given wire Explain the mode of supply of energy for maintaining it in the two cases. What becomes of the energy as it continues to flow?

3 State Ohm's law.

A battery of ten cells, joined in series, yields a current of 1 ampere when the external resistance is 10 ohms, and a current of ·6 ampere when the external resistance is 20 ohms Find the E M F. and the internal resistance of one of the cells (these being the same for all).

4. You are provided with a suitable voltaic cell and a suitable galvanometer, a soft iron rod, and two pieces of wire, one of considerable length and the other short. Explain, with the help of a diagram, how you would arrange to demonstrate the production of induced currents.

5 A small magnet, movable about a vertical axis, is placed at the centre of a circular coil lying in the plane of the magnetic meridian. (*a*) At first no current passes. (*b*) A current is passed. (*c*) The number of turns in the coils is increased, the current being unchanged in strength. (*d*) The coil is slowly rotated about the vertical diameter. Explain what happens in all these cases.

6. Describe the various ways of magnetising a piece of soft iron.

How would you trace the lines of force in the neighbourhood of a bar-magnet? Indicate how the shape of the lines you get depends on the earth's magnetism.

ANSWERS TO NUMERICAL EXERCISES

Chapter I. (p. 16.)

1 1093 yd. 1 ft. 10 in. **2.** 4808 2 m. **3.** 24 ; 0 6 m.

4. 29 92 in. **5.** $\frac{1}{25\ 4}$; $\frac{1}{3\ 047}$; $\frac{1}{2\ 54}$.

Chapter II. (p. 24.)

1. 9000 sq. cm. ; 150 sq in. , 8 976 sq m. **2** 6 metres ; 15 ft.
3. 96 sq. cm. ; 24 sq cm **4.** 78° 7, 57°, 44° 4 ; 14 7 sq. cm
5. 450 sq. ft. **6.** 24854 8 miles **7.** $47\frac{3}{4}$ **8.** 79
9. 706 9 lb. **10.** 346 9 sq. yd. **11** 744 sq ft.
12 1 82 sq ft. **13.** 1018 sq. in. **14** 21913 sq ft.

Chapter III. (p. 32.)

1. 28·316. **2** 11 0; 219 9. **3** 64969 0.
4. 13 c. ft. ; $81\frac{1}{4}$; 812 lb. **5.** 2262 0 c c. **6.** 9352 0 c. ft
7. 500 sq ft ; 645 3 c. ft. **8.** 117290 c. ft **9.** 532 c. in.
10 648,000 **11.** 59 55 c c. **12.** 1 2 : 3.
16. $\frac{2}{3}$

Chapter IV. (p. 48)

1. 708 lb. per c. ft. **2.** 112 3 cm. **3.** 0 7055 gm per c c.
4 4 5 cm **5.** 27 : 10 **6.** 68 48 gm **7.** 2 mm
13. 486 9 lb. per c. ft. **14.** 77 2 gm.

Chapter VI. (p. 74.)

1. 100 gm per sq. cm . 1360 gm. per sq cm. **2** $4\ 273 \times 10^4$ gm.
3. 4100 gm. per sq cm ; 3203 lb. per sq ft. **4.** 138 2 ft.

5. 49 1 ft **8.** 11 54; 0 88. **9.** 1 06. **10.** 0 5; 93 lb. **13.** 82 c. ft.; 104 3 c. ft. **18.** 36 42 c.c.; 7 55 gm per c.c. **21.** 16 gm **22** 0 91; 1 045 **24.** 10 5 c.c. **25.** 3 lb; 2 67; 83 c in.; 4 25 lb **26.** 6437 5 c. yd. **27.** 7 59. **28.** 0 794.

Chapter VII. (p. 90)

1. 34 64 in **2** 34 ft., 13 6 in. **3.** 1 053 kg per sq cm. **4.** 1401 lb per sq in **5** 306 c.c. **6** 1 327 gm **7.** 33 124 gm.; 18 004 gm **8.** 76 cm; 561 7 cm **9.** 6 5 gm. per sq. cm; 0 0925 lb per sq in **18** 3 675 lb.; 41,270 tons.

Chapter VIII. (p 108)

4 $10\sqrt{2}$ lb; direction, S.W. **7** 156 lb **8.** 10 lb **12** 90°, 143° 1, 126° 9 **13.** 72°, 158°, 130° **14.** $\sqrt{3}$ lb. **15** $10\sqrt{3}$ gm **17** 10 ft. per sec. **18** 3870 lb **19.** 10 sec.; 150 ft.

Chapter IX. (p. 119.)

2. 5600 ft lb. **3.** 240,000 ft lb **4** $1400\sqrt{3}$ ft. lb **5.** $9\frac{1}{11}$ h p **6** 400 h p. **11** 12,000 ft. lb.; $\frac{5}{12}$. **12.** 200 ft lb **13** 16 hours **14** 118,800 ft. lb. **15.** 24×10^3 ft lb. **16** 250 ft poundals. **17.** 300 ft. lb

Chapter X. (p 130.)

1. 4 lb. **4** 33 3 cm from fulcrum **5.** 30 lb. **6** 2 cwt; $1\frac{1}{4}$ cwt.

Chapter XI. (p. 140.)

8. 4 5. **9** 90 6 % **10** 45°; 0 084 kgm. metres. **11.** 12,000 ft. lb. **12.** $\frac{2}{25}$.

Chapter XII. (p. 152.)

9. 21° 7 C; −17° 8 C; −40° C. **10.** 181° 4 F.; 59° F.; 23° F. **11.** (*a*) −435° 1 F., −421° 6 F.; (*b*) −38° 56 F., 674° 6 F.; (*c*) 207° 7 F., 1517° F. **14.** +0° 7 C; −0° 7 C.

Chapter XIII. (p. 171.)

3. 2 0066 m.; 153° 6 C. **4.** 1 112 ft. **5** 300 926 sq. cm.
6. 20 8 sec. **7.** 13 29 gm. per c.c. **8.** 0 000178.
9. 99 75 gm approximately. **10.** 0 00015. **11** 7 5 cm.
12. 0 000734. **13** It will rise through 6 3 mm. **14** 110 24 c c
15. 1 litre. **16.** 340 10 c c. **17.** 0 00366 **19** 496 8 kgm.

Chapter XIV. (p. 181.)

4. 77916 calories. **5.** 0 033. **6** 0 055.
7. $14\frac{4}{9}$ gallons of cold water, $5\frac{5}{9}$ gallons of hot water **8** 704° 9 C
10. 96° 3 C. **14.** 20° C. **15.** 0 089.

Chapter XV. (p 202)

2. 14 4 mm. **9.** 54 7 %. **10.** 8 62 kgm.

Chapter XVI. (p. 213)

1. 12 5 lb. **4.** 63° 6 C. **5** Final temperature = 10° C.
6. 41 2 gm.; 5 87 gm. **8.** 522 5; 534 8. **12** 2 h. 41 m.

Chapter XVII. (p. 230.)

7. 126 kgm.

Chapter XVIII. (p. 245.)

5. 7 8 ft. **6.** ($91\ 7 \times 10^4$) miles. **8.** 78 5 sq. ft.
9. 7 8 c.p. **10.** 16 : 9. **11.** 25 : 1.
12. 42 cm. from the candle; 70 cm. on the distant side of the candle.
13. The 100 c.p lamp. **14.** 2 56%. **15.** 75%.

Chapter XIX. (p. 256.)

4. 60°.

Chapter XXI. (p. 287.)

3. 2 ft. from the mirror; relative size = 1/[illegible].
4. 15 cm. behind the mirror; 1·87 cm. long. **6.** Diameter = 0 03 ft.
17. 46·5 cm.; 28 cm.; 26 cm.

Chapter XXII. (p. 301.)

4. – 15 inches. **5** – 13 3 inches. **8.** – 7 5 cm.
9. 7 5 cm. in front of lens; 0 5 cm. long.
10. 22 5 cm. from lens; 52 in. × 38 in. **11.** 1 ft. or 2 ft. from candle.
13. + 16 cm. **14.** + 33 3 cm. **15.** 4 5 in. from lens
17. 10 cm.; 11 1 cm.; + 9 dioptres. **18.** – 16 cm. **19.** 11 ft; 11 in.
23. Convex; 6 15 in; 10 56 ft. square.

Chapter XXIII. (p. 313.)

1. – 2 4 in **2.** Concave; + 5 14 in **3.** 28 8 cm.
4. 120 cm. **8.** – 13 8 in **9** $f = +6$ in.; 6 inches. **11.** 4 times.

Chapter XXV. (p. 335)

2. 15 ft.; 2 5 ft per sec. **5.** 4 times. **8.** 1104 ft. per sec.

Chapter XXVI. (p 341.)

3. The bullet; 0 31 sec. **7.** (ii) 4 : 9.

Chapter XXVII. (p. 351.)

1. 292 6. **2.** 4 lb. **3.** $\frac{1}{4}$ of its length from the end. **4.** 259.
6. 46 8. **9.** 5 **10.** 600. **16.** 269 5 and 243 8

Chapter XXVIII. (p 360.)

1. 360, approximately. **2.** 558. **3.** 307 8. **5.** 560; 6 inches.

Chapter XXXI. (p. 397.)

20. 216 units.
21. (i) 0 03 dyne, parallel to the magnet's axis. (ii) Translatory force, 0 03 dyne, in direction of axis; and a rotational couple, moment $= 0\ 3\sqrt{3}$ units.

Chapter XXXII. (p. 409.)

19. 0 5 unit.

Chapter XXXVII. (p. 490)

9. 0 153 10 0 089 amp 11 3 . 1.
12 0 1005 unit. 13 0 069 absolute unit, 0 69 amp
15 4 775 amp. 16 Greater in the case of the smaller coil, 30°.
17. 496.

Chapter XXXVIII. (p 511)

5 200 ohms 8. 8 . 10 11 9 ohms and 6 ohms
12 820 ohms. 13 0 52 amp. 15 0 615 volt.
16. (i) In parallel 13 34 amp. and 1 34 volt, 1 90 amp and 1 90 volt
(ii) In series. 8 89 amp and 0 889 volt; 4 45 amp. and 4 45 volt.
17. 0 2 ohm, 0 1 ohm 18 $\frac{1}{3}$ ohm. 19. 1 36 1.
20 Reduced from 0 34 amp. to 0 25 amp 21. 1 ohm; 1 : 1 19.
22. (i) 16 amps (ii) 20 amps. (iii) 16 amps
23 9 ohms. 24 0 234 amp., 0 176 amp ; 0 058 amp.
25 $\frac{1}{3}$ volt 26 58 27. $6 8 \times 10^{-8}$ amp.
28. $1 622 \times 10^{-6}$ ohm. 29. $94 07 \times 10^{-6}$ ohm.
30 240. 31. 59.

Chapter XXXIX. (p. 524)

11 0 001 gm per coulomb. 12 0 3354 gm.; 50′ 37″. 13. 122′ 22″
14 0 796 15 0 0095 mm 16 0 505 amp
17. 31 4 hours. 18 0 427 amp.

Chapter XL. (p. 537.)

4 Equal quantities of heat developed
6. Rate at which heat is developed is twice as great
7. Heat developed in lower resistance is twice as great.
8. Each is reduced 75%. 9. One-half the length.
10 $\frac{1}{3}$ amp, $\frac{1}{4}$ amp ; 4 · 3 ; 2 58 volts. 11. 1 : 1 414.
12. 90 gms. 13. 28° 67 C. 14. 0 9165 amp 15. 1 66 amp ; 2 66.

Typical Examination Papers.

MADRAS SECONDARY SCHOOL LEAVING CERTIFICATE (p 567.)

3. 43 5 c c

PUNJAB UNIVERSITY MATRICULATION (p. 569.)

PHYSICS. (1st Paper.)

7. 1 : 0 34?

PHYSICS. (2nd Paper)

2 $3 012 \times 10^6$ ft.-pdals ; $9 41 \times 10^4$ ft. lbs. 3 1 8 lbs.

CALCUTTA UNIVERSITY INTERMEDIATE EXAMINATION (p 572)

PHYSICS. (1st Paper · Group A)

2. 1 134 × 10^6 kilograms **3.** 100 gms **4** 0 13 c c.

(Group B.)

7. 1066° C. **9.** 2 14 gms.

PHYSICS (2nd Paper Group A)

1. 16 cms **2** 13 3 cms , 1 67 cm.

(Group B.)

3. 1 5 volt ; 0 5 ohm

INDEX

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CHEMISTRY.—XXII. Introductory. Simple Chemical Operations—XXIII. Burning and Rusting—XXIV. Nitrogen and Oxygen as Constituents of Air—XXV. Water and Hydrogen—XXVI. Carbon and Some of its Compounds—XXVII. Common Salt. Hydrochloric Acid. Chlorine—XXVIII. Sulphur and Sulphuric Acid—XXIX. Phosphorus. Compounds of Nitrogen; Nitric Acid and Ammonia—XXX. Calcium Carbonate and Sulphate Silica—XXXI. Sodium, Potassium and their Compounds—XXXII. Some Common Metals—INDEX

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